How to Interpret the Alabama High School Mathematics Performance Level Descriptors

These Performance Level Descriptors (PLDs) give an empirical view of student mathematics performance in Alabama, a view related to the kinds of ACT mathematics problems students in Alabama score bands demonstrate they can solve. Alabama determined these score bands in 2019, considering the likelihood of different outcomes and achievements associated with ACT mathematics scores. After choosing score bands, Alabama educators developed these PLDs based on generalizations from hundreds of ACT mathematics questions. The skills are organized according to categories from the *Alabama Course of Study: Mathematics (pdf)*. Skills are assigned to a performance level if at least 2/3 of students at the high end of the score band consistently demonstrate the skill. Note that these descriptors apply to groups of students; there are some students who do not fit these patterns.

This empirical view of mathematics achievement can help students understand what it takes to move to higher performance levels and reap the benefits of increased college and career readiness. Teachers can benefit by ensuring that none of these skills is being neglected. While there is some truth in the idea that a student at a level should be focusing on skills at the next-higher level, teachers should take a holistic view. Sometimes it is more appropriate to teach at the highest level.

Alabama High School Mathematics Performance Level Descriptors

The following descriptors represent the range of student performances on the ACT mathematics test that are associated with each performance level. Descriptors are organized according to categories from the Alabama Course of Study: Mathematics (pdf).

	Level 2 (ACT mathematics score 16–18)	Level 3 (ACT mathematics score 19–24)	Level 4 (ACT mathematics score 25–36)
Number & Quantity The Real Number System Quantities The Complex Number System Vector and Matrix Quantities	Students scoring 16 and higher can likely demonstrate some of the following skills: • Use properties of exponents: multiply and divide monomials with whole number exponents (e.g., $3^7a^3b^6$) to get a monomial with whole number exponents. • Use structure to add and subtract complex numbers in $a + bi$ form where a and b are integers.	Students scoring 19 and higher can likely demonstrate the mathematics skills in Level 2 of **Number & Quantity** and can likely demonstrate some of the following skills: • Multiply the square root of a whole number by itself. • Bound the square root of a whole number between two whole numbers. • Approximate the square root of a whole number, perhaps using a calculator. • Rewrite a variable to a positive fractional power in radical form. • Use properties of exponents: multiply and divide monomials with integer exponents (emphasizing negative exponents) (e.g., $3^{-7}a^{-3}b^6$) and express in this form or in fractional form with whole number exponents. • Apply the power rule for exponents where exponents are small whole numbers or variables. • Use structure to add and subtract 2×2 matrices over the integers. • Use structure to multiply complex numbers in $a + bi$ form where a and b are integers, given $i^2 = -1$.	Students scoring 25 and higher can likely demonstrate the mathematics skills in Level 2 and Level 3 of **Number & Quantity** and can likely demonstrate some of the following skills: • Express radical expressions using rational exponents. • Recognize that an expression like $(2 + x)^{-10}$ can be written as $\frac{1}{(2+x)^{10}}$ but not as $\frac{1}{2^{10}} + \frac{1}{x^{10}}$ • Recognize that an expression like x^{-2} is $\frac{1}{x^2}$ and not $\frac{1}{\sqrt{x}}$ • Simplify cube-root expressions. • Decide whether numerical expressions with square roots are rational or irrational. • Find inverse functions for power functions (e.g., x^n) and radical functions. • Solve for a given variable when exponents and radicals are involved. • Perform matrix subtraction. • Find the determinant and inverse of a 2×2 matrix. • Multiply a vector by a matrix. • Recognize when a matrix product exists by knowing the dimensions of each matrix. • Represent complex numbers algebraically and graphically. • Perform operations on complex numbers. • Recognize quadratic equations/functions that have non-real solutions/zeros. • Identify a quadratic equation over the real numbers that has a given complex number as a solution.
Algebra Seeing Structure in Expressions Arithmetic with Polynomials and Rational Expressions Creating Equations Reasoning with Equations and Inequalities	Students scoring 16 and higher can likely demonstrate some of the following skills: • Find values of variables from a real-world context and substitute into an expression when the variables appear in the same order in the expression and the context. • Use structure to know to substitute the values of given variables. • Evaluate a quadratic expression at a whole number value, where coefficients and constants are whole numbers. • Understand that a solution to an equation is a value of the variable(s) where the left and right sides of the equation are equal, and identify values as solutions through simple computations. • Identify a variable pair that satisfies two linear conditions such as equations or inequalities, (e.g., cost, quantity, profit).	Students scoring 19 and higher can likely demonstrate the mathematics skills in Level 2 of **Algebra** and can likely demonstrate some of the following skills: • Evaluate a quadratic expression at an integer value, where coefficients and constants are integers. • Evaluate an absolute value expression at an integer value, where coefficients and constants are integers. • Identify a variable pair that satisfies three linear conditions such as a system of equations or inequalities. • Determine whether a rational expression is undefined at a particular value of x. • Add and subtract polynomials, including higher order polynomials, where coefficients are integers or decimals to tenths. • Multiply two linear expressions where coefficients are integers. • Apply the distributive property when multiplying a constant and a binomial. • Recognize a common factor of a polynomial in one or two variables. • Given a factor of a quadratic polynomial, find the other factor, where coefficients are integers.	Students scoring 25 and higher can likely demonstrate the mathematics skills in Level 2 and Level 3 of **Number & Quantity** and can likely demonstrate some of the following skills: • Treat parameters as numbers when performing algebraic operations. • Perform operations on polynomial and rational expressions. • Rewrite an expression involving square roots in a form with a rational denominator. • Interpret structure to identify equivalent expressions. • Recognize difference of squares and use the structure to factor. • Solve quadratic equations using a variety of methods, where solutions can be real numbers or complex numbers. • Solve nonroutine problems using basic algebraic tools. • Solve problems that require multiple applications of the distributive property. • Capture geometric reasoning through algebraic relationships. • Represent a real-world context as a system of two equations in two variables. • Interpret structure to identify equations and systems of equations with no solutions. • Find the minimum or maximum value of a linear expression in two variables subject to linear constraints. • Model a quantity with a quadratic equation in order to minimize or maximize the quantity.

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	• Add polynomials, including higher order polynomials, where coefficients are integers or decimals to tenths. (continued)	• Solve linear inequalities in one variable where coefficients and constants are rational numbers and computations are simple. (continued)	• Connect a factorization of a higher-order polynomial with its graph. • (*Should*) Analyze and construct an argument.
Algebra (continued)	• Solve linear equations in one variable with integer coefficients and constants (including one use of the distributive property and including when the solution is a fraction). • Translate verbal expressions into algebraic expressions when the order of quantities and operations within the verbal description is the same as the order within the algebraic expression, and including cases with multiple variables.	• Solve a literal equation for one of the variables, where coefficients and constants are rational numbers and computations are simple. • Solve quadratic equations of the form $ax^2 = b$ to find the positive solution. • Solve a system of two linear equations in two variables where one of the equations is in one variable. • Set up expressions, equations, or inequalities in one variable to represent arithmetic relationships in contexts such as distance, price, time, or average. • Use structure to find a solution to an equation that has the same structure on both sides but a single part of the structure differs: equate the corresponding parts and test values or solve a linear equation (e.g., $100^{m+3} = 100^{10}$) • Find slope given two points on the graph or in a table. • Find slope from a linear equation in variables x and y, where coefficients and constants are rational numbers and computations are simple.	
Functions Interpreting Functions Building Functions Linear, Quadratic, and Exponential Models Trigonometric Functions	Students scoring 16 and higher can likely demonstrate some of the following skills: • Evaluate a quadratic function at a whole number value, where coefficients and constants are whole numbers • Rewrite an equation by adding terms to both sides to achieve a goal (e.g., putting into slope-intercept form). • Given the first few terms in an arithmetic sequence, find one of the next few terms. • Given the first few terms in a geometric sequence, find one of the next few terms when the next term is found by multiplying or dividing by a whole number.	Students scoring 19 and higher can likely demonstrate the mathematics skills in Level 2 of **Functions** and can likely demonstrate some of the following skills: • Evaluate a quadratic function at an integer value, where coefficients and constants are integers. • Evaluate an absolute value function at an integer value, where coefficients and constants are integers. • Evaluate nested (composite) functions when all computations are with integers. • Use function notation. • Interpret key features of a graph/table: find minimum and maximum. • Interpret key features of a graph/table: find range over a closed interval. • Identify key features of a graph/table: find slope given two points. • Identify key features of a graph/table: find change, average rate of change, and vertical shift. • Associate uniform/constant rate of change with a linear graph. • Given two side lengths of a right triangle, relate an angle measure to the ratio of those two side lengths through sine, cosine, or tangent, or through an inverse function.	Students scoring 25 and higher can likely demonstrate the mathematics skills in Level 2 and Level 3 of **Functions** and can likely demonstrate some of the following skills: • Recognize and use the connection between unit rate and slope. • Use relationships between slopes and intercepts to build linear functions with specified properties (e.g., parallel and perpendicular lines). • Find equations of functions that have been translated or reflected across an axis. • Given the graph of $f(x)$, identify the graph of $\lvert f(x)\rvert$. • Identify the graph of a piecewise-defined function given key characteristics. • Determine the nature of the zeros of a quadratic function. • Determine asymptotes and end behavior of rational functions. • Find an expression for a composition of two functions, and use the notation $f \circ g(x)$. • Build inverse functions. • Evaluate logarithmic functions using properties of exponents and logarithms. • Use logarithmic functions to solve exponential equations, and use exponential functions to solve logarithmic equations. • Find the domain of rational functions, radical functions, logarithmic functions, trigonometric functions, and composite functions. • Interpret domain with respect to the context. • Use the shape of the data to choose a type of function for modeling.

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			• Given a recursive expression involving subscripts for the next term in a sequence, find the next term. • Build a function for a sequence and use it to find other terms. • Use trigonometric ratios to find missing sides and angles of right triangles. • Apply the law of sines and the law of cosines. • Find the area of a triangle when given the length of two sides and the measure of the included angle. • Identify key characteristics of trigonometric functions from equations or graphs. • Rewrite trigonometric functions in terms of different trigonometric functions by applying identities.
Geometry Congruence Similarity, Right Triangles, and Trigonometry Circles Expressing Geometric Properties with Equations Geometric Measurement and Dimension Modeling with Geometry	Students scoring 16 and higher can likely demonstrate some of the following skills: • Use geometric formulas to find area or volume when values are given directly for all needed variables. • For a point, figure, or graph, identify coordinates, figure, or graph when translated horizontally or vertically—or both if horizontal is given first.	Students scoring 19 and higher can likely demonstrate the mathematics skills in Level 2 of **Geometry** and can likely demonstrate some of the following skills: • Use geometric formulas to find area or volume when values are NOT given directly for all needed variables. • Find the length, width, height/depth, or volume of a rectangular prism when given the other three quantities. • Identify the reflection of a point across an axis. • Apply standard congruence relationships based on translation, rotation, and reflection, such as for base angles of isosceles triangles, angle relationships formed when parallel lines are crossed by a transversal, and vertical angles. • Find angle measures using the relationship between interior angles in a triangle and the relationship between adjacent angles. • Find the length of a midsegment in a triangle.	Students scoring 25 and higher can likely demonstrate the mathematics skills in Level 2 and Level 3 of **Geometry** and can likely demonstrate some of the following skills: • Add and subtract volumes to find the volume of a composite shape. • Find the area of an equilateral triangle in terms of side length or altitude. • For a figure or graph, identify the reflection across an axis. • For a point, figure, or graph, identify coordinates, figure, or graph when translated vertically, horizontally, or both. • Recognize congruences that appear through the use of auxiliary lines. • Recognize that congruent triangles are similar but similar triangles are not necessarily congruent. • Use the relationship between similar triangles to find side lengths and angle measures. • Recognize and apply similarity in conjunction with other relationships between lengths. • Recognize similarity of triangles determined by two transversals that intersect between parallel lines. • Determine the relationship between surface areas of similar figures and between volumes of similar figures. • Recognize that a triangle must be a right triangle before $a^2 + b^2 = c^2$ applies. • Use the converse of the Pythagorean theorem to know whether angle measures are more, less, or equal to $90°$ • Recognize properties of diagonals in quadrilaterals. • Use the relationship between the area of a sector, the central angle measure, and the radius to find any one given the others. • Use chord-radius relationships in a circle. • Recognize the relationship between a tangent and a circle. • Identify and describe relationships among inscribed and circumscribed angles. • Interpret key features of a conic section from an equation or graph. • Identify the shapes of two-dimensional cross-sections of three-dimensional objects. • Identify three-dimensional objects generated by rotations of two-dimensional objects.
Statistics & Probability Interpreting Categorical and Quantitative Data Making Inferences and Justifying Conclusions	Students scoring 16 and higher can likely demonstrate some of the following skills: • Given the probability of each simple event, where events are defined by a single trait (e.g., color), find the probability of a compound event described with “and,” “or,” and “NOT.”	Students scoring 19 and higher can likely demonstrate the mathematics skills in Level 2 of **Statistics & Probability** and can likely demonstrate some of the following skills: • Find the probability of a compound event formed from two independent events both happening, such as coin toss, roll of die, or single drawing. • Find a geometric probability where the sample space and the event are represented by rectangles.	Students scoring 25 and higher can likely demonstrate the mathematics skills in Level 2 and Level 3 of **Statistics & Probability** and can likely demonstrate some of the following skills: • Find the probability of a compound event with frequencies/counts coming from a two-way table. • Find the probability of a compound event defined by a certain sum of outcome values from independent events. • Find the probability of a compound event using random draws with replacement or without replacement.

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Conditional Probability and Rules of Probability Using Probability to Make Decisions		• Find a conditional probability when it is straightforward to apply the condition to the situation and then find a simple probability within the new situation.	• Differentiate between empirical and theoretical probability. • Find conditional probability. • Calculate permutations and combinations. • Recognize that the greater the standard deviation, the greater the spread from the mean. • Calculate expected value. • Model data with nonlinear functions.
Integrating Essential Skills: Growth in understanding and skills beyond grade 8 expectations	Students scoring 16 and higher can likely demonstrate some of the following skills: • Given two whole number quantities, perhaps in a real-world context, form their ratio in the same order as the quantities were given. • Use proportional reasoning: given a ratio or scale expressed as one whole number to another, and given one of the quantities that is a whole number multiple of its base in the ratio, find the other quantity. • Perform multiple operations to solve simple arithmetic problems in real-world contexts. • Find the least common denominator. • Find the perimeter of polygons when all side lengths are given. • Find length of a rectangle given area and width that divide nicely. • Use the structure of triangle congruence notation to determine which angles and sides correspond. • Find a median for an odd number of data points. • Use the Fundamental Counting Principle: given the number of each of three types of items, find the number of combinations of one item of each type. • Find the probability of a simple event when the sample space is clear. • Interpret a frequency from a Venn diagram in terms of “and” and “only.”	Students scoring 19 and higher can likely demonstrate the mathematics skills in Level 2 of **Integrating Essential Skills** and can likely demonstrate some of the following skills: • Express numbers in scientific notation (e.g., “million” and 1,000,000). • Multiply and divide with scientific notation. • Solve arithmetic problems that could be modeled as $y = mx + b$. • Evaluate a simple square-root equation to determine if a given whole number is a solution. • Find the area of an L-shaped figure given the whole number lengths of two horizontal and two vertical sides. • Add and subtract areas to find the area of a composite shape. • Given an amount per unit length, find the total amount required to surround a rectangle. • Given an amount per unit area, find the total amount for a rectangular area. • Find the area of a right triangle given the whole number lengths of all sides. • Find side lengths for a triangle or quadrilateral with a given perimeter and with relationships between side lengths. • Apply a scale factor for lengths and distances such as with similar triangles or scale drawings. • Given a midpoint and an endpoint, find the other endpoint when coordinates are integers. • Use the Pythagorean theorem to find the length of the hypotenuse given whole number lengths of the legs of a right triangle. • Find a missing data value when given the average/mean and all but one data value. • Divide a total to find average/mean without needing to know each data value. • Identify a bar graph by matching frequency counts.	Students scoring 25 and higher can likely demonstrate the mathematics skills in Level 2 and Level 3 of **Integrating Essential Skills** and can likely demonstrate some of the following skills: • Convert units in rate problems. • Find perimeters, areas, and volumes when given complex relationships between dimensions. • Find circumference when given diameter. • Convert between degrees and radians. • Find distance in the coordinate plane. • Find the midpoint given two points. • Strategically employ the Pythagorean theorem to find lengths when solving problems. • Identify counterexamples. • Find the number of possibilities when cases overlap and it is easy to miss or double-count if the counting method is not organized. • Find median from a frequency table or a stem-and-leaf plot. • Find average/median from a frequency table or chart. • Interpret a Venn diagram.

The ACT College and Career Readiness Standards for Mathematics perform a function similar to the Alabama Performance Level Descriptors. Score bands here do not align with Alabama mathematics performance levels. And skills are included here only when at least 80% of the students in the score band demonstrate the skill, which means skills tend to appear in Alabama performance levels at lower scores than where they appear here.

ACT Mathematics Score Band	ACT College and Career Readiness Standards—Mathematics				
	Topics in the flow to	Topics in the flow to	Topics in the flow to	Topics in the flow to	Topics in the flow to
	Number and Quantity	**Algebra**	**Functions**	**Geometry**	**Statistics and Probability**
13–15	Perform one-operation computation with whole numbers and decimals Recognize equivalent fractions and fractions in lowest terms Locate positive rational numbers (expressed as whole numbers, fractions, decimals, and mixed numbers) on the number line	Solve problems in one or two steps using whole numbers and using decimals in the context of money Exhibit knowledge of basic expressions (e.g., identify an expression for a total as $b + g$) Solve equations in the form $x + a = b$, where a and b are whole numbers or decimals	Extend a given pattern by a few terms for patterns that have a constant increase or decrease between terms	Estimate the length of a line segment based on other lengths in a geometric figure Calculate the length of a line segment based on the lengths of other line segments that go in the same direction (e.g., overlapping line segments and parallel sides of polygons with only right angles) Perform common conversions of money and of length, weight, mass, and time within a measurement system (e.g., dollars to dimes, inches to feet, and hours to minutes)	Calculate the average of a list of positive whole numbers Extract one relevant number from a basic table or chart, and use it in a single computation
16–19	Recognize one-digit factors of a number Identify a digit's place value Locate rational numbers on the number line *Note: A matrix as a representation of data is treated here as a basic table.*	Solve routine one-step arithmetic problems using positive rational numbers, such as single-step percent Solve some routine two-step arithmetic problems Relate a graph to a situation described qualitatively in terms of familiar properties such as before and after, increasing and decreasing, higher and lower Apply a definition of an operation for whole numbers (e.g., $a \square b = 3a - b$) Substitute whole numbers for unknown quantities to evaluate expressions Solve one-step equations to get integer or decimal answers Combine like terms (e.g., $2x + 5x$)	Extend a given pattern by a few terms for patterns that have a constant factor between terms	Exhibit some knowledge of the angles associated with parallel lines Compute the perimeter of polygons when all side lengths are given Compute the area of rectangles when whole number dimensions are given Locate points in the first quadrant	Calculate the average of a list of numbers Calculate the average given the number of data values and the sum of the data values Read basic tables and charts Extract relevant data from a basic table or chart and use the data in a computation Use the relationship between the probability of an event and the probability of its complement
20–23	Exhibit knowledge of elementary number concepts such as rounding, the ordering of decimals, pattern identification, primes, and greatest common factor Write positive powers of 10 by using exponents Comprehend the concept of length on the number line, and find the distance between two points Understand absolute value in terms of distance Find the distance in the coordinate plane between two points with the same x-coordinate or y-coordinate Add two matrices that have whole number entries	Solve routine two-step or three-step arithmetic problems involving concepts such as rate and proportion, tax added, percentage off, and estimating by using a given average value in place of actual values Perform straightforward word-to-symbol translations Relate a graph to a situation described in terms of a starting value and an additional amount per unit (e.g., unit cost, weekly growth) Evaluate algebraic expressions by substituting integers for unknown quantities Add and subtract simple algebraic expressions Solve routine first-degree equations Multiply two binomials Match simple inequalities with their graphs on the number line (e.g., $x \geq -\frac{3}{5}$) Exhibit knowledge of slope	Evaluate linear and quadratic functions, expressed in function notation, at integer values	Use properties of parallel lines to find the measure of an angle Exhibit knowledge of basic angle properties and special sums of angle measures (e.g., 90°, 180°, and 360°) Compute the area and perimeter of triangles and rectangles in simple problems Find the length of the hypotenuse of a right triangle when only very simple computation is involved (e.g., 3-4-5 and 6-8-10 triangles) Use geometric formulas when all necessary information is given Locate points in the coordinate plane Translate points up, down, left, and right in the coordinate plane	Calculate the missing data value given the average and all data values but one Translate from one representation of data to another (e.g., a bar graph to a circle graph) Determine the probability of a simple event Describe events as combinations of other events (e.g., using *and*, *or*, and *not*) Exhibit knowledge of simple counting techniques
24–27	Order fractions Find and use the least common multiple Work with numerical factors Exhibit some knowledge of the complex numbers Add and subtract matrices that have integer entries	Solve multistep arithmetic problems that involve planning or converting common derived units of measure (e.g., feet per second to miles per hour) Build functions and write expressions, equations, or inequalities with a single variable for common pre-algebra settings (e.g., rate and distance problems and problems that can be solved by using proportions) Match linear equations with their graphs in the coordinate plane Recognize that when numerical quantities are reported in real-world contexts, the numbers are often rounded Solve real-world problems by using first-degree equations Solve first-degree inequalities when the method does not involve reversing the inequality sign Match compound inequalities with their graphs on the number line (e.g., $-10.5 < x \leq 20.3$) Add, subtract, and multiply polynomials Identify solutions to simple quadratic equations Solve quadratic equations in the form $(x + a)(x + b) = 0$, where a and b are numbers or variables Factor simple quadratics (e.g., the difference of squares and perfect square trinomials) Work with squares and square roots of numbers Work with cubes and cube roots of numbers Work with scientific notation Work problems involving positive integer exponents Determine when an expression is undefined Determine the slope of a line from an equation	Evaluate polynomial functions, expressed in function notation, at integer values Find the next term in a sequence described recursively Build functions and use quantitative information to identify graphs for relations that are proportional or linear Attend to the difference between a function modeling a situation and the reality of the situation Understand the concept of a function as having a well-defined output value at each valid input value Understand the concept of domain and range in terms of valid input and output, and in terms of function graphs Interpret statements that use function notation in terms of their context Find the domain of polynomial functions and rational functions Find the range of polynomial functions Find where a rational function's graph has a vertical asymptote Use function notation for simple functions of two variables	Use several angle properties to find an unknown angle measure Count the number of lines of symmetry of a geometric figure Use symmetry of isosceles triangles to find unknown side lengths or angle measures Recognize that real-world measurements are typically imprecise and that an appropriate level of precision is related to the measuring device and procedure Compute the perimeter of simple composite geometric figures with unknown side lengths Compute the area of triangles and rectangles when one or more additional simple steps are required Compute the area and circumference of circles after identifying necessary information Given the length of two sides of a right triangle, find the third when the lengths are Pythagorean triples Express the sine, cosine, and tangent of an angle in a right triangle as a ratio of given side lengths Determine the slope of a line from points or a graph Find the midpoint of a line segment Find the coordinates of a point rotated 180° around a given center point	Calculate the average given the frequency counts of all the data values Manipulate data from tables and charts Compute straightforward probabilities for common situations Use Venn diagrams in counting Recognize that when data summaries are reported in the real world, results are often rounded and must be interpreted as having appropriate precision Recognize that when a statistical model is used, model values typically differ from actual values

ACT College and Career Readiness Standards—Mathematics

ACT Mathematics Score Band	Topics in the flow to **Number and Quantity**	Topics in the flow to **Algebra**	Topics in the flow to **Functions**	Topics in the flow to **Geometry**	Topics in the flow to **Statistics and Probability**
	Students who achieve the 28–32 level are likely able to use variables fluently so that they can solve problems with variables in the same way that they can solve the problems with numbers, and they can use variables to represent general properties.				
28–32	Apply number properties involving prime factorization Apply number properties involving even/odd numbers and factors/multiples Apply number properties involving positive/negative numbers Apply the facts that π is irrational and that the square root of an integer is rational only if that integer is a perfect square Apply properties of rational exponents Multiply two complex numbers Use relations involving addition, subtraction, and scalar multiplication of vectors and of matrices	Solve word problems containing several rates, proportions, or percentages Build functions and write expressions, equations, and inequalities for common algebra settings (e.g., distance to a point on a curve and profit for variable cost and demand) Interpret and use information from graphs in the coordinate plane Given an equation or function, find an equation or function whose graph is a translation by a specified amount up or down Manipulate expressions and equations Solve linear inequalities when the method involves reversing the inequality sign Match linear inequalities with their graphs on the number line Solve systems of two linear equations Solve quadratic equations Solve absolute value equations	Relate a graph to a situation described qualitatively in terms of faster change or slower change Build functions for relations that are inversely proportional Find a recursive expression for the general term in a sequence described recursively Evaluate composite functions at integer values	Use relationships involving area, perimeter, and volume of geometric figures to compute another measure (e.g., surface area for a cube of a given volume and simple geometric probability) Use the Pythagorean theorem Apply properties of 30°-60°-90°, 45°-45°-90°, similar, and congruent triangles Apply basic trigonometric ratios to solve right-triangle problems Use the distance formula Use properties of parallel and perpendicular lines to determine an equation of a line or coordinates of a point Find the coordinates of a point reflected across a vertical or horizontal line or across $y = x$ Find the coordinates of a point rotated 90° about the origin Recognize special characteristics of parabolas and circles (e.g., the vertex of a parabola and the center or radius of a circle)	Calculate or use a weighted average Interpret and use information from tables and charts, including two-way frequency tables Apply counting techniques Compute a probability when the event and/or sample space are not given or obvious Recognize the concepts of conditional and joint probability expressed in real-world contexts Recognize the concept of independence expressed in real-world contexts
33–36	Analyze and draw conclusions based on number concepts Apply properties of rational numbers and the rational number system Apply properties of real numbers and the real number system, including properties of irrational numbers Apply properties of complex numbers and the complex number system Multiply matrices Apply properties of matrices and properties of matrices as a number system	Solve complex arithmetic problems involving percent of increase or decrease or requiring integration of several concepts (e.g., using several ratios, comparing percentages, or comparing averages) Build functions and write expressions, equations, and inequalities when the process requires planning and/or strategic manipulation Analyze and draw conclusions based on properties of algebra and/or functions Analyze and draw conclusions based on information from graphs in the coordinate plane Identify characteristics of graphs based on a set of conditions or on a general equation such as $y = ax^2 + c$ Given an equation or function, find an equation or function whose graph is a translation by specified amounts in the horizontal and vertical directions Solve simple absolute value inequalities Match simple quadratic inequalities with their graphs on the number line Apply the remainder theorem for polynomials, that $P(a)$ is the remainder when $P(x)$ is divided by $(x – a)$	Compare actual values and the values of a modeling function to judge model fit and compare models Build functions for relations that are exponential Exhibit knowledge of geometric sequences Exhibit knowledge of unit circle trigonometry Match graphs of basic trigonometric functions with their equations Use trigonometric concepts and basic identities to solve problems Exhibit knowledge of logarithms Write an expression for the composite of two simple functions	Use relationships among angles, arcs, and distances in a circle Compute the area of composite geometric figures when planning and/or visualization is required Use scale factors to determine the magnitude of a size change Analyze and draw conclusions based on a set of conditions Solve multistep geometry problems that involve integrating concepts, planning, and/or visualization	Distinguish between mean, median, and mode for a list of numbers Analyze and draw conclusions based on information from tables and charts, including two-way frequency tables Understand the role of randomization in surveys, experiments, and observational studies Exhibit knowledge of conditional and joint probability Recognize that part of the power of statistical modeling comes from looking at regularity in the differences between actual values and model values