2019 Alabama Alternate Achievement Standards

MATHEMATICS

For information regarding the

Alabama Alternate Achievement Standards contact

Special Education Services

Alabama State Department of Education

3345 Gordon Persons Building,

50 North Ripley Street, Montgomery, Alabama 36104; or by mail to P.O. Box 302101, Montgomery, Alabama36130-2101; or by telephone at (334) 694-4782; or by email at:

mailto:Speced@alsde.edu

Alabama State Department of Education
Eric G. Mackey, State Superintendent of Education

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Ms. Taylor Paige Biggs, MS.	**Ms. Jessica Johnson**, B.S.	**Mrs. Kimberly Baker**, M.S. Special Education Services
Mrs. Emily Bivens, M.Ed.	**Ms. Hailey King**, B.S.	**Mr. Tod Beers**, M.Ed., AMSTI
Mrs. Teresa Burden, Ed.S.	**Ms. Tammy King**, M.Ed.	**Ms. Karma Clarke**, Ed.S., AMSTI
Mrs. Becky Cornelius, Ed.S.	**Ms. Julia Kochan**, M.Ed.	**Ms. Diane Duncan**, Ed.S., ALSDE
Ms. Tiffany Countryman, Ed.S.	**Mrs. Kristy Mann,** B.S.	**Mrs. Susan Goldthwaite**, M.Ed., Special Education Services
Ms. Kim Daily, M.Ed.	**Ms. Meagan McDonald**, B.S.	**Mrs. Elizabeth Greene**, M.Ed., Special Education Services
Mrs. Dana Davis, M.Ed.	**Mrs. Shelly Munger**, M.Ed.	**Ms. Elizabeth Hammonds**, M.Ed., AMSTI
Mr. Derallus Davis, M.Ed.	**Miss Michele Murray**, M.Ed.	**Mrs. Pamela Ivey**, M.Ed., Special Education Services
Ms. Anna Laura Dyer, Ed.S.	**Mrs. Chelsea Newels**, M.Ed.	**Mrs. Elisabeth Newell,** Ed.S. Special Education Services
Mrs. Erica Dunwoody, M.Ed.	**Mrs. Brandy Quattlebaum**, M.Ed.	**Mrs. Phenicia Nunn**, M.Ed., AMSTI
Ms. Ali Grace Eiland, M.Ed.	**Mrs. Anna Laura Reid**, Ed.S.	**Ms. Nannette Pence**, M.Ed., Assessment
Ms. Michelle Gibbons, B.S.	**Mr. John Rice**, B.S.	**Ms. Gwendolyn Jordan Preston**, M.Ed., Special Education Services
Ms. Briana Gibson, M.Ed.	**Mr. Terry Chad Sorrells**, Ed.S.	**Mrs. Tina Sanders**, Ed.S., Special Education Services
Mrs. Lori Ann Gibson, Ed.S.	**Dr. Marilyn Strutchens**, Ph.D.	**Mrs. Teri Shriver**, Ed.S., Special Education Services
Mrs. Ann-Marie Grav, M.Ed.	**Ms. Lateatrice Thomas**, Ed.S.	**Ms. Kristie Taylor,** M.Ed., AMSTI
Ms. Adriane Howard, B.S.	**Mrs. Lori White**, M.A.	**Mr. Joel White**, Ed.S., AMSTI
Ms. Amy Johnson, M.Ed.	**Mrs. Tamika Whitt-Wright,** Ed.S. J.D.	**Mrs. Kathy Wilkins**, M.Ed., Special Education Services

Overview

Introduction

The Mathematics Alternate Achievement Standards are directly aligned to the Alabama Mathematics Standards. The Mathematics Alternate Achievement Standards in this document were developed by general and special education teachers in Alabama to guide and direct instruction for students with the most significant intellectual disabilities.

Students with Significant Intellectual Disabilities

In the United States, approximately 1% of school-aged children have an intellectual disability that is "characterized by significant limitations both in intellectual functioning and adaptive behavior as expressed in conceptual, social, and practical adaptive skills." (U.S. Department of Education, 2002 and American Association of Intellectual and Developmental Disabilities, 2009) These students show evidence of cognitive functioning in the range of severe to profound and need extensive or pervasive support. In addition to significant intellectual disabilities, students may also have co-occurring communication, motor, sensory, or other impairments.

Students with the most significant intellectual disabilities first gained mandated access to the general curriculum through the Individuals with Disabilities Education Act Amendments of 1997 (PL 105-17), with further access guaranteed following the passage of the No Child Left Behind Act (NCLB) of 2001 (PL 107-110). Today, Every Student Succeeds Act (ESSA) says a state may, through a documented and validated standards-setting process, adopt alternate academic achievement standards for students with the most significant cognitive disabilities, provided those standards:

- are aligned with the challenging State academic content standards.

- promote access to the general education curriculum, consistent with the Individuals with Disabilities Education Act (IDEA).

- reflect professional judgment as to the highest possible standards achievable by such students; are designated in the IEP developed under section 614(d)(3) of IDEA for each such student as the academic achievement standards that will be used for the student.

- and are aligned to ensure that a student who meets the alternate academic achievement standards is on track to pursue postsecondary education or employment, consistent with the purposes of the Workforce Innovation and Opportunity Act to maximize opportunities for individuals with significant disabilities for competitive integrated employment. (ESSA, Section 1111 (b)(1)(E))

Format of the Alternate Achievement Standards

The format of the Alabama Alternate Achievement Standards includes the grade, general education standard, alternate achievement standard, and the topic area.

Content Areas are large groups of related clusters and content standards. In this example, the Alabama Content Area is "Operations and Algebraic Thinking." Content areas are used to group standards that are closely related.

Content Standards, listed to the right of each cluster, contain the minimum required content, and define what students should know and be able to do at the conclusion of a course or grade. Some have sub-standards, indicated with a, b, c, d, which are extensions of the content standards and are also required. Some standards are followed by examples, which are not required to be taught. When standards indicate that drawings may be used, the drawings need not show details but should show the mathematics in the problem. The order in which standards are listed within a course or grade is not intended to convey a sequence for instruction. Each content standard completes the stem "Students will..."

Proportional Reasoning

Cluster	2019 Math COS Standard	2019 AAS Standard
Develop an understanding of ratio concepts and use reasoning about ratios to solve problems.	1. Use appropriate notations [a/b, a to b, a:b] to represent a proportional relationship between quantities and use ratio language to describe the relationship between quantities.	M.AAS.6.1 Demonstrate a simple ratio relationship using ratio notation given a real-world problem.
	2. Use unit rates to represent and describe ratio relationships.	
	3. Use ratio and rate reasoning to solve mathematical and real-world problems (including but not limited to percent, measurement conversion, and equivalent ratios) using a variety of models, including tables of equivalent ratios, tape diagrams, double number lines, and equations.	

Content clusters group related content standards. The cluster in the example is "Extend the counting sequence." Because mathematics is a connected subject, standards from different clusters may sometimes be closely related.

Alternate Achievement Standard is a lesser complexity, standard that is selected based off the priority standards and aligned to the general education curriculum.

Augmentative/Alternate Devices

The Alabama Alternate Achievement Standards are to be taught using the student's communication modality (e.g., voice, sign language, augmentative/alternative communication device). This does not mean an augmentative/alternative device should be programmed to do the cognition for the student.

Accommodations

Please be familiar with accommodations. Accommodations are available for students with disabilities to level the playing field and lessen the impact of their disability in the teaching/learning and testing environments. It is important that the accommodations in the student's IEP are being appropriately determined, documented, and implemented. An example of an appropriate accommodation for a student with a visual impairment who is working toward the Alabama Alternate Achievement Standards would be using a tactile representation in place of an illustration or picture. The augmentative/alternative communication devices discussed above are an appropriate accommodation for students with communication difficulties when a verbal response is desired or needed.

Kindergarten

Foundations of Counting

Cluster	2019 Math COS Standard	2019 AAS Standard
Know number names and the count sequence.	1. Count forward orally from 0 to 100 by ones and by tens. Count backward orally from 10 to 0 by ones. 2. Count to 100 by ones beginning with any given number between 0 and 99. 3. Write numerals from 0 to 20. a. Represent 0 to 20 using concrete objects when given a written numeral from 0 to 20 (with 0 representing a count of no objects).	**M.AAS.K.1** Using vocalization, sign language, augmentative communication, or assistive technology, count to 15 by ones starting with one.
Count to tell the number of objects.	4. Connect counting to cardinality using a variety of concrete objects. a. Say the number names in consecutive order when counting objects. b. Indicate that the last number name said tells the number of objects counted in a set. c. Indicate that the number of objects in a set is the same regardless of their arrangement or the order in which they were counted. d. Explain that each successive number name refers to a quantity that is one larger.	**M.AAS.K.4** Demonstrate one-to-one correspondence, pairing each object with one, and only one, number and each number with one, and only one, object (limit numbers and objects to five).
Count to tell the number of objects.	5. Count to answer "how many?" questions. a. Count using no more than 20 concrete objects arranged in a line, a rectangular array, or a circle. b. Count using no more than 10 concrete objects in a scattered configuration. c. Draw the number of objects that matches a given numeral from 0 to 20.	**M.AAS.K.5** Using vocalization, sign language, augmentative communication, or assistive technology, count out up to five objects from a larger set, pairing each object with one, and only one, number name to tell how many.
Compare numbers.	6. Orally identify whether the number of objects in one group is greater/more than, less/fewer than, or equal/the same as the number of objects in another group, in groups containing up to 10 objects, by using matching, counting, or other strategies. 7. Compare two numbers between 0 and 10 presented as written numerals (without using inequality symbols).	**M.AAS.K.6** Identify whether the number of objects in one group is more or less than (e.g., when the quantities are clearly different) or equal to the number of objects in another group.

Operations and Algebraic Thinking

Cluster	2019 Math COS Standard	2019 AAS Standard
Understand addition as putting together and adding to and understand subtraction as taking apart and taking from.	8. Represent addition and subtraction up to 10 with concrete objects, fingers, pennies, mental images, drawings, claps or other sounds, acting out situations, verbal explanations, expressions, or equations.	**M.AAS.K.8** Demonstrate an understanding of addition as "putting together" or subtraction as "taking from" in everyday activities, limited to 5 objects.
	9. Solve addition and subtraction word problems, and add and subtract within 10, by using concrete objects or drawings to represent the problem.	
	10. Decompose numbers less than or equal to 10 into pairs of smaller numbers in more than one way, by using concrete objects or drawings, and record each decomposition by a drawing or equation. Example: 5 = 2 + 3 and 5 = 4 + 1	
	11. For any number from 0 to 10, find the number that makes 10 when added to the given number, by using concrete objects or drawings, and record the answer with a drawing or equation.	
	12. Fluently add and subtract within 5.	
Understand simple patterns	13. Duplicate and extend simple patterns using concrete objects.	**M.AAS.K.13** Using vocalization, sign language, augmentative communication, or assistive technology, duplicate and extend simple patterns using concrete objects.

Operations with Numbers

Cluster	2019 Math COS Standard	2019 AAS Standard
Work with numbers 11- 19 to gain foundations for place value.	14. Compose and decompose numbers from 11 to 19 by using concrete objects or drawings to demonstrate understanding that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.	**M.AAS.K.14** Compose numbers from 11-15 by using concrete objects or drawings to demonstrate understanding that these numbers are composed of ten ones and one, two, three, four, or five ones.

Data Analysis

Cluster	2019 Math COS Standard	2019 AAS Standard

Collect and analyze data and interpret results. representations.	15. Classify objects into given categories of 10 or fewer; count the number of objects in each category and sort the categories by count. a. Categorize data on Venn diagrams, pictographs, and "yes-no" charts using real objects, symbolic representations, or pictorial representations.	**M.AAS.K.15** Explore a simple pictograph (limited to two categories and limit a combined quantity of 5 for both categories).

Measurement

Cluster	2019 Math COS Standard	2019 AAS Standard
Describe and compare measurable attributes.	16. Identify and describe measurable attributes (length, weight, height) of a single object using vocabulary such as long/short, heavy/light, or tall/short.	**M.AAS.K.16** Classify objects according to attributes (e.g., big/small, heavy/light, tall/short).
	17. Directly compare two objects with a measurable attribute in common to see which object has "more of" or "less of" the attribute and describe the difference. Example: Directly compare the heights of two children and describe one child as "taller" or "shorter."	

Geometry

Cluster	2019 Math COS Standard	2019 AAS Standard
Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).	18. Describe objects in the environment using names of shapes and describe the relative positions of these objects using terms such as above, below, besides, in front of, behind, and next to.	**M.AAS.K.18** Recognize and match shapes of the same size and orientation and describe the relative positions using in front of and behind (limited to circle, square, rectangle, and triangle).
	19. Correctly name shapes regardless of their orientations or overall sizes.	
	20. Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid").	
Analyze, compare, create, and compose shapes.	21. Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (number of sides and vertices or "corners"), and other attributes. Example: having sides of equal length	**M.AAS.K.21** Match a shape to common objects in the same or different sizes and orientations (real or picture; limited to circle, square, rectangle, and triangle).
	22. Model shapes in the world by building them from sticks, clay balls, or other components and by drawing them.	

Analyze, compare, create, and compose shapes.	23. Use simple shapes to compose larger shapes. Example: Join two triangles with full sides touching to make a rectangle.	**M.AAS.K.23** Using a model of a larger shape outline, use simple shapes to compose larger shapes. Example: Join two triangles with full sides touching to make a rectangle.

Grade 1

Operations and Algebraic Thinking

Cluster	2019 Math COS Standard	2019 AAS Standard
Represent and solve problems involving addition and subtraction.	1. Use addition and subtraction to solve word problems within 20 by using concrete objects, drawings, and equations with a symbol for the unknown number to represent the problem. a. Add to with change unknown to solve word problems within 20. b. Take from with change unknown to solve word problems within 20. c. Put together/take apart with addend unknown to solve word problems within 20. d. Compare quantities, with difference unknown, bigger unknown, and smaller unknown while solving word problems within 20.	**M.AAS.1.1** Represent addition as "add to/put together" and subtraction as "take from/take apart" with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, or verbal explanations (limited to 10).
	2. Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20 by using concrete objects, drawings, or equations with a symbol for the unknown number to represent the problem.	
Understand and apply properties of operations and the relationship between addition and subtraction.	3. Apply properties of operations as strategies to add and subtract. Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known (commutative property of addition). To add 2 + 6 + 4, the second and third numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12 (associative property of addition). When adding 0 to a number, the result is the same number (identity property of zero for addition).	**M.AAS.1.3** Demonstrate “putting together” two sets of objects to solve the problem.
	4. Explain subtraction as an unknown-addend problem. Example: subtracting 10 - 8 by finding the number that makes 10 when added to 8	

Add and subtract within 20.	5. Relate counting to addition and subtraction. Example: counting on 2 to add 2.	**<u>M.AAS.1.5</u>** Use manipulatives or visual representations to indicate the number that results when adding one more. Apply knowledge of "one less" to subtract one from a number.
Add and subtract within 20.	6. Add and subtract within 20. a. Demonstrate fluency with addition and subtraction facts with sums or differences to 10 by counting on. b. Demonstrate fluency with addition and subtraction facts with sums or differences to 10 by making ten. c. Demonstrate fluency with addition and subtraction facts with sums or differences to 10 by decomposing a number leading to a ten. Example: 13 - 4 = 13 - 3 - 1 = 10 - 1 = 9 d. Demonstrate fluency with addition and subtraction facts with sums or differences to 10 by using the relationship between addition and subtraction. Example: Knowing that 8 + 4 = 12, one knows 12 - 8 = 4. e. Demonstrate fluency with addition and subtraction facts with sums or differences to 10 by creating equivalent but easier or known sums. Example: adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13	**<u>M.AAS.1.6</u>** Add and subtract numbers 1 to 15 using objects, pictures, and fingers.
Work with addition and subtraction equations.	7. Explain that the equal sign means "the same as." Determine whether equations involving addition and subtraction are true or false. Example: determining which of the following equations are true and which are false: 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2	**<u>M.AAS.1.7</u>** Given three related whole numbers, construct a number sentence that is true, in relation to addition and subtraction.
	8. Solve for the unknown whole number in various positions in an addition or subtraction equation, relating three whole numbers that would make it true. Example: determining the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = ? – 3, and 6 + 6 = ?	
Understand simple patterns	9. Reproduce, extend, and create patterns and sequences of numbers using a variety of materials.	**<u>M.AAS.1.9</u>** Using vocalization, sign language, augmentative communication, or assistive technology, duplicate, extend, and create simple patterns using concrete objects.

Operations with Numbers: Base Ten

Cluster	2019 Math COS Standard	2019 AAS Standard
Extend the counting sequence.	10. Extend the number sequence from 0 to 120. a. Count forward and backward by ones, starting at any number less than 120. b. Read numerals from 0 to 120. c. Write numerals from 0 to 120. d. Represent a number of objects from 0 to 120 with a written numeral.	**M.AAS.1.10** Count forward to 30 by ones, starting with any number less than 30. Recognize numerals 0 through 15 as written. When given a numeral 0 to 15, represent the numeral with objects.
Understand place value.	11. Explain that the two digits of a two-digit number represent amounts of tens and ones. a. Identify a bundle of ten ones as a "ten." b. Identify the numbers from 11 to 19 as composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones. c. Identify the numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 as one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).	**M.AAS.1.11** Recognize and create sets of ten (limit to three sets).
	12. Compare pairs of two-digit numbers based on the values of the tens and ones digits, recording the results of comparisons with the symbols >, =, and < and orally with the words "is greater than," "is equal to," and "is less than."	**M.AAS.1.12** Using vocalization, sign language, augmentative communication, or assistive technology, compare two groups of 10 or fewer items using appropriate vocabulary (e.g., more, less, equal) when the number of items in each group is similar.
Use place value understanding and properties of operations to add and subtract.	13. Add within 100, using concrete models or drawings and strategies based on place value. a. Add a two-digit number and a one-digit number. b. Add a two-digit number and a multiple of 10. c. Demonstrate that in adding two-digit numbers, tens are added to tens, ones are added to ones, and sometimes it is necessary to compose a ten. d. Relate the strategy for adding a two-digit number and a one-digit number to a written method and explain the reasoning used.	**M.AAS.1.13** Compose and decompose numbers from 1 to 15 into one ten and ones using objects, drawings, or pictures.

	14. Given a two-digit number, mentally find 10 more or 10 less than the number without having to count, and explain the reasoning used.	
	15. Subtract multiples of 10 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. Relate the strategy to a written method and explain the reasoning used.	

Data Analysis

Cluster	2019 Math COS Standard	2019 AAS Standard
Collect and analyze data and interpret results.	16. Organize, represent, and interpret data with up to three categories. a. Ask and answer questions about the total number of data points in organized data. b. Summarize data on Venn diagrams, pictographs, and "yes-no" charts using real objects, symbolic representations, or pictorial representations. c. Determine "how many" in each category using up to three categories of data. d. Determine "how many more" or "how many less" are in one category than in another using data organized into two or three categories.	**M.AAS.1.16** Sort objects or pictures into common categories (e.g., shapes, pets, fruits; limited to two categories and a combined total of 15 objects/pictures for the categories).

Measurement

Cluster	2019 Math COS Standard	2019 AAS Standard
Describe and compare measurable attributes.	17. Order three objects by length; compare the lengths of two objects indirectly by using a third object. 18. Determine the length of an object using nonstandard units with no gaps or overlaps, expressing the length of the object with a whole number.	**M.AAS.1.17** Compare and determine lengths of objects using non-standard units of measurements (real or pictures) in terms of longer/shorter and taller/shorter.
Work with time and money.	19. Tell and write time to the hours and half hours using analog and digital clocks.	**M.AAS.1.19** Demonstrate an understanding of the concept of time using words such as yesterday, today, tomorrow, morning, afternoon, day, and night; identify activities that come before, next, and after on a daily schedule using a clock limited to time in hours.

	20. Identify pennies and dimes by name and value.	**M.AAS.1.20** Using vocalization, sign language, augmentative communication, or assistive technology, identify U.S. coins by name (e.g., penny & dime).

Geometry

Cluster	2019 Math COS Standard	2019 AAS Standard
Reason with shapes and their attributes.	21. Build and draw shapes which have defining attributes. a. Distinguish between defining attributes and non-defining attributes. Examples: Triangles are closed and three-sided, which are defining attributes; color, orientation, and overall size are nondefining attributes.	**M.AAS.1.21** Determine similarities and differences among shapes of the same size or different sizes and orientations (limited to circle, square, rectangle, and triangle).
	22. Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape and compose new shapes from the composite shape.	**M.AAS.1.22** Sort shapes of the same size and orientation (limited to circle, square, rectangle, and triangle).
	23. Partition circles and rectangles into two and four equal shares and describe the shares using the words halves, fourths, and quarters, and use the phrases half of, fourth of, and quarter of. a. Describe "the whole" as two of or four of the shares of circles and rectangles partitioned into two or four equal shares. b. Explain that decomposing into more equal shares creates smaller shares of circles and rectangles.	**M.AAS.1.23** Put together two equal size pieces to make a shape that relates to a whole (e.g., two semicircles to make a circle, two squares to make a rectangle).

Grade 2

Operations and Algebraic Thinking

Cluster	2019 Math COS Standard	2019 AAS Standard

Represent and solve problems involving addition and subtraction.	1. Use addition and subtraction within 100 to solve one- and two-step word problems by using drawings and equations with a symbol for the unknown number to represent the problem.	**<u>M.AAS.2.2</u>** Represent addition as "add to/put together" and subtraction as "take from/take apart" with objects, drawings, fingers, or sounds (within 30).
Add and subtract within 20.	2. Fluently add and subtract within 20 using mental strategies such as counting on, making ten, decomposing a number leading to ten, using the relationship between addition and subtraction, and creating equivalent but easier or known sums. a. State automatically all sums of two one-digit numbers.	
Work with equal groups of objects to gain foundations for multiplication.	3. Use concrete objects to determine whether a group of up to 20 objects is even or odd. a. Write an equation to express an even number as a sum of two equal addends.	**<u>M.AAS.2.3</u>** Equally distribute even numbers of up to 20 objects between two groups.
	4. Using concrete and pictorial representations and repeated addition, determine the total number of objects in a rectangular array with up to 5 rows and up to 5 columns. a. Write an equation to express the total number of objects in a rectangular array with up to 5 rows and up to 5 columns as a sum of equal addends.	**<u>M.AAS.2.4</u>** Use repeated addition to find the sum of objects arranged in equal groups up to 10.
Understand simple patterns.	5. Reproduce, extend, create, and describe patterns and sequences using a variety of materials.	**<u>M.AAS.2.5</u>** Using vocalization, sign language, augmentative communication, or assistive technology, duplicate, extend, create, and describe simple patterns using concrete objects.

Operations with Numbers: Base Ten

Cluster	2019 Math COS Standard	2019 AAS Standard
Understand place value.	6. Explain that the three digits of a three-digit number represent amounts of hundreds, tens, and ones. a. Explain the following three-digit numbers as special cases: 100 can be thought of as a bundle of ten tens, called a “hundred,” and the numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five,	**<u>M.AAS.2.6</u>** Recognize and represent numbers up to 30 with sets of tens and ones (objects, columns, arrays).
	six, seven, eight, or nine hundreds (and 0 tens and 0 ones).	

	7. Count within 1000 by ones, fives, tens, and hundreds.	**<u>M.AAS.2.7</u>** Using vocalization, sign language, augmentative communication, or assistive technology, count and recognize numerals 0 to 50 by ones. When given a numeral 0 to 25, name the next two numbers in a three-item sequence.
	8. Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.	**<u>M.AAS.2.9</u>** Using vocalization, sign language, augmentative communication, or assistive technology, compare sets of objects and numbers using appropriate vocabulary (greater than, less than, equal to; limited to thirty objects in a group).
	9. Compare two three-digit numbers based on the value of the hundreds, tens, and ones digits, recording the results of comparisons with the symbols >, =, and < and orally with the words “is greater than,” “is equal to,” and “is less than.”	
Use place value understanding and properties of operations to add and subtract.	10. Fluently add and subtract within 100, using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.	**<u>M.AAS.2.10</u>** Using vocalization, sign language, augmentative communication, or assistive technology, identify the meaning of the + sign (add, plus, put together) and the - sign (subtract, take away, take from) and the = sign (equal, the same as); compose and decompose numbers up to 20 using objects, pictures, drawings, or numbers.
	11. Use a variety of strategies to add up to four twodigit numbers.	
	12. Add and subtract within 1000 using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. a. Explain that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.	
	13. Mentally add and subtract 10 or 100 to a given number between 100 and 900.	
	14. Explain why addition and subtraction strategies work, using place value and the properties of operations. Note: Explanations may be supported by drawings or objects.	

Data Analysis

Cluster	2019 Math COS Standard	2019 AAS Standard

Collect and analyze data and interpret results.	15. Measure lengths of several objects to the nearest whole unit. a. Create a line plot where the horizontal scale is marked off in whole-number units to show the lengths of several measured objects.	**M.AAS.2.16** Using vocalization, sign language, augmentative communication, or assistive technology, use a graph, limited to 2 categories, to answer more/less, most/least, or equal to questions (a combined total of no more than 30 objects/pictures shown for the 2 categories).
	16. Create a picture graph and bar graph to represent data with up to four categories. a. Using information presented in a bar graph, solve simple “put-together,", "take-apart,” and “compare” problems. b. Using Venn diagrams, pictographs, and "yesno" charts, analyze data to predict an outcome.	

Measurement

Cluster	2019 Math COS Standard	2019 AAS Standard
Measure and estimate lengths in standard units.	17. Measure the length of an object by selecting and using standard units of measurement shown on rulers, yardsticks, meter sticks, or measuring tapes.	**M.AAS.2.17** Using vocalization, sign language, augmentative communication, or assistive technology, identify standard tools associated with measurement (clock, ruler, scale, measuring cup); measure the lengths of objects using nonstandard units (e.g., hands, paper clips).
	18. Measure objects with two different units and describe how the two measurements relate to each other and the size of the unit chosen.	
	19. Estimate lengths using the following standard units of measurement: inches, feet, centimeters, and meters.	**M.AAS.2.19** Order three objects by length (long/longer/longest; short/shorter/shortest).
	20. Measure to determine how much longer one object is than another, expressing the length difference of the two objects using standard units of length.	
Relate addition and subtraction to length.	21. Use addition and subtraction within 100 to solve word problems involving same units of length, representing the problem with drawings (such as drawings of rulers) and/or equations with a symbol for the unknown number.	**M.AAS.2.21** Increase or decrease length by adding or subtracting nonstandard unit(s).
	22. Create a number line diagram using whole numbers and use it to represent whole-number sums and differences within 100.	**M.AAS.2.22** Represent whole-number sums within 20 using a number line.
Work with time and money.	23. Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m. a. Express an understanding of common terms such as, but not limited to, quarter past, half past, and quarter to.	**M.AAS.2.23** Using vocalization, sign language, augmentative communication, or assistive technology, identify the time that matches a routine activity using a clock (limited to hour).

	24. Solve problems with money. a. Identify nickels and quarters by name and value. b. Find the value of a collection of quarters, dimes, nickels, and pennies. c. Solve word problems by adding and subtracting within one dollar, using the $ and ¢ symbols appropriately (not including decimal notation). Example: 24¢ + 26¢ = 50¢	**M.AAS.2.24** Using vocalization, sign language, augmentative communication, or assistive technology, identify and demonstrate knowledge that money has value; limited to penny = 1 cent, nickel = 5 cents, dime = 10 cents.

Geometry

Cluster	2019 Math COS Standard	2019 AAS Standard
Reason with shapes and their attributes.	25. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes. a. Recognize and draw shapes having specified attributes. Examples: a given number of angles or a given number of equal faces	**M.AAS.2.25** Using vocalization, sign language, augmentative communication, or assistive technology, identify twodimensional shapes (limited to square, circle, triangle, and rectangle).
	26. Partition a rectangle into rows and columns of same size squares, and count to find the total number of squares.	**M.AAS.2.27** Using vocalization, sign language, augmentative communication, or assistive technology, identify half as being two equal parts of a shape (limited to circle, square, rectangle, and triangle).
	27. Partition circles and rectangles into two, three, or four equal shares. Describe the shares using such terms as halves, thirds, half of, or a third of, and describe the whole as two halves, three thirds, or four fourths. b. Explain that equal shares of identical wholes need not have the same shape.	

Grade 3

Operations and Algebraic Thinking

Cluster	2019 Math COS Standard	2019 AAS Standard
Represent and solve problems involving multiplication and division.	1. Illustrate the product of two whole numbers as equal groups by identifying the number of groups and the number in each group and represent as a written expression.	**M.AAS.3.1** Using vocalization, sign language, augmentative communication, or assistive technology, model finding the sum of equal groups using repeated addition (sums within 30).
	2. Illustrate and interpret the quotient of two whole numbers as the number of objects in each group or the number of groups when the whole is partitioned into equal shares.	

	3. Solve word situations using multiplication and division within 100 involving equal groups, arrays, and measurement quantities; represent the situation using models, drawings, and equations with a symbol for the unknown number.	**M.AAS.3.3** Use strategies (arrays, equal groups, manipulatives, etc.) to model multiplication and division equations to find an unknown number.
	4. Determine the unknown whole number in a multiplication or division equation relating three whole numbers.	
Understand properties of multiplication and the relationship between multiplication and division.	5. Develop and apply properties of operations as strategies to multiply and divide.	
	6. Use the relationship between multiplication and division to represent division as an equation with an unknown factor.	
Multiply and divide within 100.	7. Use strategies based on properties and patterns of multiplication to demonstrate fluency with multiplication and division within 100. a. Fluently determine all products obtained by multiplying two one-digit numbers. b. State automatically all products of two onedigit numbers by the end of third grade.	**M.AAS.3.7** Demonstrate fluency of multiplication using skip counting, multiples of numbers, number charts, arrays, etc.
Solve problems involving the four operations and identify and explain patterns in arithmetic.	8. Determine and justify solutions for two-step word problems using the four operations and write an equation with a letter standing for the unknown quantity. Determine reasonableness of answers using number sense, context, mental computation, and estimation strategies including rounding.	
	9. Recognize and explain arithmetic patterns using properties of operations.	

Operations with Numbers: Base Ten

Cluster	2019 Math COS Standard	2019 AAS Standard
Use place value understanding and properties of operations to perform multidigit arithmetic.	10. Identify the nearest 10 or 100 when rounding whole numbers, using place value understanding.	**M.AAS.3.10** Using vocalization, sign language, augmentative communication, or assistive technology use concrete materials and pictorial models to model whole numbers.
	11. Use various strategies to add and subtract fluently within 1000.	
	12. Use concrete materials and pictorial models based on place value and properties of operations to find the product of a one-digit whole number by a multiple of ten (from 10 to 90).	

	13. Demonstrate that a unit fraction represents one part of an area model or length model of a whole that has been equally partitioned; explain that a numerator greater than one indicates the number of unit pieces represented by the fraction.	**M.AAS.3.15** Compare fractions.
Develop understanding of fractions as numbers.	14. Interpret a fraction as a number on the number line; locate or represent fractions on a number line diagram. a. Represent a unit fraction (1/b) on a number line by defining the interval from 0 to 1 as the whole and partitioning it into equal parts as specified by the denominator. b. Represent a fraction (a/b) on a number line by marking off a length of size (1/b) from zero.	**M.AAS.3.15a** Use models to identify two equivalent fractions (limit to fourths and halves).
	15. Explain equivalence and compare fractions by reasoning about their size using visual fraction models and number lines. a. Express whole numbers as fractions and recognize fractions that are equivalent to whole numbers. b. Compare two fractions with the same numerator or with the same denominator by reasoning about their size (recognizing that fractions must refer to the same whole for the comparison to be valid). Record comparisons using <, >, or = and justify conclusions.	**M.AAS.3.15b** Recognize two equivalent fractions (limit to fourths and halves). **M.AAS.3.15c** Use models of fourths and halves to make a whole.

Data Analysis

Cluster	2019 Math COS Standard	2019 AAS Standard
Represent and interpret data.	16. For a given or collected set of data, create a scaled (one-to-many) picture graph and scaled bar graph to represent a data set with several categories. a. Determine a simple probability from a context that includes a picture. b. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled graphs.	**M.AAS.3.16** Measure lengths of objects using non-standard tools (paper clips). Limit to whole numbers.
	17. Measure lengths using rulers marked with halves and fourths of an inch to generate data and create a line plot marked off in appropriate units to display the data.	**M.AAS.3.17** Using vocalization, sign language, augmentative communication, or assistive technology, represent and interpret data on a picture or bar graph when given a model or a graph to complete.

Measurement

Cluster	2019 Math COS Standard	2019 AAS Standard
Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.	18. Tell and write time to the nearest minute; measure time intervals in minutes (within 90 minutes.) a. Solve real-world problems involving addition and subtraction of time intervals in minutes by representing the problem on a number line diagram.	**M.AAS.3.18** Using vocalization, sign language, augmentative communication, or assistive technology, tell time on a digital clock (to the hour, half-hour, quarter hour).
	19. Estimate and measure liquid volumes and masses of objects using liters (l), grams (g), and kilograms (kg). a. Use the four operations to solve one-step word problems involving masses or volumes given in the same metric units.	**M.AAS.3.19** Identify the appropriate measurement tool to measure liquids and masses of a given object.
Geometric measurement: understand concepts of area and relate area to multiplication and to addition.	20. Find the area of a rectangle with whole number side lengths by tiling without gaps or overlays and counting unit squares.	**M.AAS.3.20** Find the area of a given shape using arrays (unit cubes and tiles) in relationship to multiplication. Limit units to 25.
	21. Count unit squares (square cm, square m, square in, square ft, and improvised or non-standard units) to determine area.	
	22. Relate area to the operations of multiplication using real-world problems, concrete materials, mathematical reasoning, and the distributive property.	
	23. Decompose rectilinear figures into smaller rectangles to find the area, using concrete materials.	
Geometric measurement: Recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.	24. Decompose rectilinear figures into smaller rectangles to find the area, using concrete materials.	
	25. Decompose rectilinear figures into smaller rectangles to find the area, using concrete materials.	

Geometry

Cluster	2019 Math COS Standard	2019 AAS Standard

Reason with shapes and their attributes.	26. Recognize and describe polygons (up to 8 sides), triangles, and quadrilaterals (rhombuses, rectangles, and squares) based on the number of sides and the presence or absence of square corners. a. Draw examples of quadrilaterals that are and are not rhombuses, rectangles, and squares.	**M.AAS.3.26** Using vocalization, sign language, augmentative communication, or assistive technology, recognize and sort polygons by their attributes (triangle, rectangle, square).

Grade 4

Operations and Algebraic Thinking

Cluster	2019 Math COS Standard	2019 AAS Standard
Solve problems with whole numbers using the four operations.	1. Interpret and write equations for multiplicative comparisons.	**M.AAS.4.1** Solve one-step word problems involving real-life situations using the four operations within 100 without regrouping and select the appropriate method of computation when problem solving.
	2. Solve word problems involving multiplicative comparison using drawings and write equations to represent the problem, using a symbol for the unknown number.	
	3. Determine and justify solutions for multi-step word problems, including problems where remainders must be interpreted. a. Write equations to show solutions for multistep word problems with a letter standing for the unknown quantity. b. Determine reasonableness of answers for multi-step word problems, using mental computation and estimation strategies including rounding.	
Gain familiarity with factors and multiples.	4. For whole numbers in the range 1 to 100, find all factor pairs, identifying a number as a multiple of each of its factors. a. Determine whether a whole number in the range 1 to 100 is a multiple of a given onedigit number. b. Determine whether a whole number in the range 1 to 100 is prime or composite.	
Generate and analyze patterns.	5. Generate and analyze a number or shape pattern that follows a given rule.	**M.AAS.4.5** Use repeating patterns to make predictions.

Operations with Numbers: Base Ten

Cluster	2019 Math COS Standard	2019 AAS Standard
Generalize place value understanding for multi- digit whole numbers.	6. Using models and quantitative reasoning, explain that in a multi-digit whole number, a digit in any place represents ten times what it represents in the place to its right.	**M.AAS.4.6** Compare whole number values to 50 using symbols (e.g.,<,>,=).
	7. Read and write multi-digit whole numbers using standard form, word form, and expanded form.	
	8. Use place value understanding to compare two multi-digit numbers using >, =, and < symbols.	

	9. Round multi-digit whole numbers to any place using place value understanding.	**M.AAS.4.9** Round a whole number from 1 to 49 to the nearest ten (using a number line and hundreds chart).

Use place value understanding and properties of operations to perform multi-digit arithmetic with whole numbers.	10. Use place value strategies to fluently add and subtract multi-digit whole numbers and connect strategies to the standard algorithm.	**M.AAS.4.11** Add and subtract one- and two-digit numbers up to 49 with regrouping using concrete manipulatives and visual models.
	11. Find the product of two factors (up to four digits by a one-digit number and two two-digit numbers), using strategies based on place value and the properties of operations. a. Illustrate and explain the product of two factors using equations, rectangular arrays, and area models.	
	12. Use strategies based on place value, properties of operations, and/or the relationship between multiplication and division to find whole-number quotients and remainders with one-digit divisors and up to four-digit dividends. a. Illustrate and/or explain quotients using equations, rectangular arrays, and/or area models.	

Operations with Numbers: Fractions

Cluster	2019 Math COS Standard	2019 AAS Standard
Extend understanding of fraction equivalence and ordering.	13. Using area and length fraction models, explain why one fraction is equivalent to another, taking into account that the number and size of the parts differ even though the two fractions themselves are the same size. a. Apply principles of fraction equivalence to recognize and generate equivalent fractions. Example: a/b is equivalent to $(n \times a)/(n \times b)$.	**M.AAS.4.13** Identify and compare models of a whole (1), one-half (1/2), one-third (1/3), and one fourth (1/4) using models, manipulatives, numbers lines, and a clock.
	14. Compare two fractions with different numerators and different denominators using concrete models, benchmarks (0, ½, 1), common denominators, and/or common numerators, recording the comparisons with symbols >, =, or <, and justifying the conclusions. a. Explain that comparison of two fractions is valid only when the two fractions refer to the same whole.	

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.	15. Model and justify decompositions of fractions and explain addition and subtraction of fractions as joining or separating parts referring to the same whole. a. Decompose a fraction as a sum of unit fractions and as a sum of fractions with the same denominator in more than one way using area models, length models, and equations. b. Add and subtract fractions and mixed numbers with like denominators using fraction equivalence, properties of operations, and the relationship between addition and subtraction. c. Solve word problems involving addition and subtraction of fractions and mixed numbers	**M.AAS.4.15** Model decomposing fractions having like denominators, using visual fraction models (limit to halves and fourths).
	having like denominators, using drawings, visual fraction models, and equations to represent the problem.	
	16. Apply and extend previous understandings of multiplication to multiply a whole number times a fraction. a. Model and explain how a non-unit fraction can be represented by a whole number times the unit fraction. Example: 9/8=9 ×1/8 b. Extend previous understanding of multiplication to multiply a whole number times any fraction less than one. Example: 4 ×2/(3)= (4 × 2)/3= 8/3 c. Solve word problems involving multiplying a whole number times a fraction using visual fraction models and equations to represent the problem. Examples: 3 × 1/2, 6 × 1/8	
Understand decimal notation for fractions and compare decimal fractions.	17. Express, model, and explain the equivalence between fractions with denominators of 10 and 100. a. Use fraction equivalency to add two fractions with denominators of 10 and 100.	**M.AAS.4.17** Model equivalence between fractions of a whole, halves and fourths using visual models.
	18. Use models and decimal notation to represent fractions with denominators of 10 and 100.	
	19. Use visual models and reasoning to compare two decimals to hundredths (referring to the same whole), recording comparisons using symbols >, =, or <, and justifying the conclusions.	**M.AAS.4.19** Compare fractions of a whole, halves and fourths using symbols (>,<,=).

Data Analysis/ Measurement/ Geometry

Cluster	2019 Math COS Standard	2019 AAS Standard

Represent and interpret data.	20. Interpret data in graphs (picture, bar, and line plots) to solve problems using numbers and operations. a. Create a line plot to display a data set of measurements in fractions of a unit (1/2,1/4,1/8). b. Solve problems involving addition and subtraction of fractions using information presented in line plots.	**<u>M.AAS.4.20</u>** Using vocalization, sign language, augmentative communication, or assistive technology, represent and interpret data on a picture or bar graph when given a model or a graph to complete.
Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.	21. Select and use an appropriate unit of measurement for a given attribute (length, mass, liquid volume, time) within one system of units: metric - km, m, cm; kg, g, l, ml; customary - lb., oz; time - hr., min, sec. a. Within one system of units, express measurements of a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table.	**<u>M.AAS.4.21</u>** Given an object determine the appropriate measurement tool and units of measure using vocalization, sign language, augmentative communication, or assistive technology.

	22. Use the four operations to solve measurement word problems with distance, intervals of time, liquid volume, mass of objects, and money. a. Solve measurement problems involving simple fractions or decimals. b. Solve measurement problems that require expressing measurements given in a larger unit in terms of a smaller unit. c. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.	**<u>M.AAS.4.22</u>** Using vocalization, sign language, augmentative communication, or assistive technology, tell time on a digital and analog clock (to the hour, half-hour, quarter hour). **<u>M.AAS.4.22a</u>** Measure mass, volume, or lengths of an object when given a measurement tool. **<u>M.AAS.4.22b</u>** Using vocalization, sign language, augmentative communication, or assistive technology, identify and determine the value of penny, nickel, dime, and quarter.
	23. Apply area and perimeter formulas for rectangles in real-world and mathematical situations.	**M.AAS.4.23** Determine the area of a square or rectangle by counting units of measurement (e.g., unit squares).
Geometric measurement: understand concepts of angle and	24. Identify an angle as a geometric shape formed wherever two rays share a common endpoint. 25. Use a protractor to measure angles in wholenumber degrees and sketch angles of specified measure.	**<u>M.AAS.4.24</u>** Recognize and Identify angles in geometric shapes as larger or smaller.

measure angles.	26. Decompose an angle into non-overlapping parts to demonstrate that the angle measure of the whole is the sum of the angle measures of the parts. a. Solve addition and subtraction problems on a diagram to find unknown angles in real-world or mathematical problems.	
Draw and identify lines and angles and identify shapes by properties of their lines and angles.	27. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines, and identify these in twodimensional figures.	**M.AAS.4.27** Recognize parallel lines, intersecting lines, and angles (right, acute, obtuse).
	28. Identify two-dimensional figures based on the presence or absence of parallel or perpendicular lines or the presence or absence of angles of a specified size. a. Describe right triangles as a category and identify right triangles.	**M.AAS.4.28** Using vocalization, sign language, augmentative communication, or assistive technology, describe the defining attributes of twodimensional shapes (e.g., number of sides, number of angles).
	29. Define a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. a. Identify line-symmetric figures and draw lines of symmetry.	**M.AAS.4.29** Given a drawing of a shape with a line drawn across the shape, identify if it is divided symmetrically.

Grade 5

Operations and Algebraic Thinking

Cluster	2019 Math COS Standard	2019 AAS Standard
Write and interpret numerical expressions.	1. Write, explain, and evaluate simple numerical expressions involving the four operations to solve up to two-step problems. Include expressions involving parentheses, brackets, or braces, using commutative, associative, and distributive properties.	**M.AAS.5.1** Evaluate simple numerical expressions involving the four operations.
Analyze patterns and relationships.	2. Generate two numerical patterns using two given rules and complete an input/output table for the data. a. Use data from an input/output table to identify apparent relationships between corresponding terms. b. Form ordered pairs from values in an input/output table. c. Graph ordered pairs from an input/output table on a coordinate plane.	**M.AAS.5.2** Given a rule, Identify and extend numerical patterns (e.g., given the rule “Add 3” and the starting number 0).

Operations with Numbers: Base Ten

Cluster	2019 Math COS Standard	2019 AAS Standard
Understand the place value system.	3. Using models and quantitative reasoning, explain that in a multi-digit number, including decimals, a digit in any place represents ten times what it represents in the place to its right and 1/10 of what it represents in the place to its left. a. Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, using whole-number exponents to denote powers of 10. b. Explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10, using whole-number exponents to denote powers of 10.	**M.AAS.5.3** Compare base-10 models up to 99 and whole numbers up to 100 to determine symbol (<, >,=).
	4. Read, write, and compare decimals to thousandths. a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form. Example: $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (1/10) + 9 \times (1/100) + 2 \times (1/1000)$. b. Compare two decimals to thousandths based on the meaning of the digits in each place, using >, =, and < to record the results of comparisons.	
	5. Use place value understanding to round decimals to thousandths.	

Perform operations with multidigit whole numbers and decimals to hundredths.	6. Fluently multiply multi-digit whole numbers using the standard algorithm.	**M.AAS.5.6** Model multiplication with two-digit whole numbers by one-digit whole numbers with regrouping using strategies such as arrays, decomposition, and manipulatives.
	7. Use strategies based on place value, properties of operations, and/or the relationship between multiplication and division to find whole-number quotients and remainders with up to four-digit dividends and two-digit divisors. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	
	8. Add, subtract, multiply, and divide decimals to hundredths using strategies based on place value, properties of operations, and/or the relationships between addition/subtraction and multiplication/division; relate the strategy to a written method, and explain the reasoning used. a. Use concrete models and drawings to solve problems with decimals to hundredths. b. Solve problems in a real-world context with decimals to hundredths.	**M.AAS.5.8** Add decimals to tenths using concrete models, drawings, and manipulatives without regrouping.

Operations with Numbers: Fractions

Cluster	2019 Math COS Standard	2019 AAS Standard
Use equivalent fractions as a strategy to add and subtract fractions.	9. Model and solve real-word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. Example: Recognize an incorrect result 2/5 + 1/2= 3/7 by observing that 3/7 < 1/2. 10. Add and subtract fractions and mixed numbers with unlike denominators, using fraction equivalence to calculate a sum or difference of fractions or mixed numbers with like denominators.	**<u>M.AAS.5.9</u>** Illustrate equivalent fractions using models of wholes, halves, thirds, and fourths to add fractions with like denominators.
Apply and extend previous understandings of multiplication and division to multiply and divide fractions.	11. Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers. a. Model and interpret a fraction as division of the numerator by the denominator (a/b= a ÷ b) b. Use visual fraction models, drawings, or equations to represent word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers.	**<u>M.AAS.5.11</u>** Using vocalization, sign language, augmentative communication, or assistive technology, identify models of thirds (e.g., 1/3. 2/3, 3/3) and tenths (e.g., 1/10, 2/10, 3/10, 4/10, 5/10, 6/10, 7/10, 8/10, 9/10, 10/10).
	12. Apply and extend previous understandings of multiplication to find the product of a fraction times a whole number or a fraction times a fraction. a. Use a visual fraction model (area model, set model, or linear model) to show (a/b) × q and create a story context for this equation to interpret the product as a part of a partition of q into b equal parts. b. Use a visual fraction model (area model, set model, or linear model) to show (a/b) × (c/d) and create a story context for this equation to interpret the product. c. Multiply fractional side lengths to find areas of rectangles and represent fraction products as rectangular areas. d. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths to show that the area is the same as would be found by multiplying the side lengths.	**<u>M.AAS.5.12</u>** Determine the product of unit fractions with unlike denominators (limited to denominators of 2, 3, 4, 10) using visual models.

	13. Interpret multiplication as scaling (resizing). a. Compare the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. b. Explain why multiplying a given number by a fraction greater than 1 results in a product greater than the given number and relate the principle of fraction equivalence. c. Explain why multiplying a given number by a fraction less than 1 results in a product smaller than the given number and relate the principle of fraction equivalence.	
	14. Model and solve real-world problems involving multiplication of fractions and mixed numbers using visual fraction models, drawings, or equations to represent the problem.	**M.AAS.5.14** Use a model to solve multiplying a whole number by a unit fraction of 1/2, 1/3, and 1/4.
	15. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. a. Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions and illustrate using visual fraction models, drawings, and equations to represent the problem. b. Create a story context for a unit fraction divided by a whole number and use a visual fraction model to show the quotient. c. Create a story context for a whole number divided by a unit fraction and use a visual fraction model to show the quotient.	**M.AAS.5.15** Use a model to solve dividing a whole number by a unit fraction of 1/2.

Data Analysis		
Cluster	**2019 Math COS Standard**	**2019 AAS Standard**
Represent and interpret data.	16. Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). a. Add, subtract, multiply, and divide fractions to solve problems involving information presented in line plots. Note: Division is limited to unit fractions by whole numbers and whole numbers by unit fractions.	**M.AAS.5.16** Using vocalization, sign language, augmentative communication, or assistive technology, represent and interpret data on a picture, bar graph, or line plot when given a model or a graph.
Measurement/ Geometry		
Cluster	**2019 Math COS Standard**	**2019 AAS Standard**

Convert like measurement units within a given measurement system.	17. Convert among different-sized standard measurement units within a given measurement system and use these conversions in solving multistep, real-world problems.	**M.AAS.5.17** Using vocalization, sign language, augmentative communication, or assistive technology, to tell time using an analog or digital clock to the half or quarter hour.
		M.AAS.5.17a Use standard units to measure the weight and length of objects.
		M.AAS.5.17b Sort a collection of coins according to their value.
Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.	18. Identify volume as an attribute of solid figures, and measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised (nonstandard) units. a. Pack a solid figure without gaps or overlaps using n unit cubes to demonstrate volume as n cubic units.	**M.AAS.5.18** Using vocalization, sign language, augmentative communication, or assistive technology, identify cubes, cylinders, and spheres as three-dimensional shapes.
	19. Relate volume to the operations of multiplication and addition and solve real-world and mathematical problems involving volume. a. Use the associative property of multiplication to find the volume of a right rectangular prism and relate it to packing the prism with unit cubes. Show that the volume can be determined by multiplying the three edge lengths or by multiplying the height by the area of the base. b. Apply the formulas $V = l \times w \times h$ and $V = B \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving realworld and mathematical problems. c. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the two parts, applying this technique to solve real-world problems.	**M.AAS.5.19** Determine the volume of a rectangular prism by counting units of measurement (e.g., unit cubes).
Graph points on the coordinate plane to solve real-world and mathematical problems.	20. Graph points in the first quadrant of the coordinate plane, and interpret coordinate values of points to represent real-world and mathematical problems.	**M.AAS.5.20** Identify a point on a horizontal number line representing the horizontal x-axis (no greater than 5) and identify a point on a vertical number line representing the y-axis (no greater than 5).
Classify twodimensional figures into	21. Classify triangles according to side length (isosceles, equilateral, scalene) and angle measure (acute, obtuse, right, equiangular).	

categories based on their properties.	22. Classify quadrilaterals in a hierarchy based on properties.	
	23. Explain that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. Example: All rectangles have four right angles, and squares have four right angles, so squares are rectangles.	

Grade 6

Proportional Reasoning

Cluster	2019 Math COS Standard	2019 AAS Standard
Develop an understanding of ratio concepts and use reasoning about ratios to solve problems.	1. Use appropriate notations [a/b, a to b, a:b] to represent a proportional relationship between quantities and use ratio language to describe the relationship between quantities.	**M.AAS.6.1** Demonstrate a simple ratio relationship using ratio notation given a real-world problem.
	2. Use unit rates to represent and describe ratio relationships.	
	3. Use ratio and rate reasoning to solve mathematical and real-world problems (including but not limited to percent, measurement conversion, and equivalent ratios) using a variety of models, including tables of equivalent ratios, tape diagrams, double number lines, and equations.	

Number Systems and Operations

Cluster	2019 Math COS Standard	2019 AAS Standard
Use prior knowledge of multiplication and division to divide fractions.	4. Interpret and compute quotients of fractions using visual models and equations to represent problems. a. Use quotients of fractions to analyze and solve problems.	**M.AAS.6.4** Use models to divide fractions (limit to unit fractions).
Compute multi-digit numbers fluently and determine common factors and multiples.	5. Fluently divide multi-digit whole numbers using a standard algorithm to solve real-world and mathematical problems.	**M.AAS.6.5** Apply the concepts of dividing multidigit numbers without remainders to real-world problems.
	6. Add, subtract, multiply, and divide decimals using a standard algorithm.	**M.AAS.6.6** Solve two-factor multiplication problems with products up to 100 (whole numbers only).
	7. Use the distributive property to express the sum of two whole numbers with a common factor as a multiple of a sum of two whole numbers with no common factor.	
	8. Find the greatest common factor (GCF) and least common multiple (LCM) of two or more whole numbers. a. Use factors and multiples to determine prime factorization.	

<table>
<tr><td rowspan="5">Apply knowledge of the number system to represent and use rational numbers in a variety of forms.</td><td>9. Use signed numbers to describe quantities that have opposite directions or values and to represent quantities in real-world contexts.</td><td rowspan="2"><u>M.AAS.6.9</u>
Describe quantities with positive and negative numbers (e.g., temperature, sea level, etc.).</td></tr>
<tr><td>10. Locate integers and other rational numbers on a horizontal or vertical line diagram.
a. Define opposites as numbers located on opposite sides of 0 and the same distance from 0 on a number line.
b. Use rational numbers in real-world and mathematical situations, explaining the meaning of 0 in each situation.</td></tr>
<tr><td>11. Find the position of pairs of integers and other rational numbers on the coordinate plane.
a. Identify quadrant locations of ordered pairs on the coordinate plane based on the signs of the x and y coordinates.
b. Identify (a,b) and (a,-b) as reflections across the x-axis.
c. Identify (a,b) and (-a,b) as reflections across the y-axis.
d. Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane, including finding distances between points with the same first or second coordinate.</td><td rowspan="3"><u>M.AAS.6.11</u>
Graph or identify ordered pairs in the first quadrant of the coordinate plane between 0 and 5, limited to whole numbers.</td></tr>
<tr><td>12. Explain the meaning of absolute value and determine the absolute value of rational numbers in real-world contexts.</td></tr>
<tr><td>13. Compare and order rational numbers and absolute value of rational numbers with and without a number line in order to solve real-world and mathematical problems.</td></tr>
</table>

Algebra and Functions

Cluster	2019 Math COS Standard	2019 AAS Standard
Apply knowledge of arithmetic to read, write, and evaluate algebraic expressions.	14. Write, evaluate, and compare expressions involving whole number exponents.	**M.AAS.6.14** Match an algebraic expression with one variable to its real-world situation.

Apply knowledge of arithmetic to read, write, and evaluate algebraic expressions.	15. Write, read, and evaluate expressions in which letters represent numbers in real-world contexts. a. Interpret a variable as an unknown value for any number in a specified set, depending on the context. b. Write expressions to represent verbal statements and real-world scenarios.	**<u>M.AAS.6.15</u>** Evaluate algebraic expressions when given specific values for the variables (e.g. x + 2, where x = 4).
	c. Identify parts of an expression using mathematical terms such as sum, term, product, factor, quotient, and coefficient. d. Evaluate expressions (which may include absolute value and whole number exponents) with respect to order of operations.	
	16. Generate equivalent algebraic expressions using the properties of operations, including inverse, identity, commutative, associative, and distributive.	
	17. Determine whether two expressions are equivalent and justify the reasoning.	
Use equations and inequalities to represent and solve realworld or mathematical problems.	18. Determine whether a value is a solution to an equation or inequality by using substitution to conclude whether a given value makes the equation or inequality true.	**<u>M.AAS.6.19</u>** Match equations and inequalities to real-world situations.
	19. Write and solve an equation in the form of x+p=q or px=q for cases in which p, q, and x are all nonnegative rational numbers to solve real-world and mathematical problems. a. Interpret the solution of an equation in the context of the problem.	
	20. Write and solve inequalities in the form of x>c, x<c, x≥ c, or x ≤ c to represent a constraint or condition in a real-world or mathematical problem. a. Interpret the solution of an inequality in the context of a problem. b. Represent the solutions of inequalities on a number line and explain that the solution set may contain infinitely many solutions.	
Identify and analyze relationships between independent and dependent variables.	21. Identify, represent, and analyze two quantities that change in relationship to one another in realworld or mathematical situations. a. Use tables, graphs, and equations to represent the relationship between independent and dependent variables.	**<u>M.AAS.6.21</u>** Identify the independent and dependent variables among two quantities that change in relationship to one another in real-world situations (e.g. purchase total depends on number of items purchased).

Data Analysis, Statistics, and Probability

Cluster	2019 Math COS Standard	2019 AAS Standard
Use real-world and mathematical problems to analyze data and demonstrate an understanding of statistical variability and measures of center.	22. Write examples and non-examples of statistical questions, explaining that a statistical question anticipates variability in the data related to the question. 23. Calculate, interpret, and compare measures of center (mean, median, mode) and variability (range and interquartile range) in real-world data sets. a. Determine which measure of center best represents a real-world data set.	**M.AAS.6.23** Find the range and median (when given an odd number of data points) and mean (involving one- or twodigit numbers) in real-world situations.
	b. Interpret the measures of center and variability in the context of a problem.	
	24. Represent numerical data graphically, using dot plots, line plots, histograms, stem and leaf plots, and box plots. a. Analyze the graphical representation of data by describing the center, spread, shape (including approximately symmetric or skewed), and unusual features (including gaps, peaks, clusters, and extreme values). b. Use graphical representations of real-world data to describe the context from which they were collected.	**M.AAS.6.24** Interpret graphical representations of a data set (e.g., line plots, dot plots, bar graphs, stem and leaf plots, or line graphs).

Geometry and Measurement

Cluster	2019 Math COS Standard	2019 AAS Standard
Graph polygons in the coordinate plane to solve real-world and mathematical problems.	25. Graph polygons in the coordinate plane given coordinates of the vertices to solve real-world and mathematical problems. a. Determine missing vertices of a rectangle with the same x-coordinate or the same ycoordinate when graphed in the coordinate plane. b. Use coordinates to find the length of a side between points having the same x-coordinate or the same y-coordinate. c. Calculate perimeter and area of a polygon graphed in the coordinate plane (limiting to polygons in which consecutive vertices have the same x-coordinate or the same ycoordinate).	**M.AAS.6.25** Graph squares, rectangles, and triangles in the first quadrant of the coordinate plane.

Solve real-world and mathematical problems to determine area, surface area, and volume.	26. Calculate the area of triangles, special quadrilaterals, and other polygons by composing and decomposing them into known shapes. a. Apply the techniques of composing and decomposing polygons to find area in the context of solving real-world and mathematical problems. 27. Determine the surface area of three-dimensional figures by representing them with nets composed of rectangles and triangles to solve real-world and mathematical problems.	**M.AAS.6.27** Calculate problems involving perimeter of squares, triangles, rectangles, and other polygons with sides up to 10 units and calculate problems involving the area of rectangles and squares with sides up to 10 units (whole numbers only).
Solve real-world and mathematica problems to determine area, surface area, and volume.	28. Apply previous understanding of volume of right rectangular prisms to those with fractional edge lengths to solve real-world and mathematical problems. a. Use models (cubes or drawings) and the volume formulas ($V = lwh$ and $V = Bh$) to find and compare volumes of right rectangular prisms.	**M.AAS.6.28** Solve real-world and mathematical problems involving the volume of cubes and rectangular prisms.

Grade 7

Proportional Reasoning

Cluster	2019 Math COS Standard	2019 AAS Standard
Analyze proportional relationships and use them to solve realworld and mathematical problems.	1. Calculate unit rates of length, area, and other quantities measured in like or different units that include ratios or fractions.	**M.AAS.7.1** Calculate a unit rate (limited to whole numbers under 100).

Analyze proportional relationships and use them to solve realworld and mathematical problems.	2. Represent a relationship between two quantities and determine whether the two quantities are related proportionally. a. Use equivalent ratios displayed in a table or in a graph of the relationship in the coordinate plane to determine whether a relationship between two quantities is proportional. b. Identify the constant of proportionality (unit rate) and express the proportional relationship using multiple representations including tables, graphs, equations, diagrams, and verbal descriptions. c. Explain in context the meaning of a point (x,y) on the graph of a proportional relationship, with special attention to the points (0,0) and (1, r) where r is the unit rate.	**M.AAS.7.2** Use a ratio to model or describe a real-world relationship.
Analyze proportional relationships and use them to solve realworld and mathematical problems.	3. Solve multi-step percent problems in context using proportional reasoning, including simple interest, tax, gratuities, commissions, fees, markups and markdowns, percent increase, and percent decrease.	**M.AAS.7.3** Calculate 10%, 20%, 25%, and 50% of a number up to 100.

Number Systems and Operations

Cluster	2019 Math COS Standard	2019 AAS Standard

Apply and extend prior knowledge of addition, subtraction, multiplication, and division to operations with rational numbers.	4. Apply and extend knowledge of operations of whole numbers, fractions, and decimals to add, subtract, multiply, and divide rational numbers including integers, signed fractions, and decimals. a. Identify and explain situations where the sum of opposite quantities is 0 and opposite quantities are defined as additive inverses. b. Interpret the sum of two or more rational numbers, by using a number line and in realworld contexts. c. Explain subtraction of rational numbers as addition of additive inverses. d. Use a number line to demonstrate that the distance between two rational numbers on the number line is the absolute value of their difference and apply this principle in real-world contexts. e. Extend strategies of multiplication to rational numbers to develop rules for multiplying signed numbers, showing that the properties of the operations are preserved. f. Divide integers and explain that division by zero is undefined. Interpret the quotient of integers (with a non-zero divisor) as a rational number. g. Convert a rational number to a decimal using long division, explaining that the decimal form of a rational number terminates or eventually repeats.	**M.AAS.7.5** Solve multiplication problems up to fifteen with whole number factors.
	5. Apply and extend prior knowledge of addition, subtraction, multiplication, and division to operations with rational numbers.	

Algebra and Functions

Cluster	2019 Math COS Standard	2019 AAS Standard
Create equivalent expressions using the properties of operations.	6. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.	**M.AAS.7.7** Match equivalent expressions using the properties of operations.
	7. Generate expressions in equivalent forms based on context and explain how the quantities are related.	**M.AAS.7.7a** Identify a pattern in a sequence of whole numbers with a whole number common difference (e.g. when skip counting by 5, the whole number common difference is 5).
Solve realworld and mathematical	8. Solve multi-step real-world and mathematical problems involving rational numbers (integers, signed fractions and decimals), converting between	**M.AAS.7.8** Add and subtract integers in a realworld situation.

problems using numerical and algebraic expressions, equations, and inequalities.	forms as needed. Assess the reasonableness of answers using mental computation and estimation strategies.	
Solve realworld and mathematical problems using numerical and algebraic expressions, equations, and inequalities.	9. Use variables to represent quantities in real-world or mathematical problems and construct algebraic expressions, equations, and inequalities to solve problems by reasoning about the quantities. a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.	**M.AAS.7.9** Use the properties of operations to solve one-step equations and inequalities from real-world and mathematical problems.

Data Analysis, Statistics, and Probability

Cluster	2019 Math COS Standard	2019 AAS Standard
Make inferences about a population using random sampling.	10. Examine a sample of a population to generalize information about the population. a. Differentiate between a sample and a population. b. Compare sampling techniques to determine whether a sample is random and thus representative of a population, explaining that random sampling tends to produce representative samples and support valid inferences. c. Determine whether conclusions and generalizations can be made about a population based on a sample. d. Use data from a random sample to draw inferences about a population with an unknown characteristic of interest, generating multiple samples to gauge variation and making predictions or conclusions about the population. e. Informally explain situations in which statistical bias may exist.	**M.AAS.7.10** Find the range and median (when given an odd number of data points), and mean (involving one- or twodigit numbers) in real-world situations.

Make inferences from an informal comparison of two populations.	11. Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability.	**M.AAS.7.11** Make inferences from graphical representations of a data set (e.g., line plots, dot plots, histograms, bar graphs, stem and leaf plots, or line graphs).
	12. Make informal comparative inferences about two populations using measures of center and variability and/or mean absolute deviation in context.	**M.AAS.7.12** Compare two sets of data within a single data display such as a picture graph, line plot, or bar graph.
Investigate probability models.	13. Use a number from 0 to 1 to represent the probability of a chance event occurring, explaining that larger numbers indicate greater likelihood of the event occurring, while a number near zero indicates an unlikely event.	**M.AAS.7.14** Describe the probability of events occurring as possible or impossible.
	14. Define and develop a probability model, including models that may or may not be uniform, where uniform models assign equal probability to all outcomes and non-uniform models involve events that are not equally likely. a. Collect and use data to predict probabilities of events. b. Compare probabilities from a model to observed frequencies, explaining possible sources of discrepancy.	
	15. Approximate the probability of an event using data generated by a simulation (experimental probability) and compare it to the theoretical probability. a. Observe the relative frequency of an event over the long run, using simulation or technology, and use those results to predict approximate relative frequency.	**M.AAS.7.15** Given a data set that represents a series of events, identify the event most likely to occur.

	16. Find probabilities of simple and compound events through experimentation or simulation and by analyzing the sample space, representing the probabilities as precents, decimals, or fractions. a. Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams, and determine the probability of an event by finding the fraction of outcomes in the sample space for which the compound event occurred. b. Design and use a simulation to generate frequencies for compound events. c. Represent events described in everyday language in terms of outcomes in the sample space which composed the event.	

Geometry and Measurement

Cluster	2019 Math COS Standard	2019 AAS Standard
Construct and describe geometric figures, analyzing relationships among them. .	17. Solve problems involving scale drawings of geometric figures, including computation of actual lengths and areas from a scale drawing and reproduction of a scale drawing at a different scale.	**M.AAS.7.18** Construct and analyze a geometric figure using manipulatives.
	18. Construct geometric shapes (freehand, using a ruler and a protractor, and using technology), given a written description or measurement constraints with an emphasis on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.	
	19. Describe the two-dimensional figures created by slicing three-dimensional figures into plane sections.	**M.AAS.7.19** Match two similar geometric shapes that are proportional in size and orientation.
Solve realworld and mathematical problems involving angle measure, circumference, area, surface area, and volume.	20. Explain the relationships among circumference, diameter, area, and radius of a circle to demonstrate understanding of formulas for the area and circumference of a circle. a. Informally derive the formula for area of a circle. b. Solve area and circumference problems in realworld and mathematical situations involving circles.	**M.AAS.7.20** Identify the radius, diameter, and circumference of a circle.

Solve realworld and mathematical problems involving angle measure, circumference, area, surface area, and volume.	21. Use facts about supplementary, complementary, vertical, and adjacent angles in multi-step problems to write and solve simple equations for an unknown angle in a figure.	**M.AAS.7.21** Classify angles as acute, obtuse, right, or straight.
Solve realworld and mathematical problems involving angle measure, circumference, area, surface area, and volume.	22. Solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right rectangular prisms.	**M.AAS.7.22** Determine the area of regular, twodimensional figures. Determine the volume of rectangular prisms, limited to whole numbers.

Grade 8

Number Systems and Operations

Cluster	2019 Math COS Standard	2019 AAS Standard
Understand that the real number system is composed of rational and irrational numbers.	1. Define the real number system as composed of rational and irrational numbers. a. Explain that every number has a decimal expansion; for rational numbers, the decimal expansion repeats or terminates. b. Convert a decimal expansion that repeats into a rational number.	**M.AAS.8.1** Add and subtract fractions with like denominators (e.g., halves, thirds, fourths, tenths).
		M.AAS.8.1a Add and subtract decimals to the hundredths place.
		M.AAS.8.1b Convert a fraction with a denominator of 100 to a decimal.
	2. Locate rational approximations of irrational numbers on a number line, compare their sizes, and estimate the values of the irrational numbers.	**M.AAS.8.2** Compare quantities represented as decimals in real-world examples to the hundredths place.

Algebra and Functions

Cluster	2019 Math COS Standard	2019 AAS Standard

<table>
<tr>
<td rowspan="4">Apply concepts of integer exponents and radicals.</td>
<td>3. Develop and apply properties of integer exponents to generate equivalent numerical and algebraic expressions.</td>
<td rowspan="2"><u>M.AAS.8.4</u>
Calculate the square of numbers 1 through 10.</td>
</tr>
<tr>
<td>4. Use square root and cube root symbols to represent solutions to equations.
a. Evaluate square roots of perfect squares (less than or equal to 225) and cube roots of perfect cubes (less than or equal to 1000).
b. Explain that the square root of a non-perfect square is irrational.</td>
</tr>
<tr>
<td>5. Estimate and compare very large or very small numbers in scientific notation.</td>
<td><u>M.AAS.8.5</u>
Find the square root of the perfect squares up to 100.</td>
</tr>
<tr>
<td>6. Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used.
a. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities.
b. Interpret scientific notation that has been generated by technology.</td>
<td><u>M.AAS.8.6</u>
Identify irrational numbers as nonperfect squares (e.g., discriminate between perfect and non-perfect squares).</td>
</tr>
</table>

<table>
<tr>
<td rowspan="2">Analyze the relationship between proportional and nonproportional situations.</td>
<td>7. Determine whether a relationship between two variables is proportional or non-proportional.</td>
<td rowspan="2"><u>M.AAS.8.8</u>
Using a real-world scenario, match a table with its graph. Identify proportional or nonproportional relationships.</td>
</tr>
<tr>
<td>8. Graph proportional relationships.
a. Interpret the unit rate of a proportional relationship, describing the constant of proportionality as the slope of the graph which goes through the origin and has the equation $y = mx$ where m is the slope.</td>
</tr>
</table>

	9. Interpret y = mx + b as defining a linear equation whose graph is a line with m as the slope and b as the y-intercept. a. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in a coordinate plane. b. Given two distinct points in a coordinate plane, find the slope of the line containing the two points and explain why it will be the same for any two distinct points on the line. c. Graph linear relationships, interpreting the slope as the rate of change of the graph and the y-intercept as the initial value. d. Given that the slopes for two different sets of points are equal, demonstrate that the linear equations that include those two sets of points may have different yintercepts.	
	10. Compare proportional and non-proportional linear relationships represented in different ways (algebraically, graphically, numerically in tables, or by verbal descriptions) to solve real-world problems.	
Analyze and solve linear equations and systems of two linear equations.	11. Solve multi-step linear equations in one variable, including rational number coefficients, and equations that require using the distributive property and combining like terms. a. Determine whether linear equations in one variable have one solution, no solution, or infinitely many solutions of the form x = a, a = a, or a = b (where a and b are different numbers). b. Represent and solve real-world and mathematical problems with equations and interpret each solution in the context of the problem.	**M.AAS.8.12** Solve two-step linear equations where coefficients are less than 10 and answers are integers.
	12. Solve systems of two linear equations in two variables by graphing and substitution.	

	a. Explain that the solution(s) of systems of two linear equations in two variables corresponds to points of intersection on their graphs because points of intersection satisfy both equations simultaneously. b. Interpret and justify the results of systems of two linear equations in two variables (one solution, no solution, or infinitely many solutions) when applied to real-world and mathematical problems.	
Explain, evaluate, and compare functions.	13. Determine whether a relation is a function, defining a function as a rule that assigns to each input (independent value) exactly one output (dependent value), and given a graph, table, mapping, or set of ordered pairs.	**M.AAS.8.13** Determine whether a relation is a function given a graph or a table.
	14. Evaluate functions defined by a rule or an equation, given values for the independent variable.	
	15. Compare properties of functions represented algebraically, graphically, numerically in tables, or by verbal descriptions. a. Distinguish between linear and nonlinear functions.	**M.AAS.8.15** Identify linear and nonlinear functions graphically.
Use functions to model relationships between quantities.	16. Construct a function to model a linear relationship between two variables. a. Interpret the rate of change (slope) and initial value of the linear function from a description of a relationship or from two points in a table or graph.	**M.AAS.8.17** Given a simple scatter plot of points in a straight line, describe the relationship between the two quantities.
	17. Analyze the relationship (increasing or decreasing, linear or non-linear) between two quantities represented in a graph.	

Data Analysis, Statistics, and Probability

Cluster	2019 Math COS Standard	2019 AAS Standard
Investigate patterns of association in bivariate data.	18. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities, describing patterns in terms of positive, negative, or no association, linear and nonlinear association, clustering, and outliers.	*(Standards addressed in grade before)*
	19. Given a scatter plot that suggests a linear association, informally draw a line to fit the data, and assess the model fit by judging the closeness of the data points to the line.	

	20. Use a linear model of a real-world situation to solve problems and make predictions. a. Describe the rate of change and y-intercept in the context of a problem using a linear model of a real-world situation.	
	21. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects, using relative frequencies calculated for rows or columns to describe possible associations between the two variables.	

Geometry and Measurement

Cluster	2019 Math COS Standard	2019 AAS Standard
Understand congruence and similarity using physical models or technology.	22. Verify experimentally the properties of rigid motions (rotations, reflections, and translations): lines are taken to lines, and line segments are taken to line segments of the same length; angles are taken to angles of the same measure; and parallel lines are taken to parallel lines. a. Given a pair of two-dimensional figures, determine if a series of rigid motions maps one figure onto the other, recognizing that if such a sequence exists the figures are congruent; describe the transformation sequence that verifies a congruence relationship.	**M.AAS.8.22** Identify 3 different transformations (e.g., reflection, rotation, translation).
	23. Use coordinates to describe the effect of transformations (dilations, translations, rotations, and reflections) on two-dimensional figures.	**M.AAS.8.23** Recognize the reflection (across the x- or y-axis) and translation (across quadrants) of a two- dimensional figure on a coordinate plane (limited to non-equilateral rectangles and triangles).
	24. Given a pair of two-dimensional figures, determine if a series of dilations and rigid motions maps one figure onto the other, recognizing that if such a sequence exists the figures are similar; describe the transformation sequence that exhibits the similarity between them.	
Analyze parallel lines cut by a transversal.	25. Analyze and apply properties of parallel lines cut by a transversal to determine missing angle measures. a. Use informal arguments to establish that the sum of the interior angles of a triangle is 180 degrees.	**M.AAS.8.25** Compare any angle to a right angle using greater than, less than, or congruent to the right angle.
Understand and apply the	26. Informally justify the Pythagorean Theorem and its converse.	**M.AAS.8.26** Identify vertical angles given two parallel lines cut by a transversal.

Pythagorean Theorem.	27. Apply the Pythagorean Theorem to find the distance between two points in a coordinate plane.	**M.AAS.8.27** Use the Pythagorean Theorem to find the hypotenuse when given the measures of two legs in a real-world context. Limit to Pythagorean triples.
	28. Apply the Pythagorean Theorem to determine unknown side lengths of right triangles, including real-world applications	
Solve realworld and mathematical	29. Informally derive the formulas for the volume of cones and spheres by experimentally comparing the volumes of cones and spheres with the same	**M.AAS.8.30** Use the formulas for perimeter, area, and volume to solve real-world and
problems involving volume of cylinders, cones, and spheres.	radius and height to a cylinder with the same dimensions.	mathematical problems (where volume problems are limited to finding the volume of cylinders and rectangular prisms).
	30. Use formulas to calculate the volumes of threedimensional figures (cylinders, cones, and spheres) to solve real-world problems.	

Grade 9- Geometry with Data Analysis

Number and Quantity

	Cluster	2019 Math COS Standard	2019 AAS Standard
Numbers and Quantity	Together, irrational numbers and rational numbers complete the real number system, representing all points on the number line, while there exist numbers beyond the real numbers called complex numbers.	1. Extend understanding of irrational and rational numbers by rewriting expressions involving radicals, including addition, subtraction, multiplication, and division, in order to recognize geometric patterns.	**M.G.AAS.9.1** Solve real world problems involving addition and/or subtraction of rational numbers (whole numbers of decimals) using models when needed.
	Quantitative reasoning includes and mathematical modeling requires attention to units of measurement.	2. Use units as a way to understand problems and to guide the solution of multi-step problems. a. Choose and interpret units consistently in formulas. b. Choose and interpret the scale and the origin in graphs and data displays. c. Define appropriate quantities for the purpose of descriptive modeling. d. Choose a level of accuracy appropriate to limitations of measurements when reporting quantities.	**M.G.AAS.9.2** Given a real-world scenario, identify the appropriate unit to obtain the most accurate measurement. (Ex: When baking a cake, should you measure 1 cup of sugar with a teaspoon or a measuring cup?)

Algebra and Functions

	Cluster	2019 Math COS Standard	2019 AAS Standard
Focus 1: Algebra	The structure of an equation or inequality (including, but not limited to, one-variable linear and quadratic equations, inequalities, and systems of linear equations in two variables) can be purposefully analyzed (with and without technology) to determine an efficient strategy to find a solution, if one exists, and then to justify the solution.	3. Find the coordinates of the vertices of a polygon determined by a set of lines, given their equations, by setting their function rules equal and solving, or by using their graphs.	**M.G.AAS.9.4** Solve one-step equations or inequalities.
	Expressions, equations, and inequalities can be used to analyze and make predictions, both within mathematics and as mathematics is applied in different contexts – in particular, contexts that arise in relation to linear, quadratic, and exponential situations.	4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. Example: Rearrange the formula for the area of a trapezoid to highlight one of the bases.	

Focus 2: Connecting Algebra to Functions	Graphs can be used to obtain exact or approximate solutions of equations, inequalities, and systems of equations and inequalities—including systems of linear equations in two variables and systems of linear and quadratic equations (given or obtained by using technology).	5. Verify that the graph of a linear equation in two variables is the set of all its solutions plotted in the coordinate plane, which forms a line.	**M.G.AAS.9.5** Interpret the meaning of a point on the graph of a line. (Ex.: On a graph of milkshake purchases, trace the graph to a point and tell the number of milkshakes purchased and the total cost.
	Graphs can be used to obtain exact or approximate solutions of equations, inequalities, and systems of equations and inequalities—including systems of linear equations in two variables and systems of linear and quadratic equations (given or obtained by using technology).	6. Derive the equation of a circle of given center and radius using the Pythagorean Theorem. a. Given the endpoints of the diameter of a circle, use the midpoint formula to find its center and then use the Pythagorean Theorem to find its equation. b. Derive the distance formula from the Pythagorean Theorem.	**M.G.AAS.9.6** Using real world models (Ex. Pizza or Pie) on a coordinate grid, determine the length of the radius.

Data Analysis, Statistics, and Probability

	Cluster	2019 Math COS Standard	2019 AAS Standard
Focus 1: Quantitative Literacy	Mathematical and statistical reasoning about data can be used to evaluate conclusions and assess risks.	7. Use mathematical and statistical reasoning with quantitative data, both univariate data (set of values) and bivariate data (set of pairs of values) that suggest a linear association, in order to draw conclusions and assess risk. Example: Estimate the typical age at which a lung cancer patient is diagnosed and estimate how the typical age differs depending on the number of cigarettes smoked per day.	**M.G.AAS.9.9** After collecting data, or with given data, construct a simple graph (line, pie, bar, picture, etc.) or table and interpret the data in terms of range and mode.
	Data arise from a context and come in two types: quantitative (continuous or discrete) and categorical. Technology can be used to "clean" and organize data, including very large data sets, into a useful and manageable structure – a first step in any analysis of data	8. Use technology to organize data, including very large data sets, into a useful and manageable structure.	

Focus 2: Visualizing and Summarizing Data	Distributions of quantitative data (continuous or discrete) in one variable should be described in the context of the data with respect to what is typical (the shape, with appropriate measures of center and variability, including standard deviation) and what is not (outliers), and these characteristics can be used to compare two or more subgroups with respect to a variable.	9. Represent the distribution of univariate quantitative data with plots on the real number line, choosing a format (dot plot, histogram, or box plot) most appropriate to the data set, and represent the distribution of bivariate quantitative data with a scatter plot. Extend from simple cases by hand to more complex cases involving large data sets using technology.	
Focus 2: Visualizing and Summarizing Data		10. Use statistics appropriate to the shape of the data distribution to compare and contrast two or more data sets, utilizing the mean and median for center and the interquartile range and standard deviation for variability. c. Explain how standard deviation develops from mean absolute deviation. d. Calculate the standard deviation for a data set, using technology where appropriate.	
	Distributions of quantitative data (continuous or discrete) in one variable should be described in the context of the data with respect to what is typical (the shape, with appropriate measures of center and variability, including standard deviation) and what is not (outliers), and these characteristics can be used to compare two or more subgroups with respect to a variable.	11. Interpret differences in shape, center, and spread in the context of data sets, accounting for possible effects of extreme data points (outliers) on mean and standard deviation.	**M.G.AAS.9.11** Interpret general trends on a graph. (Limited to increase and decrease)
	Scatter plots, including plots over time, can reveal patterns, trends, clusters, and gaps that are useful in analyzing the association between two contextual variables.	12. Represent data of two quantitative variables on a scatter plot and describe how the variables are related. a. Find a linear function for a scatter plot that suggests a linear association and informally assess its fit by plotting and analyzing residuals, including the squares of the residuals, in order to improve its fit. b. Use technology to find the least-squares line of best fit for two quantitative variables.	

Focus 2: Visualizing and Summarizing Data			
Focus 2: Visualizing and Summarizing Data	Analyzing the association between two quantitative variables should involve statistical procedures, such as examining (with technology) the sum of squared deviations in fitting a linear model, analyzing residuals for patterns, generating a least-squares regression line and finding a correlation coefficient, and differentiating between correlation and causation.	13. Compute (using technology) and interpret the correlation coefficient of a linear relationship.	**<u>M.G.AAS.9.15</u>** When given a real-world scenario, choose the independent or dependent variable. Ex.: If I buy 5 books that cost $8.00 each, the total cost is $40. Which variable is independent?
		14. Distinguish between correlation and causation.	
	Data analysis techniques can be used to develop models of contextual situations and to generate and evaluate possible solutions to real problems involving those contexts.	14. Evaluate possible solutions to real-life problems by developing linear models of contextual situations and using them to predict unknown values. a. Use the linear model to solve problems in the context of the given data. b. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the given data.	

Grade 10- Geometry with Data Analysis

Geometry and Measurement

	Cluster	2019 Math COS Standard	2019 AAS Standard
Focus 1: Measurement	Areas and volumes of figures can be computed by determining how the figure might be obtained from simpler figures by dissection and recombination.	16. Identify the shapes of two-dimensional crosssections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.	**<u>M.G.AAS.10.16</u>** Given a cross section of a three-dimensional object, identify the shapes of twodimensional cross sections (limited to sphere, rectangular prism, or triangular prism).
		17. Model and solve problems using surface area and volume of solids, including composite solids and solids with portions removed. a. Give an informal argument for the formulas for the surface area and volume of a sphere, cylinder, pyramid, and cone using dissection arguments, Cavalieri's Principle, and informal limit arguments. b. Apply geometric concepts to find missing dimensions to solve surface area or volume problems.	**<u>M.G.AAS.10.17</u>** Compare and contrast the volume of real-world geometric figures.

	Constructing approximations of measurements with different tools, including technology, can support an understanding of measurement.	18. Given the coordinates of the vertices of a polygon, compute its perimeter and area using a variety of methods, including the distance formula and dynamic geometry software, and evaluate the accuracy of the results.	**M.G.AAS.10.18** Find the perimeter or area of a square, rectangle, or equilateral triangle to solve real-world problems when given the length of at least one side.
	When an object is the image of a known object under a similarity transformation, a length, area, or volume on the image can be computed by using proportional relationships.	19. Derive and apply the relationships between the lengths, perimeters, areas, and volumes of similar figures in relation to their scale factor.	
		20. Derive and apply the formula for the length of an arc and the formula for the area of a sector.	
Focus 2: Transformations	Applying geometric transformations to figures provides opportunities for describing the attributes of the figures preserved by the transformation and for describing symmetries by examining when a figure can be mapped onto itself.	21. Represent transformations and compositions of transformations in the plane (coordinate and otherwise) using tools such as tracing paper and geometry software. a. Describe transformations and compositions of transformations as functions that take points in the plane as inputs and give other points as outputs, using informal and formal notation. b. Compare transformations which preserve distance and angle measure to those that do not.	**M.G.AAS.10.21** Identify and/or model characteristics of a geometric figure that has undergone a transformation (reflection, rotation, translation) by drawing, explaining, or using manipulatives.

		22. Explore rotations, reflections, and translations using graph paper, tracing paper, and geometry software. a. Given a geometric figure and a rotation, reflection, or translation, draw the image of the transformed figure using graph paper, tracing paper, or geometry software. b. Specify a sequence of rotations, reflections, or translations that will carry a given figure onto another. c. Draw figures with different types of symmetries and describe their attributes.	
		23. Develop definitions of rotation, reflection, and translation in terms of angles, circles, perpendicular lines, parallel lines, and line segments.	
	Showing that two figures are congruent involves showing that there is a rigid motion (translation, rotation, reflection, or glide reflection) or, equivalently, a sequence of rigid	24. Define congruence of two figures in terms of rigid motions (a sequence of translations, rotations, and reflections); show that two figures are congruent by finding a sequence of rigid motions that maps one figure to the other. Example: $\triangle ABC$ is congruent to $\triangle XYZ$ since a reflection followed by a translation maps $\triangle ABC$ onto $\triangle XYZ$.	**M.G.AAS.10.24** When given two congruent triangles that have been transformed (limit to a translation), determine the congruent parts.

Focus 2: Transformations			
	motions that maps one figure to the other.	25. Verify criteria for showing triangles are congruent using a sequence of rigid motions that map one triangle to another. a. Verify that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. b. Verify that two triangles are congruent if (but not only if) the following groups of corresponding parts are congruent: angleside-angle (ASA), side-angle-side (SAS), side-side-side (SSS), and angle-angle-side (AAS). Example: Given two triangles with two pairs of congruent corresponding sides and a pair of congruent included angles, show that there must be a sequence of rigid motions will map one onto the other.	(Ex: Determine which leg on Triangle A is congruent to which leg on Triangle B).
		26. Verify experimentally the properties of dilations given by a center and a scale factor. a. Verify that a dilation takes a line not passing through the center of the dilation to a parallel line and leaves a line passing through the center unchanged. b. Verify that the dilation of a line segment is longer or shorter in the ratio given by the scale factor.	
		27. Given two figures, determine whether they are similar by identifying a similarity transformation (sequence of rigid motions and dilations) that maps one figure to the other.	
		28. Verify criteria for showing triangles are similar using a similarity transformation (sequence of rigid motions and dilations) that maps one triangle to another. a. Verify that two triangles are similar if and only if corresponding pairs of sides are proportional and corresponding pairs of angles are congruent. b. Verify that two triangles are similar if (but not only if) two pairs of corresponding angles are congruent (AA), the corresponding sides are proportional (SSS), or two pairs of corresponding sides are proportional, and the pair of included angles is congruent (SAS). Example: Given two triangles with two pairs of congruent corresponding sides and a pair of congruent included angles, show there must be a set of rigid motions that maps one onto the other.	

Focus 3 Geometric Arguments, Reasoning, and Proof	Using technology to construct and explore figures with constraints provides an opportunity to explore the independence and dependence of assumptions and conjectures.	29. Find patterns and relationships in figures including lines, triangles, quadrilaterals, and circles, using technology and other tools. a. Construct figures, using technology and other tools, in order to make and test conjectures about their properties. b. Identify different sets of properties necessary to define and construct figures.	
	Proof is the means by which we demonstrate whether a statement is true or false mathematically, and proofs can be communicated in a variety of ways (e.g., twocolumn, paragraph).	30. Develop and use precise definitions of figures such as angle, circle, perpendicular lines, parallel lines, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.	**M.G.AAS.10.30** Demonstrate perpendicular lines, parallel lines, line segments, angles, and circles by drawing, modeling, identifying, or creating.

		31. Justify whether conjectures are true or false in order to prove theorems and then apply those theorems in solving problems, communicating proofs in a variety of ways, including flow chart, two-column, and paragraph formats. a. Investigate, prove, and apply theorems about lines and angles, including but not limited to: vertical angles are congruent; when a transversal crosses parallel lines,	**M.G.AAS.10.31a** When given an isosceles triangle and a measure of a leg or base angle, identify the measure of the other leg or base angle.

Focus Geometric Arguments, Reasoning, and Proof		alternate interior angles are congruent and corresponding angles are congruent; the points on the perpendicular bisector of a line segment are those equidistant from the segment's endpoints. b. Investigate, prove, and apply theorems about triangles, including but not limited to: the sum of the measures of the interior angles of a triangle is 180˚; the base angles of isosceles triangles are congruent; the segment joining the midpoints of two sides of a triangle is parallel to the third side and half the length; a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem using triangle similarity. c. Investigate, prove, and apply theorems about parallelograms and other quadrilaterals, including but not limited to both necessary and sufficient conditions for parallelograms and other quadrilaterals, as well as relationships among kinds of quadrilaterals. Example: Prove that rectangles are parallelograms with congruent diagonals.	**M.G.AAS.10.31b** When given a parallelogram and the measure of one side or one angle, identify the measure of the opposite side or angle.
	Proofs of theorems can sometimes be made with transformations, coordinates, or algebra; all approaches can be useful, and in some cases, one may provide a more accessible or understandable argument than another.	32. Use coordinates to prove simple geometric theorems algebraically.	
		33. Prove the slope criteria for parallel and perpen dicular lines and use them to solve geometric problems. Example: Find the equation of a line parallel or perpendicular to a given line that passes through a given point.	

	Recognizing congruence, similarity, symmetry, measurement opportunities, and	34. Use congruence and similarity criteria for triangles to solve problems in real-world contexts.	**M.G.AAS.10.36** Use geometric shapes to describe real world objects.

Focus: 4 Solving Applied Problems and Modeling in Geometry			
	other geometric ideas, including right triangle trigonometry, in real-world contexts provides a means of building understanding of these concepts and is a powerful tool for solving problems related to the physical world in which we live.	35. Discover and apply relationships in similar right triangles. a. Derive and apply the constant ratios of the sides in special right triangles (45˚-45˚-90˚ and 30˚-60˚-90˚). b. Use similarity to explore and define basic trigonometric ratios, including sine ratio, cosine ratio, and tangent ratio. c. Explain and use the relationship between the sine and cosine of complementary angles. d. Demonstrate the converse of the Pythagorean Theorem. e. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems, including finding areas of regular polygons	
		36. Use geometric shapes, their measures, and their properties to model objects and use those models to solve problems.	
		37. Investigate and apply relationships among inscribed angles, radii, and chords, including but not limited to: the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.	
	Experiencing the mathematical modeling cycle in problems involving geometric concepts, from the simplification of the real problem through the solving of the simplified problem, the interpretation of its solution, and the checking of the solution's feasibility, introduces geometric techniques, tools, and points of view that are valuable to problem-solving.	38. Use the mathematical modeling cycle involving geometric methods to solve design problems. Examples: Design an object or structure to satisfy physical constraints or minimize cost; work with typographic grid systems based on ratios; apply concepts of density based on area and volume.	

Grade 11- Algebra with Probability

Number and Quantity

	Cluster	2019 Math COS Standard	2019 AAS Standard

	Together, irrational numbers and rational numbers complete the real number system, representing all points on the number line, while there exist numbers beyond the real numbers called complex numbers.	1. Explain how the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for an additional notation for radicals using rational exponents.	**M.A.AAS.11.1** Determine the value of a quantity that is squared or cubed. (Limited to perfect squares and perfect cubes).
		2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.	
		3. Define the imaginary number i such that i 2 = -1.	

Algebra and Functions

Focus 1: Algebra	Cluster	2019 Math COS Standard	2019 AAS Standard
	Expressions can be rewritten in equivalent forms by using algebraic properties, including properties of addition, multiplication, and exponentiation, to make different characteristics or features visible.	4. Interpret linear, quadratic, and exponential expressions in terms of a context by viewing one or more of their parts as a single entity. Example: Interpret the accrued amount of investment P(1 + r)t , where P is the principal and r is the interest rate, as the product of P and a factor depending on time t.	**M.A.AAS.11.4** Identify an algebraic expression involving addition or subtraction to represent a real-world problem.
	Expressions can be rewritten in equivalent forms by using algebraic properties, including properties of addition, multiplication, and exponentiation, to make different characteristics or features visible.	5. Use the structure of an expression to identify ways to rewrite it. Example: See x4 - y4 as (x2)2 - (y2)2, thus recognizing it as a difference of squares that can be factored as (x2 - y2)(x2 + y2).	**M.A.AAS.11.5** Solve simple algebraic equations using real world scenarios with one variable using multiplication or division.
		6. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor quadratic expressions with leading coefficients of one and use the factored form to reveal the zeros of the function it defines. b. Use the vertex form of a quadratic expression to reveal the maximum or minimum value and the axis of symmetry of the function it defines; complete the square to find the vertex form of quadratics with a leading coefficient of one. c. Use the properties of exponents to transform expressions for exponential functions. Example: Identify percent rate of change in functions such as y = (1.02)t, y = (0.97)t, y = (1.01)12t, y = (1.2)t/10, and classify them as representing exponential growth or decay.	

		7. Add, subtract, and multiply polynomials, showing that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication.	
	Finding solutions to an equation, inequality, or system of equations or inequalities requires the checking of candidate solutions, whether generated analytically or graphically, to ensure that solutions are found and that those found are not extraneous.	8. Explain why extraneous solutions to an equation involving absolute values may arise and how to check to be sure that a candidate solution satisfies an equation.	**<u>M.A.AAS.11.9</u>** Identify equivalent expressions given a linear expression using arithmetic operations.
	The structure of an equation or inequality (including, but not limited to, one-variable linear and quadratic equations, inequalities, and systems of linear equations in two variables) can be purposefully analyzed (with and without technology) to determine an efficient strategy to find a solution, if one exists, and then to justify the solution.	9. Select an appropriate method to solve a quadratic equation in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form (x – p)2 = q that has the same solutions. Explain how the quadratic formula is derived from this form. b. Solve quadratic equations by inspection (such as x2 = 49), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation, and recognize that some solutions may not be real.	
		10. Select an appropriate method to solve a system of two linear equations in two variables. a. Solve a system of two equations in two variables by using linear combinations; contrast situations in which use of linear combinations is more efficient with those in which substitution is more efficient. b. Contrast solutions to a system of two linear equations in two variables produced by algebraic methods with graphical and tabular methods.	
	Expressions, equations, and inequalities can be used to analyze and make predictions, both within mathematics and as mathematics is applied in different contexts – in particular,	11. Create equations and inequalities in one variable and use them to solve problems in context, either exactly or approximately. Extend from contexts arising from linear functions to those involving quadratic, exponential, and absolute value functions.	**<u>M.A.AAS.11.11</u>** A) Select an equation or inequality involving one operation (limit to addition or subtraction) with one variable that represents a

	contexts that arise in relation to linear, quadratic, and exponential situations.	12. Create equations in two or more variables to represent relationships between quantities in context; graph equations on coordinate axes with labels and scales and use them to make predictions. Limit to contexts arising from linear, quadratic, exponential, absolute value, and linear piecewise functions.	real-world problem. B) Solve the equation
		13. Represent constraints by equations and/or inequalities, and solve systems of equations and/or inequalities, interpreting solutions as	

		viable or nonviable options in a modeling context. Limit to contexts arising from linear, quadratic, exponential, absolute value, and linear piecewise functions.	

Data Analysis, Statistics, and Probability

	Cluster	2019 Math COS Standard	2019 AAS Standard
Focus 1: Quantitative Literacy	Mathematical and statistical reasoning about data can be used to evaluate conclusions and assess risks.	32. Use mathematical and statistical reasoning with bivariate categorical data in order to draw conclusions and assess risk. Example: In a clinical trial comparing the effectiveness of flu shots A and B, 21 subjects in treatment group A avoided getting the flu while 29 contracted it. In group B, 12 avoided the flu while 13 contracted it. Discuss which flu shot appears to be more effective in reducing the chances of contracting the flu. Possible answer: Even though more people in group A avoided the flu than in group B, the proportion of people avoiding the flu in group B is greater than the proportion in group A, which suggests that treatment B may be more effective in lowering the risk of getting the flu.	**M.A.AAS.11.32** Make predictions and draw conclusions from two variable data based on data displays and apply the results to a real-world situation.
	Making and defending informed, data-based decisions is a characteristic of a quantitatively literate person.	33. Design and carry out an investigation to determine whether there appears to be an association between two categorical variables, and write a persuasive argument based on the results of the investigation. Example: Investigate whether there appears to be an association between successfully completing a task in a given length of time and listening to music while attempting the task. Randomly assign some students to listen to music while attempting to complete the task and others to complete the task without listening to music. Discuss whether students should listen to music while studying, based on that analysis.	**M.A.AAS.11.33** When given a two-way table summarizing data on two categorical variables collected from the same subjects, identify possible association between the two variables.

	Data arise from a context and come in two types: quantitative (continuous or discrete) and categorical. Technology can be used to “clean” and organize data, including very large data sets, into a useful and manageable structure—a first step in any analysis of data. The association between two categorical variables is typically represented by using two-way tables and segmented bar graphs.	34. Distinguish between quantitative and categorical data and between the techniques that may be used for analyzing data of these two types. Example: The color of cars is categorical and so is summarized by frequency and proportion for each color category, while the mileage on each car’s odometer is quantitative and can be summarized by the mean. 35. Analyze the possible association between two categorical variables. a. Summarize categorical data for two categories in two-way frequency tables and represent using segmented bar graphs. b. Interpret relative frequencies in the context of categorical data (including joint, marginal, and conditional relative frequencies).	**<u>M.A.AAS.11.35</u>** Interpret general trends on a graph. (Limited to increase and decrease).
		c. Identify possible associations and trends in categorical data.	
	Data analysis techniques can be used to develop models of contextual situations and to generate and evaluate possible solutions to real problems involving those contexts.	36. Generate a two-way categorical table in order to find and evaluate solutions to real-world problems. a. Aggregate data from several groups to find an overall association between two categorical variables. b. Recognize and explore situations where the association between two categorical variables is reversed when a third variable is considered (Simpson's Paradox). Example: In a certain city, Hospital 1 has a higher fatality rate than Hospital 2. But when considering mildly injured patients and severely-injured patients as separate groups, Hospital 1 has a lower fatality rate among both groups than Hospital 2, since Hospital 1 is a Level 1 Trauma Center. Thus, Hospital 1 receives most of the severely injured patients who are less likely to survive overall but have a better chance of surviving in Hospital 1 than they would in Hospital 2.	**<u>M.A.AAS.11.36</u>** When given a real-world scenario, choose the independent or dependent variable. Ex.: If I buy 2 coffees that cost $2.00 each, the total cost is $4. Which variable is independent?
	Two events are independent if the occurrence of one event does not affect the probability of the other event. Determining whether two events are independent can be used for finding and understanding probabilities.	37. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or,”, “and," "not"). 38. Explain whether two events, A and B, are independent, using two-way tables or tree diagrams.	

	Conditional probabilities – that is, those probabilities that are "conditioned" by some known information – can be computed from data organized in contingency tables. Conditions or assumptions may affect the computation of a probability.	39. Compute the conditional probability of event A given event B, using two-way tables or tree diagrams.	
		40. Recognize and describe the concepts of conditional probability and independence in everyday situations and explain them using everyday language.	
		41. Explain why the conditional probability of A given B is the fraction of B's outcomes that also belong to A, and interpret the answer in context.	

Grade 12- Algebra with Probability

Algebra and Functions

	Cluster	2019 Math COS Standard	2019 AAS Standard
Focus 2: Connecting Algebra to Functions	Functions shift the emphasis from a point-by-point relationship between two variables (input/output) to considering an entire set of ordered pairs (where each first element is paired with exactly one second element) as an entity with its own features and characteristics.	14. Given a relation defined by an equation in two variables, identify the graph of the relation as the set of all its solutions plotted in the coordinate plane. Note: The graph of a relation often forms a curve (which could be a line).	**M.A.AAS.12.14** When given a relation in table form, identify the graph that represents the relation (Ex: The points (5,5); (6,4); (3,7) are given to the student along with three graphs, and the student chooses the graph that represents the relation).
	Graphs can be used to obtain exact or approximate solutions of equations, inequalities, and systems of equations and inequalities – including systems of linear equations in two variables and systems of linear and quadratic equations (given or obtained by using technology).	15. Define a function as a mapping from one set (called the domain) to another set (called the range) that assigns to each element of the domain exactly one element of the range. a. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. Note: If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. b. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. Limit to linear, quadratic, exponential, and absolute value functions.	**M.A.AAS.12.15** Use the vertical line test to determine if a given relation is a function.
		16. Compare and contrast relations and functions represented by equations, graphs, or tables that show related values; determine whether a relation is a function. Explain that a function f is a special kind of relation defined by the equation y = f(x).	

		17. Combine different types of standard functions to write, evaluate, and interpret functions in context. Limit to linear, quadratic, exponential, and absolute value functions. a. Use arithmetic operations to combine different types of standard functions to write and evaluate functions. Example: Given two functions, one representing flow rate of water and the other representing evaporation of that water, combine the two functions to determine the amount of water in a container at a given time.	
		b. Use function composition to combine different types of standard functions to write and evaluate functions.	
	Graphs can be used to obtain exact or approximate solutions of equations, inequalities, and systems of equations and inequalities – including systems of linear equations in two variables and systems of linear and quadratic equations (given or obtained by using technology).	18. Solve systems consisting of linear and/or quadratic equations in two variables graphically, using technology where appropriate. 19. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x). a. Find the approximate solutions of an equation graphically, using tables of values, or finding successive approximations, using technology where appropriate. Note: Include cases where f(x) is a linear, quadratic, exponential, or absolute value function and g(x) is constant or linear.	**<u>M.A.AAS.12.18</u>** Interpret the meaning of a point on the graph of a line. (Ex.: On a graph of football ticket purchases, trace the graph to a point and tell the number of tickets purchased and the total cost.
		20. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes, using technology where appropriate.	
	Functions can be described by using a variety of representations: mapping diagrams, function notation (e.g., f(x) = x2), recursive definitions, tables, and graphs.	21. Compare properties of two functions, each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). Extend from linear to quadratic, exponential, absolute value, and general piecewise.	**<u>M.A.AAS.12.21</u>** Given a function table, identify the missing number.

	Functions can be described by using a variety of representations: mapping diagrams, function notation (e.g., f(x) = x2), recursive definitions, tables, and graphs.	22. Define sequences as functions, including recursive definitions, whose domain is a subset of the integers. a. Write explicit and recursive formulas for arithmetic and geometric sequences and connect them to linear and exponential functions. Example: A sequence with constant growth will be a linear function, while a sequence with proportional growth will be an exponential function.	**<u>M.A.AAS.12.22</u>** Given a sequence of numbers, identify the rule that will give you the next number in the sequence. (Limit to expressions with simple arithmetic (adding or subtracting) or geometric (multiplying or dividing) operations.
	Functions that are members of the same family have distinguishing attributes (structure) common to all functions within that family.	23. Identify the effect on the graph of replacing f(x) by f(x)+k,k·f(x), f(k·x), and f(x+k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and explain the effects on the graph, using technology as appropriate. Limit to linear, quadratic, exponential, absolute value, and linear piecewise functions.	**<u>M.A.AAS.12.24</u>** Given a simple linear function on a graph, select the model that represents an increase by equal amounts over equal intervals.
		24. Distinguish between situations that can be modeled with linear functions and those that can be modeled with exponential functions.	

		a. Show that linear functions grow by equal differences over equal intervals, while exponential functions grow by equal factors over equal intervals. b. Define linear functions to represent situations in which one quantity changes at a constant rate per unit interval relative to another. c. Define exponential functions to represent situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.	
		25. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	
		26. Use graphs and tables to show that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically.	

	27. Interpret the parameters of functions in terms of a context. Extend from linear functions, written in the form mx + b, to exponential functions, written in the form abx Example: If the function V(t) = 19885(0.75)t describes the value of a car after it has been owned for t years, 19885 represents the purchase price of the car when t = 0, and 0.75 represents the annual rate at which its value decreases.	
Functions can be represented graphically and key features of the graphs, including zeros, intercepts, and, when relevant, rate of change and maximum/minimum values, can be associated with and interpreted in terms of the equivalent symbolic representation.	28. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Note: Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; maximums and minimums; symmetries; and end behavior. Extend from relationships that can be represented by linear functions to quadratic, exponential, absolute value, and linear piecewise functions.	**M.A.AAS.12.28** Given graphs that represent linear functions, identify key features (limit to y-intercept, x-intercept, increasing, decreasing) and/or interpret different rates of change (e.g., Which is faster or slower?).
	29. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. Limit to linear, quadratic, exponential, and absolute value functions.	
Functions can be represented graphically and key features of the graphs, including zeros, intercepts, and, when relevant, rate of change and maximum/minimum values, can be associated with and	30. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima.	**M.A.AAS.12.30** Given the graph of a linear function, identify the intercepts, the maxima, and minima.
interpreted in terms of the equivalent symbolic representation.	b. Graph piecewise-defined functions, including step functions and absolute value functions. c. Graph exponential functions, showing intercepts and end behavior.	
Functions model a wide variety of real situations and can help students understand the processes of making and changing assumptions, assigning variables, and finding solutions to contextual problems.	31. Use the mathematical modeling cycle to solve real-world problems involving linear, quadratic, exponential, absolute value, and linear piecewise functions.	**M.A.AAS.12.31** Choose the graph of the linear function that represents a solution in a real-world scenario. (Ex: Choose the graph that shows a steady increase or decrease rather than a graph with fluctuating data).