



Alabama Course of Study Mathematics K-12



Differentiated Instructional Guide (D.I.G.)





For information regarding the

Differentiated Instructional Guide

contact Special Education Services

Alabama State Department of Education

3345 Gordon Persons Building,

50 North Ripley Street, Montgomery, Alabama 36104;

or by mail to P.O. Box 302101, Montgomery, Alabama

36130-2101; or by telephone at (334) 694-4782.

Speced@alsde.edu

Alabama State Department of Education

Eric G. Mackey, State Superintendent of Education

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DIFFERENTIATED INSTRUCTIONAL GUIDE TO THE ALABAMA COURSE OF STUDY: MATHEMATICS

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PREFACE

The Differentiated Instructional Guide (DIG) to the Alabama Course of Study: Mathematics is an instructional companion to assist teachers with tailoring instruction to meet the individual needs of all students. Differentiated instruction, as a framework, allows teachers to implement a variety of strategies to maximize student growth and individual success. The format of the document is intentional to provide teachers flexibility in use and aid in targeted, meaningful instruction. The document is organized by grade to allow single to multiple grade printing. The standards are setup one per sheet to allow teachers to utilize the pages in binders, data collection notebooks, instructional tools, etc.

The *Differentiated Instructional Guide to the Alabama Course of Study: Mathematics* was developed by a diverse group of general and special educators, administrators, parents, and service providers. This Task Force used their academic content knowledge and experiential knowledge related to students with and without disabilities to produce this resource.

ACKNOWLEDGEMENTS

This document was developed by the 2019/2020 Alabama Course of Study Mathematics Task Force composed of both general and special education teachers of elementary, middle school, and high school grade levels. The Alabama State Department of Education (ALSDE) staff who assisted with the development are listed below.

Ms. Taylor Paige Biggs , MS. Mrs. Emily Bivens , M.Ed. Mrs. Teresa Burden , Ed.S Mrs. Becky Cornelius , Ed.S. Ms. Tiffany Countryman , Ed.S. Ms. Kim Daily , M.Ed. Mrs. Dana Davis , M.Ed. Mr. Derallus Davis , M.Ed. Ms. Anna Laura Dyer , Ed.S. Mrs. Erica Dunwoody , M.Ed. Ms. Ali Grace Eiland , M.Ed. Ms. Michelle Gibbons , B.S. Ms. Briana Gibson , M. Ed. Mrs. Lori Ann Gibson , Ed.S. Mrs. Ann-Marie Grav , M.Ed. Ms. Adriane Howard , B.S. Ms. Amy Johnson , M.Ed.	Ms. Jessica Johnson , B.S. Ms. Hailey King , B.S. Ms. Tammy King , M.Ed. Ms. Julia Kochan , M.Ed. Mrs. Kristy Mann , B.S. Ms. Meagan McDonald , B.S. Mrs. Shelly Munger , M.Ed. Miss Michele Murray , M.Ed. Mrs. Chelsea Newels , M.Ed. Mrs. Brandy Quattlebaum , M.Ed. Mrs. Anna Laura Reid , Ed.S Mr. John Rice , B.S. Mr. Terry Chad Sorrells , Ed.S. Dr. Marilyn Strutchens , Ph.D. Ms. Lateatrice Thomas , Ed.S. Mrs. Lori White , M.A. Mrs. Tamika Whitt-Wright , Ed.S. J.D.	Mrs. Kimberly Baker , M.S. Special Education Services Mr. Tod Beers , M.Ed., AMSTI Ms. Karma Clarke , Ed.S., AMSTI Ms. Diane Duncan , Ed.S., ALSDE Mrs. Susan Goldthwaite , M. Ed., Special Education Services Mrs. Elizabeth Greene , M.Ed., Special Education Services Ms. Elizabeth Hammonds , M.Ed., AMSTI Mrs. Pamela Ivey , M.Ed., Special Education Services Mrs. Elisabeth Newell , Ed.S. Special Education Services Mrs. Phenicia Nunn , M.Ed., AMSTI Ms. Nannette Pence , M.Ed., Assessment Ms. Gwendolyn Jordan Preston , M.Ed., Special Education Services Mrs. Tina Sanders , Ed.S., Special Education Services Mrs. Teri Shriver , Ed.S., Special Education Services Ms. Kristie Taylor , M.Ed., AMSTI Mr. Joel White , Ed.S., AMSTI Mrs. Kathy Wilkins , M.Ed., Special Education Services
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Introduction

The *Differentiated Instructional Guide* is a companion document to the 2019 Alabama Course of Study: Mathematics, for Grades K-12. Content standards contained within the course of study document may be accessed on the ALSDE website at [ALSDE | Alabama Achieves/](#). On the home page, hover over Teachers & Administrators, and scroll down and select Academic Standards. Scroll down and click on Mathematics.

Educators are reminded that content standards indicate minimum content what all students should know and be able to do by the end of each grade level or course. Local education agencies may have additional instructional or achievement expectations and may provide instructional guidelines that address content sequence, review, and remediation.

The *Differentiated Instructional Guide* identifies the progression of learning towards a content standard, as well as the prior knowledge skills necessary to construct a student's connection between old and new knowledge. Differentiated instruction "is a philosophy that enables educators to plan strategically in order to reach the needs of diverse learners in the classroom to achieve targeted standards" (Gregory & Chapman, 2007, p. 2). Through the identified progressions of learning objectives, teachers can make unique planning decisions focused on the learner's needs that align with the lesson outcomes. The prior knowledge skills identified for each content standard helps teachers develop instructional activities that build off a student's strengths and acknowledge and address their weaknesses. Using this guide to differentiate instruction and activate prior knowledge, teachers can prepare students for the study of the grade-level and course content standards at individual ability levels and may plan instruction to address the achievement gap experienced by some students. The goal of this guide is to assist teachers to react responsively to a learner's need and to enable a student to reach his or her maximum ability and promote individual success.

Organization of the Guide

The organizational components of this guide align with the organization and format progression of the 2019 Mathematics course of study that encompasses sections for content areas, content clusters, content standards, progression of learning objectives and prior knowledge skills.

Content Areas are large groups of related clusters and content standards. In this example, the Alabama Content Area is “Operations and Algebraic Thinking.” Content areas are used to group standards that are closely related.

Content clusters group related content standards. The cluster in the example is “Extend the counting sequence.” Because mathematics is a connected subject, standards from different clusters may sometimes be closely related.

Operations and Algebraic Thinking	
Cluster	2019 Math COS Standard
Understand and apply properties of operations and the relationship between addition and subtraction. <i>Note: Students need not use formal terms for these properties.</i>	3. Apply properties of operations as strategies to add and subtract. Examples: If $8 + 3 = 11$ is known, then $3 + 8 = 11$ is also known (commutative property of addition). To add $2 + 6 + 4$, the second and third numbers can be added to make a ten, so $2 + 6 + 4 = 2 + 10 = 12$ (associative property of addition). When adding 0 to a number, the result is the same number (identity property of zero for addition).
Learning Objectives	Prior Knowledge Skills
M.1.3.1: Define addition and subtraction. M.1.3.2: Recognize properties of operations. M.1.3.3: Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., $5 = 2 + 3$ and $5 = 4 + 1$). M.1.3.4: Apply signs $+$, $-$, $=$ to actions of joining and separating sets. M.1.3.5: Identify fact families to ten. M.1.3.6: Recognize the value of zero.	-Decompose numbers up to 5 using objects or drawings. -Compose numbers up to 5 using objects or drawings. -Count backward from 5. -Count forward to 10. -Write numerals from 0 to 10. -Represent a given numeral 1 to 10 with objects or drawings. -Count forward from a given number 1 to 10. -Model joining sets of objects to total 10. -Identify plus, minus, and equal signs. -Match numerals to objects or drawings. -Identify numerals 1 to 10. -Count 0 to 10. -Add and subtract numbers within 20 using objects, pictures and fingers. -Understand key words in addition and subtraction word problems. Examples: all together, how many more, how many are left, in all. -Represent numbers with objects or drawings. -Separate sets with nine or fewer objects. -Combine objects to form sets up to nine. -Count items in a set up to twenty. -Establish one-to-one correspondence between numbers and objects. -Using counting, find one less than a number 2 through 20.

Content Standards, listed to the right of each cluster, contain the minimum required content, and define what students should know and be able to do at the conclusion of a course or grade. Some have sub-standards, indicated with a, b, c, d, which are extensions of the content standards and are also required. Some standards are followed by examples, which are not required to be taught. When standards indicate that drawings may be used, the drawings need not show details but should show the mathematics in the problem. The order in which standards are listed within a course or grade is not intended to convey a sequence for instruction. Each content standard completes the stem “Students will...”

Progression of Learning Objectives are a backwards sequenced set of subskills and bodies of enabling knowledge a student must master enroute to mastering the standard. Learning objectives identify the “must learn” underlying student’s attainment of the content standards. These objectives are useful in lesson planning, classroom instruction, and Individualized Education Program (IEP) development. Utilization of a progression of instructional objectives empowers student learning towards grade-level standards while also working at individual ability levels. Also, educators develop a common language for collaborative professional opportunities to discuss, implement and build stronger experiences that meet the needs of all their learners.

Prior Knowledge Skills allow teachers to make unique planning decisions focused on the learner’s needs that align with the lesson outcomes. The prior knowledge skills identified for each content standard help teachers develop instructional activities that build off a student’s strengths and acknowledge and address his or her weaknesses. Note: the list is not an endless array or a true progression of knowledge, but rather a sample of the most significant targets to improve student achievement.

Organization of the Guide

Content standards preceded by an asterisk (*) indicate that the standard has been identified as a critical standard by Alabama Math, Science, and Technology Initiative (AMSTI) of the ALSDE. Critical standards and other related resources can be accessed by clicking each of the grade bands, or by visiting the AMSTI website:

[K-2](#)

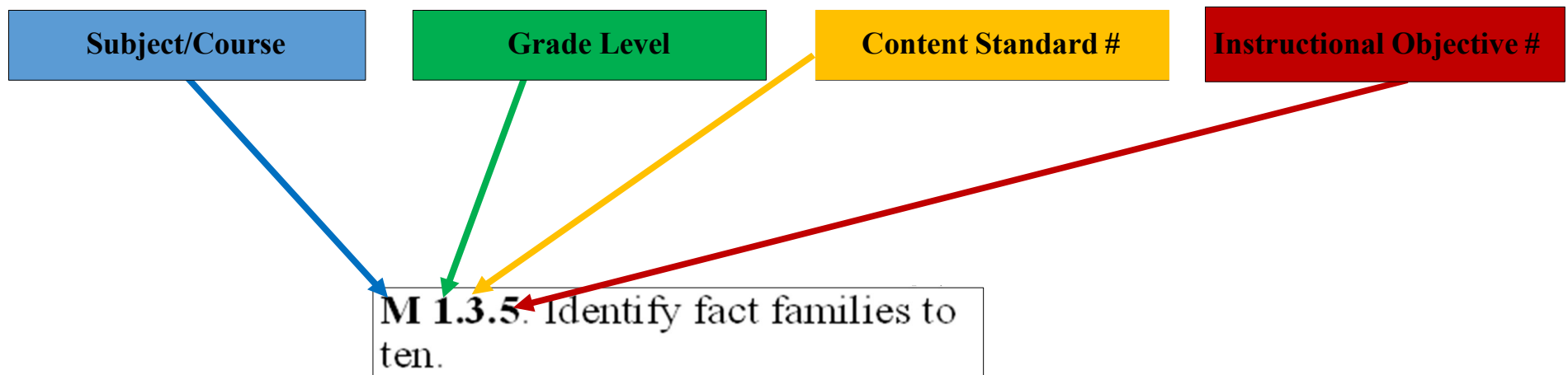
[3-5](#)

[6-8](#)

[9-12](#)

The standards marked with an asterisk are required for earning course credit for the Essentials Diploma Pathway in Grades 912. The courses are Geometry with Data Analysis, Algebra I with Probability, Algebra II with Statistics, and Mathematical Modeling. General education and special education teachers are encouraged to utilize the proficiency scales (found through the grade band links above) in working with students on this pathway, to document progress towards the standards.

The system for numbering Mathematics Objective 1. 3. 5., for example, is based upon the following:



Utilization of the Guide (This example is from Geometry with Data Analysis)

Content Area:
Used to group standards that are closely related.

Content Clusters:
This group is related to content standards. Standards from different clusters may sometimes be closely related.

Progression of Learning Objectives:
Backwards sequences set of subskills a student must master enroute to mastering the standard.

Mastered Skills:
Skills mastered by the student are checked off.

Prior Knowledge Skills:
A sample of significant targets to improve student achievement. These skills help teachers to develop instructional activities that build off a student's strengths and address weaknesses.

Content Standards:
Minimum required content for this course. They define what a student should know and be able to do at the end of the course or grade.

Number and Quantity		
Cluster	2019 Math COS Standard	
Quantitative reasoning includes, and mathematical modeling requires, attention to units of measurement.	2. Use units as a way to understand problems and to guide the solution of multi-step problems. - Choose and interpret units consistently in formulas. - Choose and interpret the scale and the origin in graphs and data displays. - Define appropriate quantities for the purpose of descriptive modeling. - Choose a level of accuracy appropriate to limitations of measurements when reporting quantities.	
	Learning Objectives	Prior Knowledge Skills
	☐ GEO.2.1: Interpret units consistently in formulas. ☐ GEO.2.2: Choose units consistently in formulas. ☐ GEO.2.3: Use units as a way to guide the solution of multi-step problems. ☐ GEO.2.4: Use units as a way to understand problems. ☐ GEO.2.5: Convert between units of measurement within the same system. ☐ GEO.2.6: Choose the scale and the origin in graphs. ☐ GEO.2.7: Interpret the scale and the origin in data displays. ☐ GEO.2.8: Define units of measurement. ☐ GEO.2.9: Identify appropriate units of measure to best describe a real-world application. ☒ GEO.2.10: Recognize the limitations for each type of measurement tool. ☒ GEO.2.11: Determine the level of precision needed for real-world measurements. ☒ GEO.2.12: Relate how rounding effects the accuracy of the measurement.	☐ Convert like measurement units within a given system. (Example: 120 min = 2 hrs). ☒ Know relative sizes of measurement units within one system of units, including km, m, cm; kg, g; lb, oz; l, ml; and hr, min, sec.

Kindergarten

Foundations of Counting

Cluster	2019 Math COS Standard	
Know number names and the count sequence. <i>Note on number reversals: Learning to write numerals is generally more difficult than learning to read them. It is common for students to reverse numerals at this stage.</i>	1. *Count forward orally from 0 to 100 by ones and by tens. Count backward orally from 10 to 0 by ones.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.K.1.1: Count backwards from 5 to 0 by ones. ☐ M.K.1.2: Mimic counting backwards from 5 to 0 by ones. ☐ M.K.1.3: Count to 50 by ones. ☐ M.K.1.4: Count to 50 by tens. ☐ M.K.1.5: Count to 20 by ones. ☐ M.K.1.6: Count to 10 by ones. ☐ M.K.1.7: Mimic counting by tens. ☐ M.K.1.8: Mimic counting by ones.	☐ Count to 20 and above. ☐ Mimic counting by ones. ☐ Recognize numbers from one to ten. ☐ Become interested in how many objects she/he has. ☐ Continue to have an interest in counting. ☐ Understand the concept of size and amount. ☐ Pair the number of objects counted with “how many.” ☐ Understand that the last number name tells the number of objects counted. ☐ Pair a group of objects with a number representing the total number of objects in the group (up to ten objects). ☐ Count objects one-by-one using only one number per object (up to ten objects). ☐ Recognize that numbers and numerals have meaning. ☐ Recognize numerals 0 (zero) through 10. ☐ Rote count to ten. ☐ Communicate some number words. ☐ Communicate the number word one. ☐ Recognize after. ☐ Recognize before. ☐ Enjoy playing with all kinds of objects. ☐ Point to matching or similar objects.

Foundations of Counting

Cluster	2019 Math COS Standard	
Know number names and the count sequence. <i>Note on number reversals: Learning to write numerals is generally more difficult than learning to read them. It is common for students to reverse numerals at this stage.</i>	2. *Count to 100 by ones beginning with any given number between 0 and 99.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.K.2.1: Count forward to 100 from a number over 50. ☐ M.K.2.2: Count forward to 100 from a number between 2 and 50. ☐ M.K.2.3: Count forward to 50 from a given number. ☐ M.K.2.4: Count to 100 by ones. ☐ M.K.2.5: Mimic counting to 100 by ones. ☐ M.K.2.6: Count to 50 by ones. ☐ M.K.2.7: Mimic counting to 50 by ones.	☐ Count to 20 and above. ☐ Mimic counting by ones. ☐ Recognize numbers from one to ten. ☐ Become interested in how many objects she/he has. ☐ Continue to have an interest in counting. ☐ Understand the concept of size and amount. ☐ Pair the number of objects counted with “how many.” ☐ Understand that the last number name tells the number of objects counted. ☐ Pair a group of objects with a number representing the total number of objects in the group (up to ten objects). ☐ Count objects one-by-one using only one number per object (up to ten objects). ☐ Recognize that numbers and numerals have meaning. ☐ Recognize numerals 0 (zero) through 10. ☐ Rote count to ten. ☐ Communicate some number words. ☐ Recognize after. ☐ Recognize before. ☐ Enjoy playing with all kinds of objects. ☐ Point to matching or similar objects.

Foundations of Counting

Cluster	2019 Math COS Standard	
Know number names and the count sequence. <i>Note on number reversals:</i> <i>Learning to write numerals is generally more difficult than learning to read them. It is common for students to reverse numerals at this stage.</i>	3. Write numerals from 0 to 20. a. Represent 0 to 20 using concrete objects when given a written numeral from 0 to 20 (with 0 representing a count of no objects).	
	Learning Objectives	Prior Knowledge Skills
	☐ M.K.3.1: Write numbers 0 to 10. ☐ M.K.3.2: Match numerals to quantity 11 to 20. ☐ M.K.3.3: Match numerals to quantity 0 to 10. ☐ M.K.3.4: Recognize written numerals 0 to 20. ☐ M.K.3.5: Demonstrate one to one correspondence for a group of objects 6 to 20. ☐ M.K.3.6: Demonstrate one to one correspondence for a group of objects 0 to 5. ☐ M.K.3.7: Trace numerals 0 to 20. ☐ M.K.3.8: Make purposeful marks such as lines and circles.	☐ Count to 20 and above. ☐ Mimic counting by ones. ☐ Recognize numbers from one to ten. ☐ Become interested in how many objects she/he has. ☐ Continue to have an interest in counting. ☐ Understand the concept of size and amount. ☐ Notice same/different and some/all. ☐ Understand that words can label sameness and differences. ☐ Understand that some have more, and some have less. ☐ Become more interested in the concept of some and all. ☐ Make purposeful marks. ☐ Given a set number of objects one through ten, answer the question "How many?". ☐ Pair the number of objects counted with "how many." ☐ Understand that the last number name tells the number of objects counted. ☐ Establish one-to-one correspondence between numbers and objects when given a picture, a drawing, or objects. ☐ Pair a group of objects with a number representing the total number of objects in the group (up to ten objects). ☐ Count objects one-by-one using only one number per object (up to ten objects). ☐ Recognize that numbers and numerals have meaning. ☐ Recognize numerals 0 (zero) through 10.

		☐ Identify the difference between written numbers and other written things. ☐ Identify the difference between written numbers and objects. ☐ Rote count to ten. ☐ Communicate some number words. ☐ Recognize after. ☐ Recognize before. ☐ Enjoy playing with all kinds of objects. ☐ Point to matching or similar objects.
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Foundations of Counting

Cluster	2019 Math COS Standard	
Know number names and the count sequence. <i>Note on number reversals: Learning to write numerals is generally more difficult than learning to read them. It is common for students to reverse numerals at this stage.</i>	4. *Connect counting to cardinality using a variety of concrete objects. - Say the number names in consecutive order when counting objects. - Indicate that the last number name said tells the number of objects counted in a set. - Indicate that the number of objects in a set is the same regardless of their arrangement or the order in which they were counted. - Explain that each successive number name refers to a quantity that is one larger.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.K.4.1: Define number and counting. ☐ M.K.4.2: Identify correct number of objects for a given number up to 20. ☐ M.K.4.3: Identify different size groups of objects up to 10. ☐ M.K.4.4a: Count to 20 by ones. ☐ M.K.4.5a: Mimic counting objects. ☐ M.K.4.6b: Know that the last number tells how many when counting 0 to 5 objects. ☐ M.K.4.7b: Mimic counting objects up to 20. ☐ M.K.4.8b: Count to 20 by ones. ☐ M.K.4.9b: Mimic counting to 20 by ones. ☐ M.K.4.10c: Define one larger/one more. ☐ M.K.4.11c: Count objects in a group and identify total after adding one more. ☐ M.K.4.12c: Count in sequential order. ☐ M.K.4.13c: Mimic counting in sequential order.	☐ Count to 20 and above. ☐ Mimic counting by ones. ☐ Recognize numbers from one to ten. ☐ Become interested in how many objects she/he has. ☐ Continue to have an interest in counting. ☐ Understand the concept of size and amount. ☐ Notice same/different and some/all. ☐ Understand that words can label sameness and differences. ☐ Understand that some have more, and some have less. ☐ Become more interested in the concept of some and all. ☐ Given a set number of objects one through ten, answer the question "How many?". ☐ Pair the number of objects counted with "how many." ☐ Understand that the last number name tells the number of objects counted. ☐ Establish one-to-one correspondence between numbers and objects when given a picture, a drawing, or objects. ☐ Pair a group of objects with a number representing the total number of objects in the group (up to ten objects).

Foundations of Counting

Cluster	2019 Math COS Standard	
Know number names and the count sequence. <i>Note on number reversals: Learning to write numerals is generally more difficult than learning to read them. It is common for students to reverse numerals at this stage.</i>	5. *Count to answer “how many?” questions. - Count using no more than 20 concrete objects arranged in a line, a rectangular array, or a circle. - Count using no more than 10 concrete objects in a scattered configuration. - Draw the number of objects that matches a given numeral from 0 to 20.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.K.5.1: Define how many, all together, and in all. ☐ M.K.5.2: Demonstrate one to one correspondence. Example: Point to only one object when counting and stop counting when all objects have been touched. ☐ M.K.5.3: Count to 20 by ones.	☐ Understand amount words, such as more, less, and another. ☐ Begin to understand that parts of an object can make a whole. ☐ Be interested in who has more or less. ☐ Understand the concept of “less than”. ☐ Mimic counting by ones. ☐ Recognize numbers from one to ten. ☐ Become interested in how many objects she/he has. ☐ Continue to have an interest in counting. ☐ Understand the concept of size and amount. ☐ Make purposeful marks. ☐ Given a set number of objects one through ten, answer the question “How many?” ☐ Pair the number of objects counted with “how many.” ☐ Understand that the last number name tells the number of objects counted. ☐ Establish one-to-one correspondence between numbers and objects when given a picture, a drawing or objects. ☐ Pair a group of objects with a number representing the total number of objects in the group. ☐ Count objects one-by-one using only one number per object. ☐ Recognize that numbers and numerals have meaning. ☐ Recognize numerals 0 through 10. ☐ Rote count to ten. ☐ Communicate number words.

*Critical Standard

		☐ Recognize after. ☐ Recognize before.
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Foundations of Counting

Cluster	2019 Math COS Standard	
Compare numbers.	6. *Orally identify whether the number of objects in one group is greater/more than, less/fewer than, or equal/the same as the number of objects in another group, in groups containing up to 10 objects, by using matching, counting, or other strategies.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.K.6.1: Define greater than, less than, and equal to. ☐ M.K.6.2: Count to 20 by ones. ☐ M.K.6.3: Count objects up to ten.	☐ Understand amount words, such as more, less, and another. ☐ Begin to understand that parts of an object can make a whole. ☐ Become more interested in the concept of some and all. ☐ Be interested in who has more or less. ☐ Understand the concept of "less than". ☐ Mimic counting by ones. ☐ Recognize numbers from one to ten. ☐ Become interested in how many objects she/he has. ☐ Understand the concept of size and amount. ☐ Given a set number of objects one through ten, answer the question "How many?". ☐ Pair the number of objects counted with "how many." ☐ Understand that the last number name tells the number of objects counted. ☐ Establish one-to-one correspondence between numbers and objects when given a picture, a drawing, or objects. ☐ Pair a group of objects with a number representing the total number of objects in the group. ☐ Count objects one-by-one using only one number per object. ☐ Recognize that numbers and numerals have meaning. ☐ Recognize numerals 0 through 10. ☐ Rote count to ten. ☐ Communicate number words.

Foundations of Counting

Cluster	2019 Math COS Standard	
Compare numbers.	7. *Compare two numbers between 0 and 10 presented as written numerals (without using inequality symbols).	
	Learning Objectives	Prior Knowledge Skills
	☐ M.K.7.1: Compare numbers 1 to 10 using objects. ☐ M.K.7.2: Name numerals 1 to 10. ☐ M.K.7.3: Identify numerals 1 to 10. ☐ M.K.7.4: Count to 10 by ones.	☐ Recognize numbers from one to ten. ☐ Understand amount words, such as more, less, and another. ☐ Begin to understand that parts of an object can make a whole. ☐ Become more interested in the concept of some and all. ☐ Be interested in who has more or less. ☐ Understand the concept of “less than”. ☐ Mimic counting by ones. ☐ Recognize numbers from one to ten. ☐ Become interested in how many objects she/he has. ☐ Understand the concept of size and amount. ☐ Given a set number of objects one through ten, answer the question “How many?”. ☐ Pair the number of objects counted with “how many.” ☐ Understand that the last number name tells the number of objects counted. ☐ Establish one-to-one correspondence between numbers and objects when given a picture, a drawing, or objects. ☐ Pair a group of objects with a number representing the total number of objects in the group. ☐ Recognize less/fewer. ☐ Recognize greater/more. ☐ Recognize same/equal.

Operations and Algebraic Thinking

Cluster	2019 Math COS Standard	
Understand addition as putting together and adding to and subtraction as taking apart and taking from. <i>Note: Drawings need not be detailed but should show the mathematics in the problem.</i>	8. *Represent addition and subtraction up to 10 with concrete objects, fingers, pennies, mental images, drawings, claps or other sounds, acting out situations, verbal explanations, expressions, or equations.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.K.8.1: Define addition as combining groups of objects. ☐ M.K.8.2: Define subtraction as separating groups of objects. ☐ M.K.8.3: Represent numbers with objects or drawings. ☐ M.K.8.4: Separate sets with nine or fewer objects. ☐ M.K.8.5: Combine objects to form sets up to nine.	☐ Notice same/different and some/all. ☐ Subtract one from a set of objects (up to 10 objects). ☐ Add one to a set of objects (up to 10 objects). ☐ Given a group of objects (ten or less), divide the group into smaller groups in various ways. ☐ Given small groups of objects, create larger groups by combining the small groups. ☐ Take away objects from a large group to create two smaller groups. ☐ Put together two small groups of objects to create a larger group. ☐ Establish one-to-one correspondence between numbers and objects when given a picture a drawing or objects. ☐ Rote count to ten. ☐ Enjoy playing with all kinds of objects. ☐ Point to matching or similar objects.

Operations and Algebraic Thinking

Cluster	2019 Math COS Standard	
Understand addition as putting together and adding to and subtraction as taking apart and taking from. <i>Note: Drawings need not be detailed but should show the mathematics in the problem.</i>	9. *Solve addition and subtraction word problems, and add and subtract within 10, by using concrete objects or drawings to represent the problem.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.K.9.1: Understand key words in addition and subtraction word problems. ☐ Examples: all together, how many more, how many are left, in all. ☐ M.K.9.2: Represent numbers with objects or drawings. ☐ M.K.9.3: Separate sets with nine or fewer objects. ☐ M.K.9.4: Combine objects to form sets up to nine.	☐ Notice same/different and some/all. ☐ Use models, solve word problems with two given sets (e.g., objects, drawings); using “putting together”; add within nine. ☐ Use models, solve word problems with two given sets (e.g., objects, drawings); using “putting together”; add within five. ☐ Represent addition and subtraction with objects, pictures, fingers, or sounds within nine. ☐ Understand addition as putting together and subtraction as taking from. ☐ Establish one-to-one correspondence between numbers and objects. ☐ Rote count to 10. ☐ Enjoy playing with all kinds of objects. ☐ Point to matching or similar objects.

Operations and Algebraic Thinking

Cluster	2019 Math COS Standard	
Understand addition as putting together and adding to and subtraction as taking apart and taking from. <i>Note: Drawings need not be detailed but should show the mathematics in the problem.</i>	10. *Decompose numbers less than or equal to 10 into pairs of smaller numbers in more than one way, by using concrete objects or drawings, and record each decomposition by a drawing or equation. <i>Example: $5 = 2 + 3$ and $5 = 4 + 1$</i>	
	Learning Objectives	Prior Knowledge Skills
	☐ M.K.10.1: Identify plus, minus, and equal signs. ☐ M.K.10.2: Match numerals to objects or drawings. ☐ M.K.10.3: Identify numerals 1 to 10. ☐ M.K.10.4: Count 0 to 10.	☐ Notice same/different and some/all. ☐ Recognize numbers from one to ten. ☐ Subtract one from a set of objects (up to five objects). ☐ Add one to a set of objects (up to five objects). ☐ Given a group of objects (ten or less), divide the group into smaller groups in various ways. ☐ Given small groups of objects, create larger groups by combining the small groups. ☐ Take away objects from a large group to create two smaller groups. ☐ Put together two small groups of objects to create a larger group. ☐ Establish one-to-one correspondence between numbers and objects when given a picture a drawing or objects. ☐ Rote count to ten.

Operations and Algebraic Thinking

Cluster	2019 Math COS Standard	
Understand addition as putting together and adding to and subtraction as taking apart and taking from. <i>Note: Drawings need not be detailed but should show the mathematics in the problem.</i>	11. *For any number from 0 to 10, find the number that makes 10 when added to the given number, by using concrete objects or drawings, and record the answer with a drawing or equation.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.K.11.1: Write numerals from 0 to 10. ☐ M.K.11.2: Represent a given numeral 1 to 10 with objects or drawings. ☐ M.K.11.3: Count forward from a given number 1 to 10. ☐ M.K.11.4: Model joining sets of objects to total 10.	☐ Notice same/different and some/all. ☐ Recognize numbers from one to ten. ☐ Add one to a set of objects (up to five objects). ☐ Given small groups of objects, create larger groups by combining the small groups. ☐ Put together two small groups of objects to create a larger group. ☐ Establish one-to-one correspondence between numbers and objects when given a picture a drawing or objects. ☐ Rote count to ten. ☐ Understand number words.

Operations and Algebraic Thinking

Cluster	2019 Math COS Standard	
Understand addition as putting together and adding to and understand subtraction as taking apart and taking from. <i>Note: Drawings need not be detailed but should show the mathematics in the problem.</i>	12. *Fluently add and subtract within 5.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.K.12.1: Decompose numbers up to 5 using objects or drawings. ☐ M.K.12.2: Compose numbers up to 5 using objects or drawings. ☐ M.K.12.3: Count backward from 5. ☐ M.K.12.4: Count forward to 5.	☐ Notice same/different and some/all. ☐ Subtract one from a set of objects (up to five objects). ☐ Given a group of objects (ten or less), divide the group into smaller groups in various ways. ☐ Take away objects from a large group to create two smaller groups. ☐ Establish one-to-one correspondence between numbers and objects when given a picture a drawing or objects. ☐ Understand number words.

Operations and Algebraic Thinking

Cluster	2019 Math COS Standard	
Understand simple patterns.	13. Duplicate and extend simple patterns using concrete objects.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.K.13.1: Define ones and tens. ☐ M.K.13.2: Match the number in the ones and tens position to a pictorial representation or manipulative of the value. ☐ M.K.13.3: Add numbers 1-9 to ten to create teen numbers using manipulatives or place value blocks. ☐ M K.13.4: Count objects up to 10.	☐ Notice same/different and some/all. ☐ Establish one-to-one correspondence between numbers and objects when given a picture a drawing or objects. ☐ Understand first and next.

Operations with Numbers

Cluster	2019 Math COS Standard	
Work with numbers 11- 19 to gain foundations for place value.	14. Compose and decompose numbers from 11 to 19 by using concrete objects or drawings to demonstrate understanding that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.K.14.1: Define ones and tens. ☐ M.K.14.2: Match the number in the ones and tens position to a pictorial representation or manipulative of the value. ☐ M.K.14.3: Add numbers 1-9 to ten to create teen numbers using manipulatives or place value blocks. ☐ M.K.14.4: Count objects up to 10.	☐ Notice same/different and some/all. ☐ Recognize numbers from zero to ten. ☐ Add one to a set of objects (up to 10 objects). ☐ Given small groups of objects, create larger groups by combining the small groups. ☐ Subtract one from a set of objects (up to five objects). ☐ Put together two small groups of objects to create a larger group. ☐ Given a group of objects (ten or less), divide the group into smaller groups in various ways. ☐ Take away objects from a large group to create two smaller groups. ☐ Establish one-to-one correspondence between numbers and objects when given a picture a drawing or objects. ☐ Rote count to ten. ☐ Understand number words.

Data Analysis

Cluster	2019 Math COS Standard	
Collect and analyze data and interpret results.	15. Classify objects into given categories of 10 or fewer; count the number of objects in each category and sort the categories by count. a. Categorize data on Venn diagrams, pictographs, and "yes-no" charts using real objects, symbolic representations, or pictorial representations.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.K.15.1: Identify more and less when given two groups of objects. ☐ M.K.15.2: Identify object attributes. Examples: color, shape, size, texture, use. ☐ M.K.15.3: Count objects up to ten. ☐ M.K.15.4: Count to 10 by ones.	☐ Participate in creating charts or graphs to represent data collection. ☐ Notice same/different and some/all. ☐ Recognize numbers from one to ten. ☐ Given a group of objects (ten or less), divide the group into smaller groups in various ways. ☐ Given small groups of objects, create larger groups by combining the small groups. ☐ Take away objects from a large group to create two smaller groups. ☐ Put together two small groups of objects to create a larger group. ☐ Establish one-to-one correspondence between numbers and objects when given a picture a drawing or objects. ☐ Rote count to ten. ☐ Begin to name and match colors, sizes, and shapes. ☐ Enjoy playing with all kinds of objects. ☐ Point to matching or similar objects. ☐ Understand that words can label sameness and differences. ☐ Understand that some have more, and some have less. ☐ Sort objects based on shape or color. ☐ Name and match primary colors.

Measurement

Cluster	2019 Math COS Standard	
Describe and compare measurable attributes.	16. Identify and describe measurable attributes (length, weight, height) of a single object using vocabulary such as long/short, heavy/light, or tall/short.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.K.16.1: Define length and weight. ☐ M.K.16.2: Explore objects in relationship to length and weight.	☐ Notice same/different and some/all. ☐ Begin to name and match colors, sizes, and shapes. ☐ Enjoy playing with all kinds of objects. ☐ Point to matching or similar objects. ☐ Understand that words can label sameness and differences. ☐ Understand that some have more, and some have less. ☐ Sort a variety of objects in a group that have one thing in common. ☐ Recognize and sort familiar objects with the same size. ☐ Have an interest in the order of things. ☐ Understand the concept of smallest and shortest. ☐ Understand the concept of same shape and size. ☐ Understand the concept of smallest and shortest. ☐ Understand the concept of light and heavy. ☐ Understand the concept long and short. ☐ Classify common objects according to height (tall/short). ☐ Classify common objects according to length (long/short). ☐ Classify common objects according to weight (heavy/light).

Measurement

Cluster	2019 Math COS Standard	
Describe and compare measurable attributes.	17. *Directly compare two objects with a measurable attribute in common to see which object has “more of” or “less of” the attribute and describe the difference. Example: Directly compare the heights of two children and describe one child as “taller” or “shorter”.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.K.17.1: Use vocabulary related to length and weight. Example: longer, shorter, heavier, lighter. ☐ M.K.17.2: Identify objects by length and weight. Example: shortest pencil, heaviest rock. ☐ M.K.17.3: Sort objects according to measurable attributes.	☐ Notice same/different and some/all. ☐ Begin to name and match colors, sizes, and shapes. ☐ Enjoy playing with all kinds of objects. ☐ Point to matching or similar objects. ☐ Understand that words can label sameness and differences. ☐ Understand that some have more, and some have less. ☐ Sort objects based on shape or color. ☐ Name and match primary colors. ☐ Sort objects based on both color and shape. ☐ Sort a variety of objects in a group that have one thing in common. ☐ Recognize and sort familiar objects with the same color, shape, or size. ☐ Understand the concept of same shape and size. ☐ Understand the concept of smallest and shortest. ☐ Understand the concept of light and heavy. ☐ Understand the concept long and short. ☐ Classify common objects according to height (tall/short). ☐ Classify common objects according to length (long/short). ☐ Classify common objects according to weight (heavy/light). ☐ Classify common objects according to size (big/small). ☐ Communicate long, short, heavy, light, big, small.

Geometry

Cluster	2019 Math COS Standard	
Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).	18. Describe objects in the environment using names of shapes and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind, and next to.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.K.18.1: Recognize location and position. Examples: above, below, besides, in front of, behind, next to. ☐ M.K.18.2: Identify cubes, cones, cylinders, and spheres. ☐ M.K.18.3: Imitate actions to place items. Examples: in, on, under. ☐ M.K.18.4: Match shapes.	☐ Notice same/different and some/all. ☐ Begin to name and match colors, sizes, and shapes. ☐ Enjoy playing with all kinds of objects. ☐ Point to matching or similar objects. ☐ Understand that words can label sameness and differences. ☐ Understand that some have more, and some have less. ☐ Sort objects based on shape or color. ☐ Name and match primary colors. ☐ Sort objects based on both color and shape. ☐ Sort a variety of objects in a group that have one thing in common. ☐ Recognize and sort familiar objects with the same color, shape, or size. ☐ Understand and point to a triangle, a circle, and a square, rectangle. ☐ Understand the concept of same shape and size. ☐ Have an interest in the order of things. ☐ Understand the concept of smallest and shortest. ☐ Begin to learn positional words.

Geometry

Cluster	2019 Math COS Standard	
Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).	19. Correctly name shapes regardless of their orientations or overall sizes.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.K.19.1: Recognize shapes. ☐ M.K.19.2: Sort shapes with different attributes. Examples: sort different size or color squares, circles, triangles, rectangles, or hexagons.	☐ Notice same/different and some/all. ☐ Begin to name and match colors, sizes, and shapes. ☐ Enjoy playing with all kinds of objects. ☐ Point to matching or similar objects. ☐ Understand that words can label sameness and differences. ☐ Understand that some have more, and some have less. ☐ Sort objects based on shape or color. ☐ Name and match primary colors. ☐ Sort objects based on both color and shape. ☐ Sort a variety of objects in a group that have one thing in common. ☐ Recognize and sort familiar objects with the same color, shape, or size. ☐ Understand and point to a triangle, a circle, a square and rectangle. ☐ Understand the concept of same shape and size.

Geometry

Cluster	2019 Math COS Standard	
Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).	20. Identify shapes as two-dimensional (lying in a plane, “flat”) or three-dimensional (“solid”).	
	Learning Objectives	Prior Knowledge Skills
	☐ M.K.20.1: Define two-dimensional and three-dimensional. Example: two-dimensional shapes are flat, three-dimensional figures are solid. ☐ M.K.20.2: Sort flat and solid objects. ☐ M.K.20.3: Explore two-dimensional shapes and three-dimensional figures.	☐ Notice same/different and some/all. ☐ Begin to name and match sizes and shapes. ☐ Enjoy playing with all kinds of objects. ☐ Point to matching or similar objects. ☐ Understand that words can label sameness and differences. ☐ Sort objects based on shape. ☐ Recognize and sort familiar objects with the same shape or size. ☐ Understand and point to a triangle, a circle, a square and rectangle. ☐ Understand the concept of same shape and size.

Geometry

Cluster	2019 Math COS Standard	
Analyze, compare, create, and compose shapes.	21. Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (number of sides and vertices or “corners”), and other attributes. <i>Example: having sides of equal length.</i>	
	Learning Objectives	Prior Knowledge Skills
	☐ M.K.21.1: Define similar and different. ☐ M.K.21.2: Use vocabulary related to two-dimensional shapes and three-dimensional figures. Examples: vertices (corners), faces (flat surfaces), edges, sides, angles. ☐ M.K.21.3: Recognize vocabulary related to two-dimensional shapes and three-dimensional figures. ☐ M.K.21.4: Identify two-dimensional shapes and three-dimensional figures. ☐ M.K.21.5: Identify shapes.	☐ Notice same/different and some/all. ☐ Begin to name and match sizes and shapes. ☐ Enjoy playing with all kinds of objects. ☐ Point to matching or similar objects. ☐ Understand that words can label sameness and differences. ☐ Sort objects based on shape. ☐ Recognize and sort familiar objects with the same shape or size. ☐ Understand and point to a triangle, a circle, a square and rectangle. ☐ Understand the concept of same shape and size.

Geometry

Cluster	2019 Math COS Standard	
Analyze, compare, create, and compose shapes.	22. Model shapes in the world by building them from sticks, clay balls, or other components and by drawing them.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.K.22.1: Recognize attributes of shapes. ☐ M.K.22.2: Identify cubes, cones, cylinders, and spheres. ☐ M.K.22.3: Identify squares, circles, triangles, rectangles, and hexagons. ☐ M.K.22.4: Identify shapes in the environment. ☐ M.K.22.5: Trace shapes. ☐ M.K.22.6: Make purpose marks such as lines and circles.	☐ Notice same/different and some/all. ☐ Begin to name and match sizes and shapes. ☐ Enjoy playing with all kinds of objects. ☐ Point to matching or similar objects. ☐ Understand that words can label sameness and differences. ☐ Sort objects based on shape. ☐ Recognize and sort familiar objects with the same shape or size. ☐ Understand and point to a triangle, a circle, a square and rectangle. ☐ Understand the concept of same shape and size.

Geometry

Cluster	2019 Math COS Standard	
Analyze, compare, create, and compose shapes.	23. Use simple shapes to compose larger shapes. <i>Example: Join two triangles with full sides touching to make a rectangle.</i>	
	Learning Objectives	Prior Knowledge Skills
	☐ M.K.23.1: Combine shapes to fill the area of a given shape. ☐ M.K.23.2: Decompose pictures made of simple shapes. ☐ M.K.23.3: Match shapes. ☐ M.K.23.4: Match pieces by color, image, or shape to complete a puzzle.	☐ Notice same/different and some/all. ☐ Begin to name and match sizes and shapes. ☐ Enjoy playing with all kinds of objects. ☐ Point to matching or similar objects. ☐ Understand that words can label sameness and differences. ☐ Sort objects based on shape. ☐ Recognize and sort familiar objects with the same shape or size. ☐ Understand and point to a triangle, a circle, a square and rectangle. ☐ Understand the concept of same shape and size.

Grade 1

Operations and Algebraic Thinking

Cluster	2019 Math COS Standard	
Represent and solve problems involving addition and subtraction. <i>Note: Students use properties of operations and different strategies to find the sum of three whole numbers, such as counting on, making tens, decomposing numbers, doubles, and near doubles.</i>	1. *Use addition and subtraction to solve word problems within 20 by using concrete objects, drawings, and equations with a symbol for the unknown number to represent the problem. - Add to with change unknown to solve word problems within 20. - Take from with change unknown to solve word problems within 20. - Put together/take apart with addend unknown to solve word problems within 20. - Compare quantities, with difference unknown, bigger unknown, and smaller unknown while solving word problems within 20.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.1.1.1: Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem. ☐ M.1.1.2: Understand key words in addition and subtraction word problems. Examples: sum, difference, all together, how many more, how many are left, in all. ☐ M.1.1.3: Define subtraction as separating groups of objects, taking from, or taking apart. ☐ M.1.1.4: Define addition as combining groups of objects, adding to, or putting together. ☐ M.1.1.5: Represent addition and subtraction with objects, fingers, mental images, drawings, sounds, acting out situations, verbal explanations, expressions, or equations. ☐ M.1.1.6: Represent numbers with objects or drawings. ☐ M.1.1.7: Use objects to combine and separate groups.	☐ Define how many, all together, and in all. ☐ Count to 20 by ones. ☐ Demonstrate 1:1 correspondence. ☐ Mimic counting objects in sequential order arranged in a line, circle, or array. ☐ Count no more than 5 objects in a scattered configuration. ☐ Mimic counting no more than 5 objects in a scattered configuration. ☐ Count to 10 by ones. ☐ Count in sequential order. ☐ Mimic counting in sequential order. ☐ Demonstrate one to one correspondence. ☐ Make purposeful marks such as lines and circles. ☐ Understand amount words, such as more, less, and another. ☐ Begin to understand that parts of an object can make a whole. ☐ Be interested in more and less. ☐ Understand the concept of “less than”. ☐ Mimic counting by ones. ☐ Recognize numbers from one to ten.

Operations and Algebraic Thinking

Cluster	2019 Math COS Standard	
Represent and solve problems involving addition and subtraction. <i>Note: Students use properties of operations and different strategies to find the sum of three whole numbers, such as counting on, making tens, decomposing numbers, doubles, and near doubles.</i>	2. *Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20 by using concrete objects, drawings, or equations with a symbol for the unknown number to represent the problem.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.1.2.1: Solve addition word problems with sums less than or equal to 10, e.g., by using objects or drawings to represent the problem. ☐ M.1.2.2: Understand key words in addition word problems. Examples: sum, all together, how many more, in all. ☐ M.1.2.3: Define addition as combining groups of objects, adding to, or putting together. ☐ M.1.2.4: Represent addition with objects, fingers, mental images, drawings, sounds, acting out situations, verbal explanations, expressions, or equations. ☐ M.1.2.5: Represent numbers with objects or drawings. ☐ M.1.2.6: Use objects to combine groups.	☐ Understand key words in addition and subtraction word problems. Examples: all together, how many more, how many are left, in all. ☐ Represent numbers with objects or drawings. ☐ Separate sets with nine or fewer objects. ☐ Combine objects to form sets up to nine. ☐ Represent addition and subtraction with objects, pictures, fingers, or sounds within twenty. ☐ Understand addition as putting together and subtraction as taking from. ☐ Establish one-to-one correspondence between numbers and objects. ☐ Rote count to 20. ☐ Notice same/different and some/all. ☐ Establish one-to-one correspondence between numbers and objects. ☐ Point to matching or similar objects.

Operations and Algebraic Thinking

Cluster	2019 Math COS Standard	
Understand and apply properties of operations and the relationship between addition and subtraction. <i>Note: Students need not use formal terms for these properties.</i>	3. Apply properties of operations as strategies to add and subtract. <i>Examples: If $8 + 3 = 11$ is known, then $3 + 8 = 11$ is also known (commutative property of addition). To add $2 + 6 + 4$, the second and third numbers can be added to make a ten, so $2 + 6 + 4 = 2 + 10 = 12$. (Associative property of addition). When adding 0 to a number, the result is the same number (identity property of zero for addition).</i>	
	Learning Objectives	Prior Knowledge Skills
	☐ M.1.3.1: Define addition and subtraction. ☐ M.1.3.2: Recognize properties of operations. ☐ M.1.3.3: Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., $5 = 2 + 3$ and $5 = 4 + 1$). ☐ M.1.3.4: Apply signs $+$, $-$, $=$ to actions of joining and separating sets. ☐ M.1.3.5: Identify fact families to ten. ☐ M.1.3.6: Recognize the value of zero.	☐ Decompose numbers up to 5 using objects or drawings. ☐ Compose numbers up to 5 using objects or drawings. ☐ Count backward from 5. ☐ Count forward to 5. ☐ Write numerals from 0 to 10. ☐ Represent a given numeral 1 to 10 with objects or drawings. ☐ Count forward from a given number 1 to 10. ☐ Model joining sets of objects to total 10. ☐ Identify plus, minus, and equal signs. ☐ Match numerals to objects or drawings. ☐ Identify numerals 1 to 10. ☐ Count 0 to 10.- Add and subtract numbers within 20 using objects, pictures, and fingers. ☐ Understand key words in addition and subtraction word problems. Examples: all together, how many more, how many are left, in all. ☐ Represent numbers with objects or drawings. ☐ Separate sets with nine or fewer objects. ☐ Combine objects to form sets up to nine. ☐ Count items in a set up to twenty. ☐ Establish one-to-one correspondence between numbers and objects. ☐ Using counting, find one less than a number 2 through 20.

Operations and Algebraic Thinking

Cluster	2019 Math COS Standard	
Understand and apply properties of operations and the relationship between addition and subtraction. <i>Note: Students need not use formal terms for these properties.</i>	4. Explain subtraction as an unknown-addend problem. <i>Example: subtracting 10 - 8 by finding the number that makes 10 when added to 8.</i>	
	Learning Objectives	Prior Knowledge Skills
	☐ M.1.4.1: Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., $5 = 2 + 3$ and $5 = 4 + 1$). ☐ M.1.4.2: Identify fact families to ten. ☐ M.1.4.3: Recall basic addition facts to ten. ☐ M.1.4.4: Represent addition and subtraction with objects, fingers, mental images, drawings, sounds, acting out situations, verbal explanations, expressions, or equations.	☐ Decompose numbers up to 5 using objects or drawings. ☐ Compose numbers up to 5 using objects or drawings. ☐ Count backward from 5. ☐ Count forward to 5. ☐ Add and subtract numbers within 20 using objects, pictures, and fingers. ☐ Pair “taking away” with subtraction. ☐ Take a smaller set out of a larger set. ☐ Pair putting together with adding. ☐ Combine two sets to make a larger set up to twenty. ☐ Separate from a larger group to make 2 smaller groups. ☐ Count items in a set up to twenty. ☐ Establish one-to-one correspondence between numbers and objects. ☐ Understand one less than a number 2 through 20. ☐ Understand one more than a number 1 through 20. ☐ Rote count to 20.

Operations and Algebraic Thinking

Cluster	2019 Math COS Standard	
Add and subtract within 20. <i>Note: Fluency involves a mixture of “just knowing” answers, knowing answers from patterns, and knowing answers from the use of strategies. The word fluently is used in the standards to mean accurately, efficiently, and flexibly.</i>	5. Relate counting to addition and subtraction. Example: counting on 2 to add 2.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.1.5.1: Represent addition and subtraction with objects, fingers, mental images, drawings, sounds, acting out situations, verbal explanations, expressions, or equations. ☐ M.1.5.2: Count forward and backward from a given number. ☐ M.1.5.3: Count to 20 by ones.	☐ Mimic counting backwards from 5 to 0 by ones. ☐ Count to 50 by ones. ☐ Count to 50 by tens. ☐ Count to 20 by ones. ☐ Count to 10 by ones. ☐ Mimic counting by tens. ☐ Mimic counting by ones. ☐ Add and subtract numbers within 20 using objects, pictures, and fingers. ☐ Pair “taking away” with subtraction. ☐ Take a smaller set out of a larger set. ☐ Pair putting together with adding. ☐ Combine two sets to make a larger set up to twenty. ☐ Separate from a larger group to make 2 smaller groups. ☐ Count items in a set up to twenty. ☐ Establish one-to-one correspondence between numbers and objects. ☐ Understand one less than a number 2 through 20. ☐ Understand one more than a number 1 through 20. ☐ Rote count to 20.

Operations and Algebraic Thinking

Cluster	2019 Math COS Standard	
Add and subtract within 20. <i>Note: Fluency involves a mixture of “just knowing” answers, knowing answers from patterns, and knowing answers from the use of strategies. The word fluently is used in the standards to mean accurately, efficiently, and flexibly.</i>	6. *Add and subtract within 20. - Demonstrate fluency with addition and subtraction facts with sums or differences to 10 by counting on. - Demonstrate fluency with addition and subtraction facts with sums or differences to 10 by making ten. - Demonstrate fluency with addition and subtraction facts with sums or differences to 10 by decomposing a number leading to a ten. <i>Example: $13 - 4 = 13 - 3 - 1 = 10 - 1 = 9$.</i> - Demonstrate fluency with addition and subtraction facts with sums or differences to 10 by using the relationship between addition and subtraction. <i>Example: Knowing that $8 + 4 = 12$, one knows $12 - 8 = 4$.</i> - Demonstrate fluency with addition and subtraction facts with sums or differences to 10 by creating equivalent but easier or known sums. <i>Example: adding $6 + 7$ by creating the known equivalent $6 + 6 + 1 = 12 + 1 = 13$.</i>	
	Learning Objectives	Prior Knowledge Skills
	☐ M.1.6.1: Decompose numbers less than or equal to 10. ☐ M.1.6.2: Add and subtract within 5. ☐ M.1.6.3: Count forward and backward from a given number. ☐ M.1.6.4: Represent addition and subtraction with objects, fingers, mental images, drawings, sounds, acting out situations, verbal explanations, expressions, or equations.	☐ Identify plus, minus, and equal signs. ☐ Match numerals to objects or drawings. ☐ Identify numerals 0 to 10. ☐ Count 0 to 10. ☐ Understand key words in addition and subtraction word problems. Examples: all together, how many more, how many are left, in all. ☐ Represent numbers with objects or drawings. ☐ Separate sets with nine or fewer objects. ☐ Combine objects to form sets up to nine. ☐ Define addition as combining groups of objects. ☐ Define subtraction as separating groups of objects. ☐ Represent numbers with objects or drawings. ☐ Separate sets with nine or fewer objects.

		☐ Combine objects to form sets up to nine. ☐ Add and subtract numbers within 10 using objects, pictures, and fingers. ☐ Pair “taking away” with subtraction. ☐ Take a smaller set out of a larger set. ☐ Pair putting together with adding. ☐ Combine two sets to make a larger set up to twenty. ☐ Separate from a larger group to make 2 smaller groups. ☐ Count items in a set up to twenty.
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Operations and Algebraic Thinking

Cluster	2019 Math COS Standard	
Work with addition and subtraction equations.	7. *Explain that the equal sign means “the same as.” Determine whether equations involving addition and subtraction are true or false. <i>Example: determining which of the following equations are true and which are false: $6 = 6$, $7 = 8 - 1$, $5 + 2 = 2 + 5$, $4 + 1 = 5 + 2$.</i>	
	Learning Objectives	Prior Knowledge Skills
	☐ M.1.7.1: Define true, false, and equal. ☐ M.1.7.2: Demonstrate equal using manipulatives or object drawings. ☐ M.1.7.3: Recall basic addition facts to ten. ☐ M.1.7.4: Recognize equation symbols in vertical and horizontal addition and subtraction problems. ☐ M.1.7.5: Represent addition and subtraction with objects, fingers, mental images, drawings, sounds, acting out situations, verbal explanations, expressions, or equations.	☐ Identify plus, minus, and equal signs. ☐ Match numerals to objects or drawings. ☐ Identify numerals 1 to 10. ☐ Count 0 to 10. ☐ Add and subtract numbers within 20 using objects, pictures, and fingers. ☐ Understand true, false, same (equal). ☐ Take a smaller set out of a larger set. ☐ Combine two sets to make a larger set up to twenty. ☐ Count items in a set up to twenty. ☐ Establish one-to-one correspondence between numbers and objects. ☐ Using counting, find one less than a number 2 through 20. ☐ Using counting, find one more than a number 1 through 20. ☐ Rote count to 20. ☐ Understand adding numbers up and down is the same as side by side.

Operations and Algebraic Thinking

Cluster	2019 Math COS Standard	
Work with addition and subtraction equations.	8. Solve for the unknown whole number in various positions in an addition or subtraction equation, relating three whole numbers that would make it true. <i>Example: determining the unknown number that makes the equation true in each of the equations $8 + ? = 11$, $5 = ? - 3$, and $6 + 6 = ?$.</i>	
	Learning Objectives	Prior Knowledge Skills
	☐ M.1.8.1: Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., $5 = 2 + 3$ and $5 = 4 + 1$). ☐ M.1.8.2: Identify fact families as a relationship between addition and subtraction. ☐ M.1.8.3: Recall basic addition and subtraction facts to ten. ☐ M.1.8.4: Identify plus, minus, and equal signs. ☐ M.1.8.5: Represent addition and subtraction with objects, fingers, mental images, drawings, sounds, acting out situations, verbal explanations, expressions, or equations.	☐ Add numbers 1-9 to ten to create teen numbers using manipulatives or place value blocks. ☐ Count objects up to 10. ☐ Write numerals from 0 to 10. ☐ Represent a given numeral 1 to 10 with objects or drawings. ☐ Count forward from a given number 1 to 10. ☐ Model joining sets of objects to total 10. ☐ Match numerals to objects or drawings. ☐ Identify numerals 1 to 10. ☐ Count from 0 to 10. ☐ Add and subtract numbers within 20 using objects, pictures, and fingers. ☐ Understand true, false, same (equal). ☐ Take a smaller set out of a larger set. ☐ Combine two sets to make a larger set up to twenty. ☐ Count items in a set up to twenty. ☐ Establish one-to-one correspondence between numbers and objects. ☐ Understand one less than a number 2 through 20. ☐ Understand one more than a number 1 through 20. ☐ Understand positional terms with equal signs.

Operations and Algebraic Thinking

Cluster	2019 Math COS Standard	
Understand simple patterns.	9. Reproduce, extend, and create patterns and sequences of numbers using a variety of materials.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.1.9.1: Duplicate and extend simple patterns by using concrete objects. ☐ M.1.9.2: Identify simple patterns. ☐ M.1.9.3: Mimic simple patterns. ☐ M.1.9.4: Match a simple object.	☐ Define ones and tens. ☐ Match the number in the ones and tens position to a pictorial representation or manipulative of the value. ☐ Add numbers 1-9 to ten to create teen numbers using manipulatives or place value blocks. ☐ Count objects up to 10. ☐ Notice same/different and some/all. ☐ Establish one-to-one correspondence between numbers and objects when given a picture a drawing or objects. ☐ Understand first and next.

Operations with Numbers: Base Ten

Cluster	2019 Math COS Standard	
Extend the counting sequence.	10. *Extend the number sequence from 0 to 120. - Count forward and backward by ones, starting at any number less than 120. - Read numerals from 0 to 120. - Write numerals from 0 to 120. - Represent a number of objects from 0 to 120 with a written numeral.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.1.10.1: Write numerals from 0 to 20. ☐ M.1.10.2: Recognize numerals to 100. ☐ M.1.10.3: Match the numeral to the number objects or picture of objects. ☐ M.1.10.4: Count to 100 by ones. ☐ M.1.10.5: Count to 20 by ones. ☐ M.1.10.6: Identify and name numerals 0-9. ☐ M.1.10.7: Trace numerals 0-9.	☐ Count forward to 100 from a number over 50. ☐ Count forward to 100 from a number between 2 and 50. ☐ Count forward to 50 from a given number. ☐ Count to 100 by ones. ☐ Mimic counting to 100 by ones. ☐ Count to 50 by ones. ☐ Mimic counting to 50 by ones. ☐ Mimic counting backwards from 5 to 0 by ones. ☐ Count to 50 by tens. ☐ Count to 20 by ones. ☐ Count to 10 by ones. ☐ Mimic counting by tens. ☐ Mimic counting by ones. ☐ Count to 50 and above. ☐ Mimic counting forward and backward by ones. ☐ Recognize numbers from 1-50. ☐ Become interested in how many objects she/he has. ☐ Continue to have an interest in counting. ☐ Understand the concept of size and amount. ☐ Pair the number of objects counted with "how many". ☐ Understand that the last number name tells the number of objects counted. ☐ Pair a group of objects with a number representing the total number of objects in the group (up to ten objects).

*Critical Standard

		☐ Count objects one-by-one using only one number per object (up to ten objects). ☐ Recognize that numbers have meaning. ☐ Rote count to 50. ☐ Communicate number words. ☐ Recognize before and after. ☐ Trace numerals 1- 20.
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Operations with Numbers: Base Ten

Cluster	2019 Math COS Standard	
Understand place value.	11. *Explain that the two digits of a two-digit number represent amounts of tens and ones. - Identify a bundle of ten ones as a “ten”. - Identify the numbers from 11 to 19 as composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones. - Identify the numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 as one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).	
	Learning Objectives	Prior Knowledge Skills
	☐ M.1.11.1: Match the number in the ones and tens position to a pictorial representation or manipulative of the value. ☐ M.1.11.2: Represent numbers with multiple models. Examples: models, base ten blocks, number lines, linking cubes, straw bundles. ☐ M.1.11.3: Count to 100 by tens. ☐ M.1.11.4: Count 10 objects. ☐ M.1.11.5: Count to 10 by ones. ☐ M.1.11.6: Name numerals 0 to 19.	☐ Define ones and tens. ☐ Match the number in the ones and tens position to a pictorial representation or manipulative of the value. ☐ Add numbers 1-9 to ten to create teen numbers using manipulatives or place value blocks. ☐ Count objects up to 10. ☐ Notice same/different and some/all. ☐ Recognize numbers from 1-50. ☐ Add one to a set of objects (up to 10 objects). ☐ Given small groups of objects, create larger groups by combining the small groups. ☐ Understand ten and 1 (ten 1's =10). ☐ Put together two small groups of objects to create a larger group. ☐ Understand number words. ☐ Establish one-to-one correspondence between numbers and objects when given a picture a drawing or objects. ☐ Rote count to 10.

Operations with Numbers: Base Ten

Cluster	2019 Math COS Standard	
Understand place value.	12. *Compare pairs of two-digit numbers based on the values of the tens and ones digits, recording the results of comparisons with the symbols $>$, $=$, and $<$ and orally with the words “is greater than,” “is equal to,” and “is less than”.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.1.12.1: Define greater than, less than and equal to. ☐ M.1.12.2: Demonstrate greater than, less than, and equal to using manipulatives, object drawings or numbers 0 to 10. ☐ M.1.12.3: Use comparison vocabulary. Examples: greater than, equal to, and less than. ☐ M.1.12.4: Recognize symbols for greater than, less than and equal to. ☐ M.1.12.5: Determine the value of the digits in the ones and tens place. ☐ M.1.12.6: Identify sets with more, less, or equal objects. ☐ M.1.12.7: Imitate creating sets of a given size.	☐ Define ones and tens. ☐ Match the number in the ones and tens position to a pictorial representation or manipulative of the value. ☐ Add numbers 1-9 to ten to create teen numbers using manipulatives or place value blocks. ☐ Count objects up to 10. ☐ Define greater than, less than, and equal to. ☐ Count to 20 by ones. ☐ Count objects up to ten. ☐ Understand amount words, such as more, less, and another. ☐ Begin to understand that parts of an object can make a whole. ☐ Become more interested in the concept of some and all. ☐ Be interested in who has more or less. ☐ Understand the concept of “less than.” ☐ Mimic counting by ones. ☐ Recognize numbers from one to ten. ☐ Become interested in how many objects she/he has. ☐ Understand the concept of size and amount. ☐ Given a set number of objects one through ten, answer the question “How many?” ☐ Pair the number of objects counted with “How many?” ☐ Understand that the last number name tells the number of objects counted.

Operations with Numbers: Base Ten

Cluster	2019 Math COS Standard	
Use place value understanding and properties of operations to add and subtract.	13. *Add within 100, using concrete models or drawings and strategies based on place value. - Add a two-digit number and a one-digit number. - Add a two-digit number and a multiple of 10. - Demonstrate that in adding two-digit numbers, tens are added to tens, ones are added to ones, and sometimes it is necessary to compose a ten. - Relate the strategy for adding a two-digit number and a one-digit number to a written method and explain the reasoning used.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.1.13.1: Demonstrate regrouping, total sum, and solve using drawings and concrete models. ☐ M.1.13.2: Model written method for recording horizontal addition problems. ☐ M.1.13.3: Determine the value of the number in the ones and tens place. ☐ M.1.13.4: Match the number in the ones, tens, and hundreds position to a pictorial representation or manipulative of the value. ☐ M.1.13.5: Represent numbers with multiple models. Examples: models, base ten blocks, number lines, linking cubes, straw bundles. ☐ M.1.13.6: Recall single-digit addition facts.	☐ Define ones and tens. ☐ Match the number in the ones and tens position to a pictorial representation or manipulative of the value. ☐ Add numbers 0-9 to ten to create teen numbers using manipulatives or place value blocks. ☐ Count objects up to 10. ☐ Recognize numbers from 0-10. ☐ Become interested in how many objects she/he has. ☐ Understand the concept of size and amount. ☐ Given a set number of objects one through ten, answer the question "How many?". ☐ Pair the number of objects counted with "how many?". ☐ Understand that the last number name tells the number of objects counted. ☐ Understand that 10 1's = 10. ☐ Establish one-to-one correspondence between numbers and objects when given a picture, a drawing, or objects. ☐ Pair a group of objects with a number representing the total number of objects in the group. ☐ Count objects one-by-one using only one number per object. ☐ Recognize that numbers and numerals have meaning. ☐ Rote count to 10. ☐ Communicate number words.

Operations with Numbers: Base Ten

Cluster	2019 Math COS Standard	
Use place value understanding and properties of operations to add and subtract.	14. *Given a two-digit number, mentally find 10 more or 10 less than the number without having to count, and explain the reasoning used.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.1.14.1: Define more and less. ☐ M.1.14.2: Demonstrate conceptual understanding of adding or subtracting 10 using concrete models. ☐ M.1.14.3: Count backward from 100 by tens. ☐ M.1.14.4: Count forward to 100 by tens. ☐ M.1.14.5: Count to 100 by ones.	☐ Define ones and tens. ☐ Match the number in the ones and tens position to a pictorial representation or manipulative of the value. ☐ Add numbers 1-9 to ten to create teen numbers using manipulatives or place value blocks. ☐ Count objects up to 10. ☐ Count to 50 and above. ☐ Mimic counting by ones. ☐ Recognize numbers from 1-50. ☐ Understand the concept of amount. ☐ Pair the number of objects counted with “how many?”. ☐ Understand that the last number name tells the number of objects counted. ☐ Pair a group of objects with a number representing the total number of objects in the group (up to ten objects). ☐ Count objects one-by-one using only one number per object (up to ten objects). ☐ Recognize that numbers have meaning. ☐ Recognize numbers 1-10. ☐ Rote count to 50. ☐ Communicate number words. ☐ Add one to a set of objects (up to 10 objects).

Operations with Numbers: Base Ten

Cluster	2019 Math COS Standard	
Use place value understanding and properties of operations to add and subtract.	15. *Subtract multiples of 10 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. Relate the strategy to a written method and explain the reasoning used.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.1.15.1: Demonstrate conceptual understanding of subtraction using concrete models. ☐ M.1.15.2: Model written method for recording problems involving subtraction of 10 from multiples of 10. ☐ M.1.15.3: Count backward from 100 by tens. ☐ M.1.15.4: Count forward to 100 by tens. ☐ M.1.15.5: Mimic counting to 100 by tens.	☐ Define subtraction as separating groups of objects. ☐ Represent numbers with objects or drawings. ☐ Separate sets with nine or fewer objects. ☐ Combine objects to form sets up to nine. ☐ Notice same/different and some/all. ☐ Subtract one from a set of objects (up to five objects). ☐ Given a group of objects (ten or less), divide the group into smaller groups in various ways. ☐ Take away objects from a large group to create two smaller groups. ☐ Establish one-to-one correspondence between numbers and objects when given a picture a drawing or objects. ☐ Understand number words. ☐ Understand that 10 1's = 10. ☐ Count forward to 50 by tens. ☐ Count backwards from 50 by tens. ☐ Mimic counting to 50 by tens.

Data Analysis

Cluster	2019 Math COS Standard	
Collect and analyze data and interpret results.	16. Organize, represent, and interpret data with up to three categories. - Ask and answer questions about the total number of data points in organized data. - Summarize data on Venn diagrams, pictographs, and "yes-no" charts using real objects, symbolic representations, or pictorial representations. - Determine "how many" in each category using up to three categories of data. - Determine "how many more" or "how many less" are in one category than in another using data organized into two or three categories.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.1.16.1: Define more and less. ☐ M.1.16.2: Describe methods for representing data. Examples: pictographs, tally charts, bar graphs, and Venn Diagrams. ☐ M.1.16.3: Locate information on data displays. ☐ M.1.16.4: Classify objects into given categories; count the number of objects in each category and sort the categories by count. ☐ M.1.16.5: Recognize different types of data displays.	☐ Identify more and less when given two groups of objects. ☐ Identify object attributes. Examples: color, shape, size, texture, use. ☐ Count objects up to ten. ☐ Count to 10 by ones. ☐ Understand a different type of graphs (ex. Venn diagram, bar graphs and pictograph). ☐ Identify more and less when given two groups of objects of 10 or fewer. ☐ Count objects up to 10. ☐ Count to 10 by ones. ☐ Understand categories. ☐ Identify object attributes. Examples: color, shape, size, texture, purpose. ☐ Count to 1-20. ☐ Mimic counting by ones. ☐ Recognize numerals from 0-20. ☐ Understand the concept of amount. ☐ Pair the number of objects counted with "how many?". ☐ Understand that the last number name tells the number of objects counted. ☐ Pair a group of objects with a number representing the total number of objects in the group (up to ten objects).

Measurement

Cluster	2019 Math COS Standard	
Describe and compare measurable attributes.	17. Order three objects by length; compare the lengths of two objects indirectly by using a third object.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.1.17.1: Define length. ☐ M.1.17.2: Use vocabulary related to length. Examples: longer, shorter, longest, shortest, taller. ☐ M.1.17.3: Identify objects by length. Examples: shortest pencil, tallest boy. ☐ M.1.17.4: Sort objects according to length. Example: sort short pencils from long pencils. ☐ M.1.17.5: Explore objects in relationship to length.	☐ Use vocabulary related to length and weight. Example: longer, shorter, heavier, lighter. ☐ Identify objects by length and weight. Example: shortest pencil, heaviest rock. ☐ Sort objects according to measurable attributes. ☐ Define length and weight. ☐ Explore objects in relationship to length and weight. ☐ Define more, less, length. ☐ Use vocabulary related to length. Examples: longer, shorter. ☐ Identify objects by length. Examples: shortest pencil, heaviest rock. ☐ Sort objects according to measurable attributes. ☐ Use comparative language (longer/shorter, taller/shorter) for the attributes of objects related to length. ☐ Communicate long, tall, short. ☐ Recognize the length attributes of objects (long/short, tall/short). ☐ Recognize length as the measurement of something from end to end.

Measurement

Cluster	2019 Math COS Standard	
Describe and compare measurable attributes.	18. *Determine the length of an object using non-standard units with no gaps or overlaps, expressing the length of the object with a whole number.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.1.18.1: Describe gap and overlap. ☐ M.1.18.2: Describe what it means to measure using non-standard units. ☐ M.1.18.3: Model measuring using non-standard units.	☐ Use vocabulary related to length and weight. Example: longer, shorter, heavier, lighter. ☐ Identify objects by length and weight. Example: shortest pencil, heaviest rock. ☐ Sort objects according to measurable attributes. ☐ Define length and weight. ☐ Explore objects in relationship to length and weight. ☐ Define more, less, length, width, weight, and height. ☐ Use vocabulary related to length, width, weight, and height. Examples: longer, shorter, heavier, lighter, small, big. ☐ Identify objects by length, weight, and height. - Examples: shortest pencil, heaviest rock. ☐ Sort objects according to non-measurable attributes. ☐ Understanding concepts of small, big, heavy, light, tall, short. ☐ Understand concept of too much or too little.

Measurement

Cluster	2019 Math COS Standard	
Work with time and money.	19. Tell and write time to the hours and half hours using analog and digital clocks.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.1.19.1: Describe the shorthand as the hour hand and the long hand as the minute hand on an analog clock. ☐ M.1.19.2: Describe the first number as the hour, and the numbers after the colon as the minutes on a digital clock. ☐ M.1.19.3: Count to 30 by fives. ☐ M.1.19.4: Recognize numbers 1 to 12, and 30. ☐ M.1.19.5: Trace numerals 1 to 12, and 30. ☐ M.1.19.6: Associate digital and analog clocks with the measurement of time.	☐ Identify numbers 1 to 12. ☐ Count by 5s. ☐ Identify activities on a daily schedule that come before, next, after other activities. ☐ Know before, next and after. ☐ Use a daily schedule containing times (in hours) and activities (in pictures). ☐ Tell time in hours on an analog clock. ☐ Demonstrate an understanding of yesterday, today, tomorrow, morning, afternoon, day, and night. ☐ Recognize yesterday, today, tomorrow. ☐ Recognize morning, afternoon, evening/night. ☐ Recognize day and night. ☐ Understand the concept of time.

Measurement

Cluster	2019 Math COS Standard	
Work with time and money.	20. Identify pennies and dimes by name and value.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.1.20.1: Identify that a penny has a value of one cent and demonstrate that 10 pennies have the same value as 1 dime. ☐ M.1.20.2: Recognize the value of 1 and 10. ☐ M.1.20.3: Sort pennies and dimes. ☐ M.1.20.4: Count 10 objects.	☐ Count to 1-10. ☐ Understand the concept of amount. ☐ Pair the number of objects counted with “how many?”. ☐ Understand that the last number name tells the number of objects counted. ☐ Pair a group of objects with a number representing the total number of objects in the group. ☐ Count objects one-by-one using only one number per object. ☐ Recognize that numbers have meaning. ☐ Recognize numerals 1-10. ☐ Communicate number words. ☐ Point to matching or similar objects. ☐ Identify a penny, dime by attributes (color, size). ☐ Recognize a penny as 1 cent. ☐ Differentiate coins from other objects.

Geometry

Cluster	2019 Math COS Standard	
Reason with shapes and their attributes. <i>Note: Students do not need to learn formal names such as “right rectangular prism.”</i>	21. Build and draw shapes which have defining attributes. a. Distinguish between defining attributes and non-defining attributes. <i>Examples: Triangles are closed and three- sided, which are defining attributes; color, orientation, and overall size are non-defining attributes.</i>	
	Learning Objectives	Prior Knowledge Skills
	☐ M.1.21.1: Define side, angle, closed and open. ☐ M.1.21.2: Describe attributes of shapes. <i>Examples: number of sides, number of angles.</i> ☐ M.1.21.3: Identify two-dimensional shapes. ☐ M.1.21.4: Sort two-dimensional shapes. ☐ M.1.21.5: Identify basic attributes. <i>Examples: color, shape, size.</i>	☐ Recognize attributes of shapes. ☐ Identify cubes, cones, cylinders, and spheres. ☐ Identify squares, circles, triangles, rectangles, and hexagons. ☐ Identify shapes in the environment. ☐ Trace shapes. ☐ Make purpose marks such as lines and circles. ☐ Notice same/different and some/all. ☐ Begin to name and match sizes and shapes. ☐ Enjoy playing with all kinds of objects. ☐ Point to matching or similar objects. ☐ Understand that words can label sameness and differences. ☐ Sort objects based on shape or color. ☐ Understand and point to a triangle, a circle, a square and rectangle. ☐ Understand the concept of same shape and size. ☐ Understand that some have more, and some have less. ☐ Name and match primary colors. ☐ Sort objects based on both color and shape. ☐ Sort a variety of objects in a group that have one thing in common. ☐ Recognize and sort familiar objects with the same color, shape, or size.

Geometry

Cluster	2019 Math COS Standard	
Reason with shapes and their attributes. <i>Note: Students do not need to learn formal names such as “right rectangular prism.”</i>	22. *Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape and compose new shapes from the composite shape.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.1.22.1: Combine shapes to fill in the area of a given shape. ☐ M.1.22.2: Replicate composite shapes. ☐ M.1.22.3: Decompose pictures made of simple shapes. ☐ M.1.22.4: Name shapes. Examples: square, circle, triangle, rectangle, and hexagon. ☐ M.1.22.5: Recognize shapes.	☐ Combine shapes to fill the area of a given shape. ☐ Decompose pictures made of simple shapes. ☐ Match shapes. ☐ Match pieces by color, image, or shape to complete a puzzle. ☐ Define similar and different. ☐ Use vocabulary related to two-dimensional shapes and three-dimensional figures. Examples: vertices (corners), faces (flat surfaces), edges, sides, angles. ☐ Recognize vocabulary related to two-dimensional shapes and three-dimensional figures. ☐ Identify two-dimensional shapes and three-dimensional figures. ☐ Identify shapes. ☐ Notice same/different and some/all. ☐ Begin to name and match sizes and shapes. ☐ Enjoy playing with all kinds of objects. ☐ Point to matching or similar objects. ☐ Understand that words can label sameness and differences. ☐ Sort objects based on shape. ☐ Recognize and sort familiar objects with the same shape or size. ☐ Understand and point to a triangle, a circle, a square and rectangle.

Geometry

Cluster	2019 Math COS Standard	
Reason with shapes and their attributes. <i>Note: Students do not need to learn formal names such as “right rectangular prism.”.</i>	23. Partition circles and rectangles into two and four equal shares and describe the shares using the words halves, fourths, and quarters, and use the phrases half of, fourth of, and quarter of. a. Describe “the whole” as two of or four of the shares of circles and rectangles partitioned into two or four equal shares. b. Explain that decomposing into more equal shares creates smaller shares of circles and rectangles.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.1.23.1: Define halves, fourths, quarters, whole, parts (shares) and equal. ☐ M.1.23.2: Demonstrate sharing situations to show equal smaller shares. ☐ M.1.23.3: Distinguish between equal and non-equal parts. ☐ M.1.23.4: Decompose pictures made of simple shapes. ☐ M.1.23.5: Identify squares, circles, triangles, and rectangles.	☐ Combine shapes to fill the area of a given shape. ☐ Decompose pictures made of simple shapes. ☐ Match shapes. ☐ Match pieces by color, image, or shape to complete a puzzle. ☐ Define similar and different. ☐ Use vocabulary related to two-dimensional shapes and three-dimensional figures. Examples: vertices (corners), faces (flat surfaces), edges, sides, angles. ☐ Recognize vocabulary related to two-dimensional shapes and three-dimensional figures. ☐ Identify two-dimensional shapes and three-dimensional figures. ☐ Identify shapes. ☐ Notice same/different and some/all. ☐ Begin to name and match sizes and shapes. ☐ Enjoy playing with all kinds of objects. ☐ Point to matching or similar objects. ☐ Understand that words can label sameness and differences. ☐ Sort objects based on shape. ☐ Recognize and sort familiar objects with the same shape or size.

Grade 2

Operations and Algebraic Thinking

Cluster	2019 Math COS Standard	
Represent and solve problems involving addition and subtraction. <i>Note: Second grade problem types include adding to, taking from, putting together, taking apart, and comparing with unknowns in all positions.</i>	1. *Use addition and subtraction within 100 to solve one- and two-step word problems by using drawings and equations with a symbol for the unknown number to represent the problem.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.2.1.1: Solve one-step addition and subtraction word problems with an unknown by using drawings and equations with a symbol for the unknown number to represent the problem. ☐ M.2.1.2: Understand key words in addition and subtraction word problems. Examples: adding to, taking from, putting together, taking apart, sum, difference, all together, how many more, how many are left, in all. ☐ M.2.1.3: Locate the unknown regardless of position. Examples: start unknown, change unknown, and result unknown. ☐ M.2.1.4: Apply signs +, -, = to actions of joining and separating sets. ☐ M.2.1.5: Add and subtract within 50, e.g., by using objects or drawings to represent the problem. ☐ M.2.1.6: Solve addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by	☐ Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem. ☐ Understand key words in addition and subtraction word problems. Examples: sum, difference, all together, how many more, how many are left, in all. ☐ Define subtraction as separating groups of objects, taking from, or taking apart. ☐ Define addition as combining groups of objects, adding to, or putting together. ☐ Represent addition and subtraction with objects, fingers, mental images, drawings, sounds, acting out situations, verbal explanations, expressions, or equations. ☐ Represent numbers with objects or drawings. ☐ Use objects to combine and separate groups. ☐ Define how many, all together, and in all. ☐ Count to 20 by ones. ☐ Demonstrate 1:1 correspondence. ☐ Mimic counting objects in sequential order arranged in a line, circle, or array. ☐ Count no more than 5 objects in a scattered configuration. ☐ Mimic counting no more than 5 objects in a scattered configuration. ☐ Count to 10 by ones. ☐ Count in sequential order. ☐ Mimic counting in sequential order.

	using objects or drawings to represent the problem. ☐ M.2.1.7: Represent addition and subtraction with objects, fingers, mental images, drawings, sounds, acting out situations, verbal explanations, expressions, or equations. ☐ M.2.1.8: Represent numbers with objects or drawings.	☐ Demonstrate one to one correspondence. ☐ Make purposeful marks such as lines and circles.
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Operations and Algebraic Thinking

Cluster	2019 Math COS Standard	
Add and subtract within 20.	2. *Fluently add and subtract within 20 using mental strategies such as counting on, making ten, decomposing a number leading to ten, using the relationship between addition and subtraction, and creating equivalent but easier or known sums. a. State automatically all sums of two one-digit numbers.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.2.2.1: Recall single – digit subtraction facts with minuends of 10 or less. ☐ M.2.2.2: Recall single – digit addition facts with sums up to 10. ☐ M.2.2.3: Apply addition and subtraction strategies. Examples: doubles, doubles plus one, doubles minus one. ☐ M.2.2.4: Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.	○ Define addition and subtraction. ○ Recognize properties of operations. ○ Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., $5 = 2 + 3$ and $5 = 4 + 1$). ○ Apply signs +, -, = to actions of joining and separating sets. ○ Identify fact families to ten. ○ Recognize the value of zero. ○ Decompose numbers up to 5 using objects or drawings. ○ Compose numbers up to 5 using objects or drawings. ○ Count backward from 5. ○ Count forward to 5. ○ Write numerals from 0 to 10. ○ Represent a given numeral 1 to 10 with objects or drawings. ○ Count forward from a given number 1 to 10. ○ Model joining sets of objects to total 10. ○ Identify plus, minus, and equal signs. ○ Match numerals to objects or drawings. ○ Identify numerals 1 to 10. ○ Count 0 to 10. ○ Add and subtract numbers within 20 using objects, pictures, and fingers.

Operations and Algebraic Thinking

Cluster	2019 Math COS Standard	
Work with equal groups of objects to gain foundations for multiplication.	3. Use concrete objects to determine whether a group of up to 20 objects is even or odd. a. Write an equation to express an even number as a sum of two equal addends.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.2.3.1: Define pair, odd and even. ☐ M.2.3.2: Recall doubles addition facts with sums to 20. ☐ M.2.3.3: Apply signs + and = to actions of joining sets. ☐ M.2.3.4: Model written method for composing equations. ☐ M.2.3.5: Skip count by 2s.	☐ Mimic skip counting by 2s. ☐ Recognize numbers from 1-10. ☐ Become interested in how many objects she/he has. ☐ Understand the concept of size and amount. ☐ Given a set number of objects one through ten, answer the question "How many?". ☐ Pair the number of objects counted with "how many". ☐ Understand that the last number name tells the number of objects counted. ☐ Understand that 10 1's = 10. ☐ Establish one-to-one correspondence between numbers and objects when given a picture, a drawing, or objects. ☐ Pair a group of objects with a number representing the total number of objects in the group. ☐ Count objects one-by-one using only one number per object. ☐ Recognize that numbers and numerals have meaning. ☐ Rote count to 10. ☐ Identify plus, minus, and equal signs. ☐ Match numerals to objects or drawings. ☐ Communicate number words. ☐ Recognize after. ☐ Recognize before.

Operations and Algebraic Thinking

Cluster	2019 Math COS Standard	
Work with equal groups of objects to gain foundations for multiplication.	4. *Using concrete and pictorial representations and repeated addition, determine the total number of objects in a rectangular array with up to 5 rows and up to 5 columns. a. Write an equation to express the total number of objects in a rectangular array with up to 5 rows and up to 5 columns as a sum of equal addends.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.2.4.1: Distinguish between rows and columns. ☐ M.2.4.2: Use repeated addition to solve problems with multiple addends. ☐ M.2.4.3: Count forward in multiples from a given number. Examples: 3, 6, 9, 12; 4, 8, 12, 16. ☐ M.2.4.4: Recall doubles addition facts. ☐ M.2.4.5: Model written method for composing equations.	☐ Give two sets of objects repeatedly from a larger group to represent multiples. ☐ Establish one-to-one correspondence between numbers and objects. ☐ Identify the = sign as equal. ☐ Pair same and equal. ☐ Know same when comparing numbers of objects. ☐ Recognize cue words for plus (add, plus, combine). ☐ Identify the + sign as plus. ☐ Use manipulatives and counting, recognize and represent the number 20 as two sets of ten. ☐ Use manipulatives and counting, recognize and represent the numbers 1 through 40. ☐ Establish one-to-one correspondence between numbers and objects. ☐ Rote count to forty.

Operations and Algebraic Thinking

Cluster	2019 Math COS Standard	
Understand simple patterns.	5. Reproduce, extend, create, and describe patterns and sequences using a variety of materials.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.2.5.1: Describe a pattern of colors, shapes, and/or numbers using a variety of materials. ☐ M.2.5.2: Create a pattern of colors, shapes, and/or numbers using a variety of materials. ☐ M.2.5.3: Extend a pattern of colors, shapes, and/or numbers. ☐ M.2.5.4: Mimic a simple pattern of colors, shapes, and/or numbers. ☐ M.2.5.5: Match a simple pattern of colors, shapes, and/or numbers.	☐ Notice same/different and some/all. ☐ Establish one-to-one correspondence between numbers and objects when given a picture, a drawing, or objects. ☐ Understand first and next. ☐ Sort objects based on shape. ☐ Recognize and sort familiar objects with the same shape or size. ☐ Understand and point to a triangle, a circle, a square and rectangle. ☐ Understand the concept of same shape and size. ☐ Mimic a pattern presented.

Operations with Numbers: Base Ten

Cluster	2019 Math COS Standard	
Understand place value.	6. *Explain that the three digits of a three-digit number represent amounts of hundreds, tens, and ones. a. Explain the following three-digit numbers as special cases: 100 can be thought of as a bundle of ten tens, called a “hundred,” and the numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).	
	Learning Objectives	Prior Knowledge Skills
	☐ M.2.6.1: Match the number in the ones, tens, and hundreds position to a pictorial representation or manipulative of the value. ☐ M.2.6.2: Represent numbers with multiple concrete models. Examples: concrete models – base ten blocks, number lines, linking cubes, straw bundles. ☐ M.2.6.3: Count to 1000 by hundreds. ☐ M.2.6.4: Count to 100 by tens. ☐ M.2.6.5: Create groups of 10. ☐ M.2.6.6: Match the numeral in the ones and tens position to a pictorial representation or manipulative of the value. ☐ M.2.6.7: Match the numeral to the number of objects or picture of objects.	☐ Notice same/different and some/all. ☐ Recognize numbers from 1-50. ☐ Add one to a set of objects (up to 10 objects). ☐ Given small groups of objects, create larger groups by combining the small groups. ☐ Understand ten and 1 (ten 1's =10). ☐ Put together two small groups of objects to create a larger group. ☐ Understand number words. ☐ Establish one-to-one correspondence between numbers and objects when given a picture a drawing or objects. ☐ Rote count to 50 by tens. ☐ Rote count to 500 by hundreds. ☐ Mimic counting to 100 by tens. ☐ Mimic counting to 900 by hundreds.

Operations with Numbers: Base Ten

Cluster	2019 Math COS Standard	
Understand place value.	7. Count within 1000 by ones, fives, tens, and hundreds.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.2.7.1: Create a number pattern. ☐ M.2.7.2: Count backward from 100 by fives and tens. ☐ M.2.7.3: Count forward to 100 by fives and tens. ☐ M.2.7.4: Count to 100 by ones.	☐ Add and subtract numbers 0–30 using numbers. ☐ Recognize numbers 0–30 as representing quantities. ☐ Add and subtract numbers 0–30 using pictures. ☐ Add and subtract numbers 0–30 using objects. ☐ Know mathematical symbols for add and subtract. ☐ Given a set of objects, find the total number of objects when a given set is removed. ☐ Given a set of objects, find the total number of objects when another set is combined with the original set. ☐ Given two sets of objects (less than ten objects each), count the total number of objects. ☐ Count objects to thirty. ☐ Establish one-to-one correspondence between numbers and objects. ☐ Identify the = sign as equal. ☐ Pair same and equal. ☐ Know same when comparing numbers of objects. ☐ Recognize cue words for minus (subtract, take away, separate). ☐ Identify the – sign as minus. ☐ Recognize cue words for plus (add, plus, combine). ☐ Identify the + sign as plus.

Operations with Numbers: Base Ten

Cluster	2019 Math COS Standard	
Understand place value.	8. Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.2.8.1: Identify zero as a place holder in two-digit and three-digit numbers. ☐ M.2.8.2: Match the number in the ones, tens, and hundreds position to a pictorial representation or manipulative of the value. ☐ M.2.8.3: Identify the value of number in the ones, tens, and hundreds place. ☐ M.2.8.4: Identify place value for ones, tens, and hundreds. ☐ M.2.8.5: Read number names one through one hundred. ☐ M.2.8.6: Write numerals 1 to 100. ☐ M.2.8.7: Recognize number names one through twenty. ☐ M.2.8.8: Trace numerals 0 to 100.	☐ Notice same/different and some/all. ☐ Recognize numbers from 1-100. ☐ Add one to a set of objects (up to 10 objects). ☐ Given small groups of objects, create larger groups by combining the small groups. ☐ Understand ten and 1 (ten 1's =10). ☐ Understand that 10 (tens) = 100. ☐ Put together two small groups of objects to create a larger group. ☐ Understand number words. ☐ Establish one-to-one correspondence between numbers and objects when given a picture a drawing or objects. ☐ Rote count to 10. ☐ Understand the value of 0 (zero). ☐ Write numbers 1-100. ☐ Understand number words 1-100. ☐ Trace numbers 1-100.

Operations with Numbers: Base Ten

Cluster	2019 Math COS Standard	
Understand place value.	9. Compare two three-digit numbers based on the value of the hundreds, tens, and ones digits, recording the results of comparisons with the symbols $>$, $=$, and $<$ and orally with the words “is greater than,” “is equal to,” and “is less than.”	
	Learning Objectives	Prior Knowledge Skills
	☐ M.2.9.1: Define greater than, less than and equal to. ☐ M.2.9.2: Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols $>$, $=$, and $<$. ☐ M.2.9.3: Arrange two – digit numbers in order from greatest to least or least to greatest. ☐ M.2.9.4: Identify zero as a place holder in two – digit and three – digit numbers. ☐ M.2.9.5: Model using $>$, $=$, and $<$ symbols to record the results of comparisons of two two-digit numbers. ☐ M.2.9.6: Select numbers on a number line that are more than, less than or equal to a specified number. ☐ M.2.9.7: Match the words greater than, equal to and less than to the symbols $>$, $=$, and $<$. ☐ M.2.9.8: Determine the value of the digits in the ones and tens place. ☐ M.2.9.9: Identify sets with more, less, or equal objects.	☐ Understand amount words, such as more, less, and another. ☐ Become more interested in the concept of some and all. ☐ Be interested in who has more or less. ☐ Understand the concept of “less than” “more than”. ☐ Mimic counting by ones. ☐ Recognize numbers from one to 100. ☐ Become interested in how many objects she/he has. ☐ Understand the concept of size and amount. ☐ Given a set number of objects one through ten, answer the question “How many?”. ☐ Pair the number of objects counted with “how many”. ☐ Understand that the last number name tells the number of objects counted. ☐ Establish one-to-one correspondence between numbers and objects when given a picture, a drawing, or objects. ☐ Pair a group of objects with a number representing the total number of objects in the group. ☐ Count objects one-by-one using only one number per object.

Operations with Numbers: Base Ten

Cluster	2019 Math COS Standard	
Use place value understanding and properties of operations to add and subtract.	10. *Fluently add and subtract within 100, using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.2.10.1: Add within 100, including adding a two-digit number and a one-digit number and adding two two-digit numbers, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method, and explain the reasoning used. ☐ M.2.10.2: Add within 20, demonstrating fluency for addition within 10. Use strategies such as counting on; making ten; decomposing a number leading to a ten; and creating equivalent but easier or known sums. ☐ M.2.10.3: Determine the value of the number in the ones, tens, and hundreds place. ☐ M.2.10.4: Model written method for recording horizontal and vertical addition problems. ☐ M.2.10.5: Understand that the two digits of a two-digit number represent amounts of tens and ones. ☐ M.2.10.6: Match the number in the ones and tens position to a pictorial representation or manipulative of the value.	☐ Notice same/different and some/all. ☐ Subtract one from a set of objects (up to 10 objects). ☐ Given a group of objects (20 or less), divide the group into smaller groups in various ways. ☐ Take away objects from a large group to create two smaller groups. ☐ Establish one-to-one correspondence between numbers and objects when given a picture a drawing or objects. ☐ Understand number words. ☐ Understand that 10 1's = 10. ☐ Count forward to 100 by tens. ☐ Count backwards from 100 by tens. ☐ Mimic counting to 100 by tens. ☐ Recognize numbers from 1-100. ☐ Become interested in how many objects she/he has. ☐ Understand the concept of size and amount. ☐ Given a set number of objects one through ten, answer the question "How many?". ☐ Pair the number of objects counted with "how many".

Operations with Numbers: Base Ten

Cluster	2019 Math COS Standard	
Use place value understanding and properties of operations to add and subtract.	11. Use a variety of strategies to add up to four two-digit numbers.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.2.11.1: Add within 100, including adding a two-digit number and a one-digit number and adding two two-digit numbers, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method, and explain the reasoning used. ☐ M.2.11.2: Add within 20, demonstrating fluency for addition within 10. Use strategies such as counting on; making ten; decomposing a number leading to a ten; and creating equivalent but easier or known sums. ☐ M.2.11.3: Determine the value of the number in the ones, tens and hundreds place. ☐ M.2.11.4: Model written method for recording horizontal and vertical addition problems. ☐ M.2.11.5: Understand that the two digits of a two-digit number represent amounts of tens and ones. ☐ M.2.11.6: Match the number in the ones and tens position to a pictorial representation or manipulative of the value.	☐ Subtract one from a set of objects (up to 10 objects). ☐ Given a group of objects (20 or less), divide the group into smaller groups in various ways. ☐ Take away objects from a large group to create two smaller groups. ☐ Establish one-to-one correspondence between numbers and objects when given a picture a drawing or objects. ☐ Understand number words. ☐ Understand that 10 1's = 10. ☐ Count backwards from 100 by tens. ☐ Mimic counting to 100 by tens. ☐ Recognize numbers from 1-100. ☐ Understand the concept of size and amount. ☐ Understand that the last number name tells the number of objects counted. ☐ Understand that 10 1's = 10. ☐ Understand that 10 (tens) = 100. ☐ Pair a group of objects with a number representing the total number of objects in the group.

Operations with Numbers: Base Ten

Cluster	2019 Math COS Standard	
Use place value understanding and properties of operations to add and subtract.	12. *Add and subtract within 1000 using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. a. Explain that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.2.12.1: Define regrouping, total, sum, difference and solve. ☐ M.2.12.2: Add and subtract two two-digit numbers with and without regrouping. ☐ M.2.12.3: Determine the value of the number in the ones, tens, hundreds, and thousands place using concrete models or drawings and strategies based on place value. ☐ M.2.12.4: Match the number in the ones, tens, hundreds, and thousands position to a pictorial representation or manipulative of the value. ☐ M.2.12.5: Model written method for recording horizontal and vertical addition and subtraction problems. ☐ M.2.12.6: Represent two-three digit numbers with multiple models. Examples: models – base ten blocks, number lines, linking cubes, straw bundles. ☐ M.2.12.7: Recall single-digit addition and subtraction facts.	☐ Notice same/different and some/all. ☐ Recognize numerals from 1-50. ☐ Add one to a set of objects (up to 10 objects). ☐ Given small groups of objects, create larger groups by combining the small groups. ☐ Understand ten and 1 (ten 1's =10). ☐ Put together two small groups of objects to create a larger group. ☐ Subtract one from a set of objects (up to 10 objects). ☐ Given a group of objects (20 or less), divide the group into smaller groups in various ways. ☐ Take away objects from a large group to create two smaller groups. ☐ Understand number words. ☐ Establish one-to-one correspondence between numbers and objects when given a picture a drawing or objects. ☐ Rote count to 10. ☐ Understand amount words, such as more, less, and another. ☐ Begin to understand that parts of an object can make a whole.

	☐ M.2.12.8: Add and subtract within 20, e.g., by using objects or drawings to represent the problem.	
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Operations with Numbers: Base Ten

Cluster	2019 Math COS Standard	
Use place value understanding and properties of operations to add and subtract.	13. Mentally add and subtract 10 or 100 to a given number between 100 and 900.	
	Learning Objectives	Prior Knowledge Skills
	☐ M. 2.13.1: Demonstrate conceptual understanding of adding or subtracting 10 using concrete models. ☐ M.2.13.2: Recognize the place value of ones, tens, and hundreds. ☐ M.2.13.3: Count forward and backward by 100. ☐ M.2.13.4: Count forward and backward by 10. ☐ M.2.13.5: Recall single-digit subtraction facts. ☐ M.2.13.6: Recall single-digit addition facts.	☐ Recognize numerals from 1-50. ☐ Become interested in how many objects she/he has. ☐ Understand the concept of size and amount. ☐ Given a set number of objects one through ten, answer the question "How many?". ☐ Pair the number of objects counted with "how many?". ☐ Understand that the last number name tells the number of objects counted. ☐ Understand that $10 \text{ 1's} = 10$. ☐ Establish one-to-one correspondence between numbers and objects when given a picture, a drawing, or objects. ☐ Pair a group of objects with a number representing the total number of objects in the group. ☐ Count objects one-by-one using only one number per object. ☐ Recognize that numbers and numerals have meaning. ☐ Mimic counting forward and backward by 100. ☐ Mimic counting forward and backward by 10. ☐ Communicate number words. ☐ Recognize after. ☐ Recognize before. ☐ Understand one less than a number 2 through 20. ☐ Understand one more than a number 1 through 20.

Operations with Numbers: Base Ten

Cluster	2019 Math COS Standard	
Use place value understanding and properties of operations to add and subtract.	14. Explain why addition and subtraction strategies work, using place value and the properties of operations. <i>Note: Explanations may be supported by drawings or objects.</i>	
	Learning Objectives	Prior Knowledge Skills
	☐ M.2.14.1: Explain addition and subtraction problems using concrete objects, pictures. ☐ M.2.14.2: Use multiple strategies to add and subtract including counting on, counting back and using doubles. ☐ M.2.14.3: Recall single-digit subtraction facts. ☐ M.2.14.4: Recall single-digit addition facts. ☐ M.2.14.5: Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.	☐ Match numerals to objects or drawings. ☐ Identify numerals 1 to 10. ☐ Count from 0 to 10. ☐ Add and subtract numbers within 20 using objects, pictures, and fingers. ☐ Take a smaller set out of a larger set. ☐ Combine two sets to make a larger set up to twenty. ☐ Count items in a set up to twenty. ☐ Establish one-to-one correspondence between numbers and objects. ☐ Understand one less than a number 2 through 20. ☐ Understand one more than a number 1 through 20. ☐ Understand positional terms with equal signs.

Data Analysis

Cluster	2019 Math COS Standard	
Collect and analyze data and interpret results.	15. Measure lengths of several objects to the nearest whole unit. a. Create a line plot where the horizontal scale is marked off in whole-number units to show the lengths of several measured objects.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.2.15.1: Define length and line plot. ☐ M.2.15.2: Use vocabulary related to comparison of length. Examples: longer, shorter, longest, shortest, taller. ☐ M.2.15.3: Demonstrate rounding up to the nearest whole unit on measurement tools. ☐ M.2.15.4: Demonstrate measuring length using standard units. ☐ M.2.15.5: Describe a line plot. ☐ M.2.15.6: Model measuring length using standard units. ☐ M.2.15.7: Identify objects by length. ☐ M.2.15.8: Sort objects according to length. ☐ M.2.15.9: Explore objects in relationship to length.	☐ Define more, less, length, width, weight, and height. ☐ Identify objects by length and height. Examples: shortest pencil, heaviest rock. ☐ Identify objects by length. Examples: shortest pencil, heaviest rock. ☐ Sort objects according to measurable attributes. ☐ Sort objects according to non-measurable attributes. ☐ Use comparative language (longer/shorter, taller/shorter) for the attributes of objects related to length. ☐ Communicate long, tall, short. ☐ Recognize the length attributes of objects (long/short, tall/short). ☐ Recognize length as the measurement of something from end to end. ☐ Understanding concepts of small, big, tall, short. ☐ Use manipulatives and counting. ☐ Establish one-to-one correspondence between numbers and objects. ☐ Write numerals 0-20. ☐ Mimic marking an x on number line.

Data Analysis

Cluster	2019 Math COS Standard	
Collect and analyze data and interpret results.	16. *Create a picture graph and bar graph to represent data with up to four categories. - Using information presented in a bar graph, solve simple “put-together,” “take-apart,” and “compare” problems. - Using Venn diagrams, pictographs, and "yes-no" charts, analyze data to predict an outcome.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.2.16.1: Use addition and subtraction within 20 to solve addition and subtraction word problems with an unknown number. ☐ M.2.16.2: Describe picture graph and bar graph. ☐ M.2.16.3: Demonstrate conceptual understanding of adding or subtracting using a variety of materials. ☐ M.2.16.4: Use vocabulary related to comparing data. Examples: more than, less than, most, least, equal. ☐ M.2.16.5: Recognize attributes of data displays. ☐ M.2.16.6: Locate information on data displays. ☐ M.2.16.7: Classify objects into given categories. ☐ M.2.16.8: Sort the categories by count. ☐ M.2.16.9: Recognize different types of data displays. ☐ M.2.16.10: Count objects up to 50.	☐ Understand different types of graphs (ex. Venn diagram, bar graphs and pictograph). ☐ Identify more and less when given two groups of objects of 10 or fewer. ☐ Understand that words can label sameness and differences. ☐ Understand categories. ☐ Identify object attributes. Examples: color, shape, size, texture, purpose. ☐ Sort objects based on both color and shape. ☐ Sort a variety of objects in a group that have one thing in common. ☐ Recognize numerals from 0-20. ☐ Understand the concept of amount. ☐ Understand that the last number name tells the number of objects counted. ☐ Pair a group of objects with a number representing the total number of objects in the group (up to ten objects). ☐ Recognize numerals 0-10. ☐ Add one to a set of objects (up to 10 objects). ☐ Put together two small groups of objects to create a larger group to represent adding.

Measurement		
Cluster	2019 Math COS Standard	
Measure and estimate lengths in standard units.	17. Measure the length of an object by selecting and using standard units of measurement shown on rulers, yardsticks, meter sticks, or measuring tapes.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.2.17.1: Identify units of measurement for length. Examples: inches, feet, yard; centimeter, meters. ☐ M.2.17.2: Demonstrate how to use measurement tools. Example: avoiding gaps and overlaps. ☐ M.2.17.3: Identify measurement tools. ☐ M.2.17.4: Model measuring using non-standard units. ☐ M.2.17.5: Order three objects by length. ☐ M.2.17.6: Compare the lengths of two objects indirectly by using a third object. ☐ M.2.17.7: Describe measurable attributes of objects such as length or weight.	☐ Define more, less, length, width, weight, and height. ☐ Identify objects by length and height. - Examples: shortest pencil, heaviest rock. ☐ Identify objects by length. Examples: shortest pencil, heaviest rock. ☐ Sort objects according to measurable attributes. ☐ Sort objects according to non-measurable attributes. ☐ Use comparative language (longer/shorter, taller/shorter) for the attributes of objects related to length. ☐ Communicate long, tall, short. ☐ Recognize the length attributes of objects (long/short, tall/short). ☐ Recognize length as the measurement of something from end to end. ☐ Understanding concepts of small, big, tall, short.

Measurement		
Cluster	2019 Math COS Standard	
Measure and estimate lengths in standard units.	18. Measure objects with two different units and describe how the two measurements relate to each other and the size of the unit chosen.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.2.18.1: Identify units of measurement for length. Examples: inches, feet, yard; centimeter, meters. ☐ M.2.18.2: Demonstrate how to use measurement tools. Example: avoiding gaps and overlaps. ☐ M.2.18.3: Identify units of measure on measurement tools. ☐ M.2.18.4: Use vocabulary related to comparison of length. Examples: longer, shorter, longest, shortest, taller. ☐ M.2.18.5: Identify numerals one to 50.	☐ Define more, less, length, width, weight, and height. ☐ Use vocabulary related to length, width, weight, and height. Examples: longer, shorter, small, big. ☐ Identify objects by length and height. Examples: shortest pencil, heaviest rock. ☐ Identify objects by length. Examples: shortest pencil, heaviest rock. ☐ Sort objects according to measurable attributes. ☐ Sort objects according to non-measurable attributes. ☐ Use comparative language (longer/shorter, taller/shorter) for the attributes of objects related to length. ☐ Communicate long, tall, short. ☐ Recognize the length attributes of objects (long/short, tall/short). ☐ Recognize length as the measurement of something from end to end. ☐ Understanding concepts of small, big, tall, short. ☐ Identify numerals 0-25.

Measurement		
Cluster	2019 Math COS Standard	
Measure and estimate lengths in standard units.	19. *Estimate lengths using the following standard units of measurement: inches, feet, centimeters, and meters.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.2.19.1: Define estimate. ☐ M.2.19.2: Measure objects using standard and non-standard units. ☐ M.2.19.3: Identify units of measure on measurement tools. ☐ M.2.19.4: Model measuring using non-standard units. ☐ M.2.19.5: Use vocabulary related to comparison of length. Examples: longer, shorter, longest, shortest, and taller.	☐ Define more, less, length, width, weight, and height. ☐ Use vocabulary related to length, width, weight, and height. Examples: longer, shorter, small, big. ☐ Identify objects by length and height. Examples: shortest pencil, heaviest rock. ☐ Identify objects by length. Examples: shortest pencil, heaviest rock. ☐ Use comparative language (longer/shorter, taller/shorter) for the attributes of objects related to length. ☐ Communicate long, tall, short. ☐ Understanding concepts of small, big, tall, short.

Measurement		
Cluster	2019 Math COS Standard	
Measure and estimate lengths in standard units.	20. Measure to determine how much longer one object is than another, expressing the length difference of the two objects using standard units of length.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.2.20.1: Measure objects using standard units. ☐ M.2.20.2: Record lengths with appropriate units. ☐ M.2.20.3: Use subtraction within 20 to solve problems. ☐ M.2.20.4: Compare length using non-standard units to determine which is longer. ☐ M.2.20.5: Use vocabulary related to comparison of length. Examples: longer, shorter, longest, shortest, and taller.	☐ Define more, less, length. ☐ Use vocabulary related to length. Examples: longer, shorter. ☐ Identify objects by length. Examples: shortest pencil, heaviest rock. ☐ Sort objects according to measurable attributes. ☐ Use comparative language (longer/shorter, taller/shorter) for the attributes of objects related to length. ☐ Communicate long, tall, short. ☐ Recognize the length attributes of objects (long/short, tall/short). ☐ Recognize length as the measurement of something from end to end. ☐ Understand different forms of measurement (inches, centimeters). ☐ Understand ruler. ☐ Match numerals to objects or drawings. ☐ Identify numerals 0 to 20. ☐ Count from 0 to 20. ☐ Add and subtract numbers within 20 using objects, pictures, and fingers. ☐ Take a smaller set out of a larger set. ☐ Combine two sets to make a larger set up to twenty. ☐ Count items in a set up to twenty. ☐ Establish one-to-one correspondence between numbers and objects. ☐ Understand one less than a number 2 through 20. ☐ Understand one more than a number 1 through 20.

Measurement		
Cluster	2019 Math COS Standard	
Relate addition and subtraction to length.	21. *Use addition and subtraction within 100 to solve word problems involving same units of length, representing the problem with drawings (such as drawings of rulers) and/or equations with a symbol for the unknown number.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.2.21.1: Solve one-step addition and subtraction word problems with an unknown by using drawings and equations with a symbol for the unknown number to represent the problem. Examples: question mark, blank, box, or letter. ☐ M.2.21.2: Demonstrate the understanding of terms in addition and subtraction word problems involving length. Examples: adding to, taking from, putting together, taking apart, sum, difference, all together, how many more, how many are left, in all, inches, feet, yards, longer, shorter, nearer, farther, closer. ☐ M.2.21.3: Locate the unknown number regardless of position. ☐ M.2.21.4: Add and subtract within 50, e.g., by using objects or drawings to represent the problem. ☐ M.2.21.5: Model writing equations from word problems. ☐ M.2.21.6: Apply signs +, -, = to actions of joining and separating sets. ☐ M.2.21.7: Identify units of measurement for length.	☐ Establish one-to-one correspondence between numbers and objects. ☐ Point to matching or similar objects. ☐ Add and subtract numbers within 20 using objects, pictures, and fingers. ☐ Pair “taking away” with subtraction. ☐ Take a smaller set out of a larger set. ☐ Pair putting together with adding. ☐ Combine two sets to make a larger set up to twenty. ☐ Count items in a set up to twenty. ☐ Using counting, find one less than a number 2 through 20. ☐ Using counting, find one more than a number 1 through 20. ☐ Understand +, -, = and what they represent. ☐ Define more, less, length, width, weight, and height. ☐ Use vocabulary related to length, width, weight, and height.

*Critical Standard

	Examples: inches, feet, yard; centimeter, meters.	
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Measurement		
Cluster	2019 Math COS Standard	
Relate addition and subtraction to length.	22. *Create a number line diagram using whole numbers and use it to represent whole number sums and differences within 100.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.2.22.1: Recognize that each successive number name refers to a quantity that is one larger; and each previous number name refers to a quantity that is one less. ☐ M.2.22.2: Use a number line to add and subtract within 10. ☐ M.2.22.3: Write numerals 0 to 100. ☐ M.2.22.4: Trace numerals 0 to 100.	☐ Represent addition and subtraction with objects, pictures, fingers, or sounds within twenty. ☐ Understand addition as putting together and subtraction as taking from. ☐ Establish one-to-one correspondence between numbers and objects. ☐ Rote count to 25. ☐ Notice same/different and some/all. ☐ Point to matching or similar objects. ☐ Add and subtract numbers within 20 using objects, pictures, and fingers. ☐ Pair “taking away” with subtraction. ☐ Take a smaller set out of a larger set. ☐ Pair putting together with adding. ☐ Combine two sets to make a larger set up to twenty. ☐ Count items in a set up to twenty. ☐ Using counting, find one less than a number 2 through 20. ☐ Using counting, find one more than a number 1 through 20. ☐ Understand +, -, = and what they represent. ☐ Count forward to 50 by tens. ☐ Count backwards from 50 by tens. ☐ Mimic counting to 50 by tens. ☐ Trace numerals 0- 50. ☐ Mimic creating a number line with equally spaced points from 0 to 20.

Measurement		
Cluster	2019 Math COS Standard	
Work with time and money.	23. Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m. a. Express an understanding of common terms such as, but not limited to, quarter past, half past, and quarter to.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.2.23.1: Tell and write time in hours and half-hours using analog and digital clocks. ☐ M.2.23.2: Recognize vocabulary terms related to time measurements. Examples: minute, hour, half hour, o'clock, morning, evening, a.m., p.m. ☐ M.2.23.3: Illustrate time to hour and half hour. Example: Given the time 3:00, illustrate long hand and shorthand positions on a clock. ☐ M.2.23.4: Identify the shorthand as the hour hand, and the long hand as the minute hand on an analog clock. ☐ M.2.23.5: Identify the first number as the hour, and the numbers after the colon as the minutes on a digital clock. ☐ M.2.23.6: Write numerals 0 to 59. ☐ M.2.23.7: Recognize numerals 0 to 59. ☐ M.2.23.8: Count to 60 by fives. ☐ M.2.23.9: Distinguish between analog and digital clocks.	☐ Identify numerals 0 to 12. ☐ Count by 5s. ☐ Identify activities on a daily schedule that come before, next, after other activities. ☐ Know before, next and after. ☐ Use a daily schedule containing times (in hours) and activities (in pictures). ☐ Understand differences with analog and digital clocks. ☐ Understand hour is the same as 60 minutes. ☐ Know the hours, minutes, seconds on a clock. ☐ Tell time in hours on an analog clock. ☐ Demonstrate an understanding of yesterday, today, tomorrow, morning, afternoon, day, and night. ☐ Recognize yesterday, today, tomorrow. ☐ Recognize morning, afternoon, evening/night. ☐ Recognize day and night. ☐ Understand the concept of time.

Measurement		
Cluster	2019 Math COS Standard	
Work with time and money.	24. Solve problems with money. a. Identify nickels and quarters by name and value. b. Find the value of a collection of quarters, dimes, nickels, and pennies. c. Solve word problems by adding and subtracting within one dollar, using the \$ and ¢ symbols appropriately (not including decimal notation). Example: $24¢ + 26¢ = 50¢$.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.2.24.1: Determine the monetary value of a set of like and unlike bills. ☐ M.2.24.2: Determine the monetary value of a set of like and unlike coins. ☐ M.2.24.3: Apply addition and subtraction strategies. ☐ M.2.24.4: Understand key words in addition and subtraction word problems involving money. Examples: adding to, taking from, putting together, taking apart, sum, difference, all together, how much more, how much is left, in all, cents, dollar, change, paid, total. ☐ M.2.24.5: Count forward from a given number by ones, fives, tens, and twenty-fives. ☐ M.2.24.6: Identify coins and bills and their value. ☐ M.2.24.7: Identify symbols for dollar (\$), cent (¢). ☐ M.2.24.8: Identify coins by name including penny, nickel, dime, and quarter. ☐ M.2.24.9: Sort pennies, nickels, dimes, and quarters.	☐ Count to 1-25. ☐ Understand the concept of amount. ☐ Pair the number of objects counted with “how many?”. ☐ Understand that the last number name tells the number of objects counted. ☐ Pair a group of objects with a number representing the total number of objects in the group. ☐ Count objects one-by-one using only one number per object. ☐ Recognize that numbers have meaning. ☐ Recognize numerals 1-25. ☐ Communicate number words. ☐ Point to matching or similar objects. ☐ Identify a penny, dime, nickels, quarters by attributes (color, size).

*Critical Standard

	☐ M.2.24.10: Count 10 objects. Examples: pennies and dollar bills.	
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Geometry

Cluster	2019 Math COS Standard	
Reason with shapes and their attributes.	25. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes. a. Recognize and draw shapes having specified attributes. <i>Examples: a given number of angles or a given number of equal faces.</i>	
	Learning Objectives	Prior Knowledge Skills
	☐ M.2.25.1: Define side, angle, face, closed, and open. ☐ M.2.25.2: Use vocabulary related to shape attributes. Examples: sides, angles, face, closed, open. ☐ M.2.25.3: Trace shapes. ☐ M.2.25.4: Sort triangles, quadrilaterals, pentagons, hexagons, and cubes. ☐ M.2.25.5: Explore triangles, quadrilaterals, pentagons, hexagons, and cubes.	☐ Notice same/different and some/all. ☐ Begin to name and match sizes and shapes. ☐ Enjoy playing with all kinds of objects. ☐ Point to matching or similar objects. ☐ Understand that words can label sameness and differences. ☐ Sort objects based on shape or color. ☐ Understand and point to a triangle, a circle, a square and rectangle. ☐ Understand the concept of same shape and size. ☐ Understand that some have more, and some have less. ☐ Sort objects based on shape. ☐ Sort a variety of objects in a group that have one thing in common. ☐ Recognize and sort familiar objects with the same color, shape, or size ☐ Understand and point to a triangle, a circle, a square and rectangle. ☐ Understand a line and a point, angle. ☐ Count 1-6 for sides. ☐ Understand the different shapes. ☐ Draw basic shapes.

Geometry

Cluster	2019 Math COS Standard	
Reason with shapes and their attributes.	26. Partition a rectangle into rows and columns of same-size squares, and count to find the total number of squares.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.2.26.1: Define rows, columns, and total. ☐ M.2.26.2: Identify rectangle. ☐ M.2.26.3: Count to 20 by ones. ☐ M.2.26.4: Trace partitions in a rectangle.	☐ Notice same/different and some/all. ☐ Begin to name and match sizes and shapes. ☐ Enjoy playing with all kinds of objects. ☐ Point to matching or similar objects. ☐ Understand that words can label sameness and differences. ☐ Sort objects based on shape or color. ☐ Understand and point to a square or rectangle. ☐ Understand the concept of same shape and size. ☐ Understand that some have more, and some have less. ☐ Sort objects based on shape. ☐ Sort a variety of objects in a group that have one thing in common. ☐ Recognize and sort familiar objects with the same color, shape, or size. ☐ Understand and point to a square and rectangle. ☐ Understand a line and a point, row, column. ☐ Identify more and less when given two groups of objects of 10 or fewer. ☐ Count objects up to 25. ☐ Count to 10 by ones. ☐ Understand categories. ☐ Count to 0-25. ☐ Mimic counting by ones. ☐ Recognize numerals 0-20. ☐ Understand the concept of amount. ☐ Pair the number of objects counted with “how many?”. ☐ Understand that the last number name tells the number of objects counted.

		☐ Pair a group of objects with a number representing the total number of objects in the group (up to ten objects). ☐ Count objects one-by-one using only one number per object (up to ten objects). ☐ Recognize that numbers have meaning. ☐ Recognize numerals 0-10. ☐ Add one to a set of objects (up to 10 objects). ☐ Put together two small groups of objects to create a larger group to represent adding. ☐ Separate smaller groups from a larger group of objects to represent subtraction. ☐ Establish one-to-one correspondence between numbers and objects when given a picture a drawing or objects. ☐ Understand number words. ☐ Rote counting to 20. ☐ Sort objects based on both color and shape. ☐ Sort a variety of objects in a group that have one thing in common. ☐ Understand that words can label sameness and differences. ☐ Sort objects based on shape or color.
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Geometry

Cluster	2019 Math COS Standard	
Reason with shapes and their attributes.	27. *Partition circles and rectangles into two, three, or four equal shares. Describe the shares using such terms as halves, thirds, half of, or a third of, and describe the whole as two halves, three thirds, or four fourths. a. Explain that equal shares of identical wholes need not have the same shape.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.2.27.1: Define halves, thirds, fourths, quarters, whole, parts (shares) and equal. ☐ M.2.27.2: Distinguish between equal and non-equal parts. ☐ M.2.27.3: Model partitioning circles and rectangles. ☐ M.2.27.4: Decompose pictures made of simple shapes. ☐ M.2.27.5: Identify squares, circles, triangles, and rectangles. ☐ M.2.27.6: Explore shapes or figures that can be decomposed into smaller equal parts.	☐ Notice same/different and some/all. ☐ Begin to name and match sizes and shapes. ☐ Enjoy playing with all kinds of objects. ☐ Point to matching or similar objects. ☐ Understand that words can label same and differences. ☐ Sort objects based on shape. ☐ Recognize and sort familiar objects with the same shape or size. ☐ Understand and point to a triangle, a circle, a square and rectangle. ☐ Understand the concept of same shape and size. ☐ Interact with shapes. ☐ Understand a whole and half from one object. ☐ Understand grouping of objects also equal a whole. ☐ Separate whole group into 2 equal groups to show halves. ☐ Separate 2 halves into 4 equal groups to show fourths (quarters). ☐ Understand the term of equal. ☐ Understand that separating shapes can create other shapes.

Grade 3

Operations and Algebraic Thinking

Cluster	2019 Math COS Standard	
Represent and solve problems involving multiplication and division.	1. *Illustrate the product of two whole numbers as equal groups by identifying the number of groups and the number in each group and represent as a written expression.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.3.1.1: Identify and define the parts of a multiplication problem including factors, multiplier, multiplicand, and product. ☐ M.3.1.2: Use multiplication to find the total number of objects arranged in rectangular arrays based on columns and rows. ☐ M.3.1.3: Write an equation to express the product of the multipliers (factors). ☐ M.3.1.4: Relate multiplication to repeated addition and skip counting. ☐ M.3.1.5: Apply concepts of multiplication through the use of manipulatives, number stories, skip counting arrays, area of a rectangle, or repeated addition. ☐ M.3.1.6: Apply basic multiplication facts through 9 x 9 using manipulatives, solving problems, and writing number stories. ☐ M.3.1.7: Solve addition problems with multiple addends. ☐ M.3.1.8: Represent addition using manipulatives.	☐ Recall doubles addition facts. ☐ Use repeated addition to solve problems with multiple addends. ☐ Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem. ☐ Understand key words in addition and subtraction word problems. Examples: sum, difference, all together, how many more, how many are left, in all. ☐ Define subtraction as separating groups of objects, taking from, or taking apart. ☐ Define addition as combining groups of objects, adding to, or putting together. ☐ Represent addition and subtraction with objects, fingers, mental images, drawings, sounds, acting out situations, verbal explanations, expressions, or equations. ☐ Represent numbers with objects or drawings. ☐ Use objects to combine and separate groups.

Operations and Algebraic Thinking

Cluster	2019 Math COS Standard	
Represent and solve problems involving multiplication and division.	2. *Illustrate and interpret the quotient of two whole numbers as the number of objects in each group or the number of groups when the whole is partitioned into equal shares.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.3.2.1: Define the parts of a division problem including divisor, dividend, and quotient. ☐ M.3.2.2: Write a division equation. ☐ M.3.2.3: Apply the signs $\div$ and $=$ to the action of separating sets. ☐ M.3.2.4: Recognize division as either repeated subtraction, parts of a set, parts of a whole, or the inverse of multiplication. ☐ M.3.2.5: Model grouping with basic division facts partitioned equally (e.g., $8/2$). ☐ M.3.2.6: Apply properties of operations as strategies to subtract. ☐ M.3.2.7: Subtract within 20. ☐ M.3.2.8: Represent equal groups using manipulatives.	☐ Putting a larger item into smaller groups. ☐ Model writing equations from word problems. ☐ Apply signs $+$, $-$, $=$ to actions of joining and separating sets. ☐ Use of base 10 blocks to demonstrate making equal sets of a larger whole.

Operations and Algebraic Thinking

Cluster	2019 Math COS Standard	
Represent and solve problems involving multiplication and division.	3. *Solve word situations using multiplication and division within 100 involving equal groups, arrays, and measurement quantities; represent the situation using models, drawings, and equations with a symbol for the unknown number.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.3.3.1: Demonstrate computational understanding of multiplication and division by solving authentic problems with multiple representations using drawings, words, and/or numbers. ☐ M.3.3.2: Identify key vocabulary words to solve multiplication and division word problems. ☐ M.3.3.3: Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. ☐ M.3.3.4: Recall basic multiplication facts. ☐ M.3.3.5: Add and subtract within 20. ☐ M.3.3.6: Represent repeated addition, subtraction, and equal groups using manipulatives.	☐ Define pair, odd and even. ☐ Recall doubles addition facts with sums to 20. ☐ Apply signs + and = to actions of joining sets. ☐ Model written method for composing equations. ☐ Skip count by 2s. ☐ Represent addition and subtraction with objects, pictures, fingers, or sounds within twenty. ☐ Understand addition as putting together and subtraction as taking from. ☐ Establish one-to-one correspondence between numbers and objects. ☐ Rote count to 20. ☐ Notice same/different and some/all. ☐ Establish one-to-one correspondence between numbers and objects. ☐ Point to matching or similar objects.

Operations and Algebraic Thinking

Cluster	2019 Math COS Standard	
Represent and solve problems involving multiplication and division.	4. Determine the unknown whole number in a multiplication or division equation relating three whole numbers.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.3.4.1: Use arrays to show equal groups in multiplication and division. ☐ M.3.4.2: Recall basic multiplication facts. ☐ M.3.4.3: Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. ☐ M.3.4.4: Represent repeated addition, repeated subtraction, and equal groups using manipulatives.	☐ Distinguish between rows and columns. ☐ Use repeated addition to solve problems with multiple addends Pulled up from first grade. ☐ Represent addition and subtraction with objects, pictures, fingers, or sounds within twenty. ☐ Understand addition as putting together and subtraction as taking from. ☐ Establish one-to-one correspondence between numbers and objects. ☐ Rote count to 20. ☐ Notice same/different and some/all. ☐ Establish one-to-one correspondence between numbers and objects. ☐ Point to matching or similar objects.

Operations and Algebraic Thinking

Cluster	2019 Math COS Standard	
Understand properties of multiplication and the relationship between multiplication and division. <i>Note: Students need not use formal terms for these properties.</i>	5. *Develop and apply properties of operations as strategies to multiply and divide.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.3.5.1: Define properties of operations. ☐ M.3.5.2: Apply basic multiplication facts. ☐ M.3.5.3: Apply properties of operations as strategies to add and subtract. ☐ M.3.5.4: Count to answer “how many?” questions about as many as 30 things arranged in a rectangular array.	☐ Match the numeral in the ones and tens position to a pictorial representation or manipulative of the value. ☐ Count forward in multiples from a given number. ☐ Represent addition and subtraction with objects, fingers, mental images, drawings, sounds, acting out situations, verbal explanations, expressions, or equations. ☐ Represent numbers with objects or drawings.

Operations and Algebraic Thinking

Cluster	2019 Math COS Standard	
Understand properties of multiplication and the relationship between multiplication and division. <i>Note: Students need not use formal terms for these properties.</i>	6. Use the relationship between multiplication and division to represent division as an equation with an unknown factor.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.3.6.1: Apply divisibility rules for 2, 5, and 10. ☐ M.3.6.2: Apply basic multiplication facts. ☐ M.3.6.3: Understand subtraction as an unknown-addend problem. ☐ M.3.6.4: Recognize division as repeated subtraction, parts of a set, parts of a whole, or the inverse of multiplication.	☐ Add and subtract within 5. ☐ Match the numeral in the ones and tens position to a pictorial representation or manipulative of the value. ☐ Count forward in multiples from a given number. ☐ Skip count by 2s.

Operations and Algebraic Thinking

Cluster	2019 Math COS Standard	
Multiply and divide within 100.	7. *Use strategies based on properties and patterns of multiplication to demonstrate fluency with multiplication and division within 100. a. Fluently determine all products obtained by multiplying two one-digit numbers. b. State automatically all products of two one-digit numbers by the end of third grade.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.3.7.1: Name the first 10 multiples of each one-digit natural number. ☐ M.3.7.2: Recognize multiplication as repeated addition, and division as repeated subtraction. ☐ M.3.7.3: Apply properties of operations as strategies to add and subtract. ☐ M.3.7.4: Recall basic addition and subtraction facts.	☐ Recall single-digit subtraction facts. ☐ Recall single-digit addition facts. ☐ Add and subtract two two-digit numbers with and without regrouping. ☐ Determine the value of the number in the ones, tens, hundreds, and thousands place using concrete models or drawings and strategies based on place value.

Operations and Algebraic Thinking

Cluster	2019 Math COS Standard	
Solve problems involving the four operations and identify and explain patterns in arithmetic.	8. *Determine and justify solutions for two-step word problems using the four operations and write an equation with a letter standing for the unknown quantity. Determine reasonableness of answers using number sense, context, mental computation, and estimation strategies including rounding.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.3.8.1: Define the identity property of addition and multiplication. ☐ M.3.8.2: Estimating sums and differences using multiple methods, including compatible numbers and rounding, to judge the reasonableness of an answer. ☐ M.3.8.3: Apply commutative, associative, and identity properties for all operations to solve problems. ☐ M.3.8.4: Identify a rule when given a pattern. ☐ M.3.8.5: Solve addition and subtraction problems, including word problems, involving one-and two-digit numbers with and without regrouping, using multiple strategies. ☐ M 3.8.6: Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. ☐ M.3.8.7: Represent multiplication and division with manipulatives. ☐ M.3.8.8: Recall basic addition and subtraction facts.	☐ Explain addition and subtraction problems using concrete objects, pictures. ☐ Use multiple strategies to add and subtract including counting on, counting back, and using doubles. ☐ Create a number pattern. ☐ Use multiple strategies to add and subtract including counting on, counting back, and using doubles. ☐ Recall single-digit subtraction facts. ☐ Recall single-digit addition facts.

Operations and Algebraic Thinking

Cluster	2019 Math COS Standard	
Solve problems involving the four operations and identify and explain patterns in arithmetic.	9. Recognize and explain arithmetic patterns using properties of operations.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.3.9.1: Define arithmetic patterns: geometric or numeric. ☐ M.3.9.2: Explain arithmetic patterns using properties of operations. ☐ M.3.9.3: Recognize arithmetic patterns (including geometric patterns or patterns in the addition table or multiplication table). ☐ M.3.9.4: Construct repeating and growing patterns with a variety of representations. ☐ M.3.9.5: Demonstrate computational fluency, including quick recall, of addition and multiplication facts. ☐ M.3.9.6: Duplicate an existing pattern. ☐ M.3.9.7: Skip count. ☐ M.3.9.8: Represent addition and multiplication with manipulatives.	☐ Explain addition and subtraction problems using concrete objects, pictures. ☐ Use multiple strategies to add and subtract including counting on, counting back, and using doubles. ☐ Create a number pattern. ☐ Use multiple strategies to add and subtract including counting on, counting back, and using doubles.

Operations with Numbers: Base Ten

Cluster	2019 Math COS Standard	
Use place value understanding and properties of operations to perform multi-digit arithmetic.	10. Identify the nearest 10 or 100 when rounding whole numbers, using place value understanding.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.3.10.1: Define rounding. ☐ M.3.10.2: Round whole numbers from 100 to 999 using whole numbers from 10 to 99. ☐ M.3.10.3: Model rounding whole numbers to the nearest 100. ☐ M.3.10.4: Round whole numbers from 10 to 99 using whole numbers from 1 to 9. ☐ M.3.10.5: Model rounding whole numbers to the nearest 10. ☐ M.3.10.6: Identify the steps in rounding two- and three-digit numbers. Example: Identify the digit that may change and the number to the right. ☐ M.3.10.7: Round whole numbers from 1 to 9 and model to show proficiency. ☐ M.3.10.8: Understand that the two digits of a two-digit number represent amounts of tens and ones. ☐ M.3.10.9: Match the number in the ones, tens, and hundreds position to a pictorial representation or manipulative of the value.	☐ Determine the value of the number in the ones, tens, and hundreds place. ☐ Recognize the place value of ones, tens, and hundreds.

Operations with Numbers: Base Ten

Cluster	2019 Math COS Standard	
Use place value understanding and properties of operations to perform multi-digit arithmetic.	11. *Use various strategies to add and subtract fluently within 1000.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.3.11.1: Define the commutative and associative properties of addition and subtraction. ☐ M.3.11.2: Subtract within 100 using strategies and algorithms based on the relationship between addition and subtraction. ☐ M.3.11.3: Subtract within 100 using strategies and algorithms based on properties of operations. ☐ M.3.11.4: Subtract within 100 using strategies and algorithms based on place value. ☐ M.3.11.5: Add within 100 using strategies and algorithms based on the relationship between addition and subtraction. ☐ M.3.11.6: Add within 100 using strategies and algorithms based on properties of operations. ☐ M.3.11.7: Add within 100 using strategies and algorithms based on place value. ☐ M.3.11.8: Recall basic addition and subtraction facts.	☐ Add within 100, including adding a two-digit number and a one-digit number and adding two two-digit numbers. ☐ Add within 20, demonstrating fluency for addition within 10. ☐ Add and subtract within 20. ☐ Identify place value for ones, tens, and hundreds. ☐ Read number names one through one hundred.

Operations with Numbers: Base Ten

Cluster	2019 Math COS Standard	
Use place value understanding and properties of operations to perform multi-digit arithmetic.	12. Use concrete materials and pictorial models based on place value and properties of operations to find the product of a one-digit whole number by a multiple of ten (from 10 to 90).	
	Learning Objectives	Prior Knowledge Skills
	☐ M.3.12.1: Model place value by multiplying vertically. ☐ M.3.12.2: Model properties of operations by multiplying horizontally. ☐ M.3.12.3: Recall basic multiplication facts. ☐ M.3.12.4: Recall multiplication as repeated addition. ☐ M.3.12.5: Apply properties of operations as strategies to add.	☐ Identify place value for ones, tens, and hundreds. ☐ Recall basic multiplication facts. ☐ Recognize properties of operations. ☐ Demonstrate that multiplication is the same as repeated addition.

Operations with Numbers: Fractions

Cluster	2019 Math COS Standard	
Develop understanding of fractions as numbers. <i>Denominators are limited to 2, 3, 4, 6, and 8.</i>	13. *Demonstrate that a unit fraction represents one part of an area model or length model of a whole that has been equally partitioned; explain that a numerator greater than one indicates the number of unit pieces represented by the fraction.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.3.13.1: Define fraction, numerator, and denominator. ☐ M.3.13.2: Identify the parts of a fraction. ☐ M.3.13.3: Label numerator, denominator, and fraction bar. ☐ M.3.13.4: Identify parts of a whole with two, three, or four equal parts. ☐ M.3.13.5: Distinguish between equal and non-equal parts. ☐ M.3.13.6: Partition circles and rectangles into two and four equal shares; describe the shares using the words halves, fourths, and quarters; and use the phrases half of, fourth of, and quarter of.	☐ Define halves, thirds, fourths, quarters, whole, parts (shares) and equal. ☐ Distinguish between equal and non-equal parts.

Operations with Numbers: Fractions

Cluster	2019 Math COS Standard	
Develop understanding of fractions as numbers. <i>Denominators are limited to 2, 3, 4, 6, and 8.</i>	14. *Interpret a fraction as a number on the number line; locate or represent fractions on a number line diagram. - Represent a unit fraction (1 bb) on a number line by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts as specified by the denominator. - Represent a fraction (aabb) on a number line by marking off a length of size (1bb) from zero.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.3.14.1: Recognize fractions as lengths from zero to one. ☐ M.3.14.2: Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2..., and represent whole-number sums and differences within 100 on a number diagram. ☐ M.3.14.3: Identify a number line. ☐ M.3.14.4: Recognize whole numbers as lengths from zero to one. ☐ M.3.14.5: Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2..., and represent whole-number sums and differences within 100 on a number diagram. ☐ M.3.14.6: Identify a number line. ☐ M.3.14.7: Label the fractions on a pre-made number line diagram. ☐ M.3.14.8: Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2..., and represent whole-	☐ Select numbers on a number line that are more than, less than or equal to a specified number. ☐ Count to 20 by ones. ☐ Count to 10 by ones.

	number sums and differences within 100 on a number diagram. ☐ M.3.14.9: Recognize a number line diagram with equally spaced points.	
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Operations with Numbers: Fractions

Cluster	2019 Math COS Standard	
Develop understanding of fractions as numbers. <i>Denominators are limited to 2, 3, 4, 6, and 8.</i>	15. *Explain equivalence and compare fractions by reasoning about their size using visual fraction models and number lines. - Express whole numbers as fractions and recognize fractions that are equivalent to whole numbers. - Compare two fractions with the same numerator or with the same denominator by reasoning about their size (recognizing that fractions must refer to the same whole for the comparison to be valid). Record comparisons using $<$, $>$, or $=$ and justify conclusions.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.3.15.1: Define equivalent. ☐ M.3.15.2: Recognize pictorial representations of equivalent fractions. ☐ M.3.15.3: Recognize different interpretations of fractions, including parts of a set or a collection, points on a number line, numbers that lie between two consecutive whole numbers, and lengths of segments on a ruler. ☐ M.3.15.4: Recognize that equal shares of identical wholes need not have the same shape. ☐ M.3.15.5: Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape and compose new shapes from the composite shape. ☐ M.3.15.6: Label a fraction with multiple representations.	☐ Label numerator, denominator, and fraction bar. ☐ Match the number in the ones, tens, and hundreds position to a pictorial representation or manipulative of the value. ☐ Distinguish between equal and non-equal parts. ☐ Model partitioning circles and rectangles. ☐ Identify two-dimensional shapes. ☐ Sort two-dimensional shapes. ☐ Name shapes.

	☐ M.3.15.7: Recognize that a whole can be partitioned into differing equal parts (halves, fourths, eighths, etc.). ☐ M.3.15.8: Partition circles and rectangles into two and four equal shares; and describe the shares using the words halves, fourths, and quarters; and use the phrases half of, fourth of, and quarter of. ☐ M.3.15.9: Recognize different interpretations of fractions, including parts of a set or a collection, points on a number line, numbers that lie between two consecutive whole numbers, and lengths of segments on a ruler. ☐ M.3.15.10: Label a pictorial representation. ☐ M.3.15.11: Recognize that a fraction is a part of a whole. ☐ M.3.15.12: Partition circles and rectangles into two and four equal shares; describe the shares using the words halves, fourths, and quarters; and use the phrases half of, fourth of, and quarter of.	
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Data Analysis

Cluster	2019 Math COS Standard	
Represent and interpret data.	16. For a given or collected set of data, create a scaled (one-to-many) picture graph and scaled bar graph to represent a data set with several categories. a. Determine a simple probability from a context that includes a picture. b. Solve one- and two-step “how many more” and “how many less” problems using information presented in scaled graphs.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.3.16.1: Define picture graph, bar graph, and data. ☐ M.3.16.2: Interpret the data to solve problems. ☐ M.3.16.3: Identify the parts of a graph (x-axis, y-axis, title, key, equal intervals, labels). ☐ M.3.16.4: Locate the data on a picture graph and a bar graph. ☐ M.3.16.5: Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another. ☐ M.3.16.6: Directly compare two objects, with a measurable attribute in common, to see which object has “more of” or “less of” the attribute and describe the difference.	☐ Describe picture graph and bar graph. ☐ Use vocabulary related to comparing data. Examples: more than, less than, most, least, equal. ☐ Recognize attributes of data displays. ☐ Locate information on data displays. ☐ Classify objects into given categories. ☐ Sort the categories by count. ☐ Recognize different types of data displays.

Data Analysis

Cluster	2019 Math COS Standard	
Represent and interpret data.	17. Measure lengths using rulers marked with halves and fourths of an inch to generate data and create a line plot marked off in appropriate units to display the data.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.3.17.1: Define line plot. ☐ M.3.17.2: Identify the parts of a line plot. ☐ M.3.17.3: Measure objects to the nearest inch. ☐ M.3.17.4: Identify one-inch units on a ruler starting with 0. ☐ M.3.17.5: Express the length of an object as a whole number of length units by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. ☐ M.3.17.6: Directly compare two objects, with a measurable attribute in common, to see which object has “more of” or “less of” the attribute and describe the difference.	☐ Define length and line plot. ☐ Use vocabulary related to comparison of length. Examples: longer, shorter, longest, shortest, taller. ☐ Demonstrate rounding up to the nearest whole unit on measurement tools. ☐ Demonstrate measuring length using standard units. ☐ Describe a line plot. ☐ Model measuring length using standard units. ☐ Identify the object length. ☐ Explore objects in relationship to length.

Measurement		
Cluster	2019 Math COS Standard	
Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.	18. *Tell and write time to the nearest minute; measure time intervals in minutes (within 90 minutes.) a. Solve real-world problems involving addition and subtraction of time intervals in minutes by representing the problem on a number line diagram.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.3.18.1: Compare equivalent units of time using hours and minutes. Examples: 60 minutes = one hour, 30 minutes = one half of an hour. ☐ M.3.18.2: Recognize key vocabulary and/or phrases associated with time. ☐ M.3.18.3: Compare the lengths of time to complete everyday activities. ☐ M.3.18.4: Tell and write time in hours and half-hours using analog and digital clocks. ☐ M.3.18.5: Recognize hour, minute, and second hands on an analog clock. ☐ M.3.18.6: Count by 5's to 60.	☐ Write numerals 0 to 59. ☐ Recognize numerals 0 to 59. ☐ Count to 60 by fives. ☐ Distinguish between analog and digital clocks. ☐ Identify the short hand as the hour hand, and the long hand as the minute hand on an analog clock. ☐ Identify the first number as the hour, and the numbers after the colon as the minutes on a digital clock. ☐ Tell and write time in hours and half-hours using analog and digital clocks. ☐ Recognize vocabulary terms related to time measurements.

Measurement

Cluster	2019 Math COS Standard	
Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.	19. *Estimate and measure liquid volumes and masses of objects using liters (l), grams (g), and kilograms (kg). a. Use the four operations to solve one-step word problems involving masses or volumes given in the same metric units.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.3.19.1: Define liquid volume, mass, grams, kilograms, and liters. ☐ M.3.19.2: Recognize how the standard units of measure compare to one another. ☐ M.3.19.3: Identify key terms for word problems. ☐ M.3.19.4: Express the length of an object as a whole number of length units by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. ☐ M.3.19.5: Recall basic addition, subtraction, multiplication, and division facts. ☐ M.3.19.6: Describe measurable attributes of objects such as length or weight. Describe several measurable attributes of a single object.	☐ Measure objects using standard units. ☐ Recall single-digit subtraction facts. ☐ Recall single-digit addition facts.

Measurement		
Cluster	2019 Math COS Standard	
Geometric measurement: understand concepts of area and relate area to multiplication and to addition.	20. Find the area of a rectangle with whole number side lengths by tiling without gaps or overlaps and counting unit squares.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.3.20.1: Define length. ☐ M.3.20.2: Recognize that units of measure must be equal. ☐ M.3.20.3: Express the length of an object as a whole number of length units by laying multiple copies of a shorter object (the length unit) end to end. ☐ M.3.20.4: Recognize that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps.	☐ Identify units of measurement for length. Examples: inches, feet, yard; centimeter, meters. ☐ Demonstrate how to use measurement tools. Example: avoiding gaps and overlaps. ☐ Identify units of measure on measurement tools. ☐ Use vocabulary related to comparison of length. Examples: longer, shorter, longest, shortest, taller.

Measurement

Cluster	2019 Math COS Standard	
Geometric measurement: understand concepts of area and relate area to multiplication and to addition.	21. Count unit squares (square cm, square m, square in, square ft, and improvised or non-standard units) to determine area.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.3.21.1: Recognize that unit squares are equal. ☐ M.3.21.2: Define the units of measurement (cm, m, in, ft). ☐ M.3.21.3: Express the length of an object as a whole number of length units by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps.	☐ Identify units of measurement for length. ☐ Order three objects by length. ☐ Compare the lengths of two objects indirectly by using a third object.

Measurement		
Cluster	2019 Math COS Standard	
Geometric measurement: understand concepts of area and relate area to multiplication and to addition.	22. *Relate area to the operations of multiplication using real-world problems, concrete materials, mathematical reasoning, and the distributive property.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.3.22.1: Recognize arrays as multiplication or repeated addition. ☐ M.3.22.2: Recall basic addition and multiplication facts. ☐ M.3.22.3: Build and draw shapes to possess defining attributes. ☐ M.3.22.4: Compose simple shapes to form larger shapes.	☐ Recall doubles addition facts with sums to 20. ☐ Apply signs + and = to actions of joining sets. ☐ Model written method for composing equations.

Measurement

Cluster	2019 Math COS Standard	
Geometric measurement: understand concepts of area and relate area to multiplication and to addition.	23. *Decompose rectilinear figures into smaller rectangles to find the area, using concrete materials.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.3.23.1: Label pre-made arrays. ☐ M.3.23.2: Partition a rectangle into rows and columns of same-size squares, and count to find the total number of them. ☐ M.3.23.3: Recall basic addition and multiplication facts. ☐ M.3.23.4: Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles). ☐ M.3.23.5: Identify a rectangle.	☐ Define side, angle, face, closed, and open. ☐ Use vocabulary related to shape attributes. Examples: sides, angles, face, closed, open. ☐ Trace shapes. ☐ Sort triangles, quadrilaterals, pentagons, hexagons, and cubes. ☐ Explore triangles, quadrilaterals, pentagons, hexagons, and cubes.

Measurement		
Cluster	2019 Math COS Standard	
Geometric measurement: Recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.	24. Construct rectangles with the same perimeter and different areas or the same area and different perimeters.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.3.24.1: Define perimeter. ☐ M.3.24.2: Recall the formula for perimeter ($P = L + L + W + W$ or $P = 2L + 2W$). ☐ M.3.24.3: Recall basic addition and multiplication facts. ☐ M.3.24.4: Build and draw shapes to possess defining attributes. ☐ M.3.24.5: Express the length of an object as a whole number of length units by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. ☐ M.3.24.6: Describe measurable attributes of objects such as length or weight.	☐ Recall doubles addition facts. ☐ Model written method for composing equations.

Measurement		
Cluster	2019 Math COS Standard	
Geometric measurement: Recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.	25. Solve real-world problems involving perimeters of polygons, including finding the perimeter given the side lengths and finding an unknown side length of rectangles.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.3.25.1: Define perimeter. ☐ M.3.25.2: Recall the formula for perimeter ($P = L + L + W + W$ or $P = 2L + 2W$). ☐ M.3.25.3: Recall basic addition and multiplication facts. M. 3.23.4: Build and draw shapes to possess defining attributes. ☐ M.3.25.4: Express the length of an object as a whole number of length units by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. ☐ M.3.25.5: Describe measurable attributes of objects such as length or weight.	☐ Recall doubles addition facts. ☐ Model written method for composing equations.

Geometry

Cluster	2019 Math COS Standard	
Reason with shapes and their attributes.	26. *Recognize and describe polygons (up to 8 sides), triangles, and quadrilaterals (rhombuses, rectangles, and squares) based on the number of sides and the presence or absence of square corners. a. Draw examples of quadrilaterals that are and are not rhombuses, rectangles, and squares.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.3.26.1: Recall the vocabulary of shapes (labels, sides, faces, vertices, etc.). ☐ M.3.26.2: Recognize and draw shapes having specified attributes such as a given number of angles. ☐ M.3.26.3: Build and draw shapes to possess defining attributes. ☐ M.3.26.4: Sort shapes into categories.	☐ Identify squares, circles, triangles, and rectangles. ☐ Define side, angle, face, closed, and open. ☐ Use vocabulary related to shape attributes. Examples: sides, angles, face, closed, open. ☐ Trace shapes. ☐ Sort triangles, quadrilaterals, pentagons, hexagons, and cubes. ☐ Explore triangles, quadrilaterals, pentagons, hexagons, and cubes. ☐ Name shapes. ☐ Recognize shapes.

Grade 4

Operations and Algebraic Thinking

Cluster	2019 Math COS Standard	
Solve problems with whole numbers using the four operations.	1. *Interpret and write equations for multiplicative comparisons.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.4.1.1: Use arrays to show equal groups in multiplication. ☐ M.4.1.2: Recall basic multiplication facts. ☐ M.4.1.4: Demonstrate computational fluency, including quick recall of addition and subtraction facts. ☐ M.4.1.5: Recognize multiplication as repeated addition.	☐ Identify and define the parts of a multiplication problem including factors, multiplier, multiplicand, and product. ☐ Use multiplication to find the total number of objects arranged in rectangular arrays based on columns and rows. ☐ Write an equation to express the product of the multipliers (factors). ☐ Relate multiplication to repeated addition and skip counting. ☐ Apply concepts of multiplication through the use of manipulatives, number stories, skip-counting arrays, area of a rectangle, or repeated addition. ☐ Apply basic multiplication facts through 9 x 9 using manipulatives, solving problems, and writing number stories. ☐ Solve addition problems with multiple addends. ☐ Represent addition using manipulatives. ☐ Distinguish between rows and columns. ☐ Use repeated addition to solve problems with multiple addends. ☐ Count forward in multiples from a given number. Examples: 3, 6, 9, 12; 4, 8, 12, 16. ☐ Recall doubles addition facts. ☐ Model written method for composing equations.

Operations and Algebraic Thinking

Cluster	2019 Math COS Standard	
Solve problems with whole numbers using the four operations.	2. *Solve word problems involving multiplicative comparison using drawings and write equations to represent the problem, using a symbol for the unknown number.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.4.2.1: Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. ☐ M.4.2.2: Recognize key terms to solve word problems. Examples: in all, how much, how many, in each. ☐ M.4.2.3: Apply properties of operations as strategies to add. ☐ M.4.2.4: Recall basic multiplication facts. ☐ M.4.2.5: Demonstrate computational fluency, including quick recall of addition and subtraction facts.	☐ Demonstrate computational understanding of multiplication and division by solving authentic problems with multiple representations using drawings, words, and/or numbers. ☐ Identify key vocabulary words to solve multiplication and division word problems. Examples: times, every, at this rate, each, per, equal/equally, in all, total. ☐ Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. ☐ Recall basic multiplication facts. ☐ Add and subtract within 20. ☐ Represent repeated addition, subtraction, and equal groups using manipulatives. ☐ Distinguish between rows and columns. ☐ Use repeated addition to solve problems with multiple addends. ☐ Count forward in multiples from a given number. Examples: 3, 6, 9, 12; 4, 8, 12, 16. ☐ Recall doubles addition facts. ☐ Model written method for composing equations.

Operations and Algebraic Thinking

Cluster	2019 Math COS Standard	
Solve problems with whole numbers using the four operations.	3. *Determine and justify solutions for multi-step word problems, including problems where remainders must be interpreted. - Write equations to show solutions for multi-step word problems with a letter standing for the unknown quantity. - Determine reasonableness of answers for multi-step word problems, using mental computation and estimation strategies including rounding.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.4.3.1: Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. ☐ M.4.3.2: Solve single-step word problems. ☐ M.4.3.3: Recognize key terms to solve word problems. Examples: in all, how much, how many, in each. ☐ M.4.3.4: Solve division problems without remainders. ☐ M.4.3.5: Recall basic addition, subtraction, and multiplication facts.	☐ Demonstrate computational understanding of multiplication and division by solving authentic problems with multiple representations using drawings, words, and/or numbers. ☐ Identify key vocabulary words to solve multiplication and division word problems. Examples: times, every, at this rate, each, per, equal/equally, in all, total. ☐ Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. ☐ Recall basic multiplication facts. ☐ Add and subtract within 20. ☐ Represent repeated addition, subtraction, and equal groups using manipulatives. ☐ Distinguish between rows and columns. ☐ Use repeated addition to solve problems with multiple addends. ☐ Count forward in multiples from a given number. Examples: 3, 6, 9, 12; 4, 8, 12, 16. ☐ Recall doubles addition facts. ☐ Model written method for composing equations.

Operations and Algebraic Thinking

Cluster	2019 Math COS Standard	
Gain familiarity with factors and multiples.	4. For whole numbers in the range 1 to 100, find all factor pairs, identifying a number as a multiple of each of its factors. - Determine whether a whole number in the range 1 to 100 is a multiple of a given one-digit number. - Determine whether a whole number in the range 1 to 100 is prime or composite.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.4.4.1: Define factors, prime number, and composite number. ☐ M.4.4.2: Apply properties of operations as strategies to multiply and divide. ☐ M.4.4.3: Identify all factor pairs for a whole number in the range 1-20. ☐ M.4.4.4: Name the first ten multiples of each one-digit natural number. ☐ M.4.4.5: Recall basic multiplication facts. ☐ M.4.4.6: Count within 1000; skip-count by 5s, 10s, and 100s.	☐ Represent equal groups using manipulatives. ☐ Identify and define the parts of a multiplication problem including factors, multiplier, multiplicand and product. ☐ Use multiplication to find the total number of objects arranged in rectangular arrays based on columns and rows. ☐ Write an equation to express the product of the multipliers (factors). ☐ Relate multiplication to repeated addition and skip counting. ☐ Define pair, odd and even. ☐ Recall doubles addition facts with sums to 20. ☐ Apply sign+ and = to actions of joining sets. ☐ Model written method for composing equations. ☐ Skip count by 2s.

Operations and Algebraic Thinking

Cluster	2019 Math COS Standard	
Generate and analyze patterns.	5. Generate and analyze a number or shape pattern that follows a given rule.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.4.5.1: Identify arithmetic patterns, including patterns in the addition table or multiplication table; and explain them using properties of operations. ☐ M.4.5.2: Recognize arithmetic patterns (including geometric patterns or patterns in the addition table or multiplication table). ☐ M.4.5.3: Construct repeating and growing patterns with a variety of representations. ☐ M.4.5.4: Continue an existing pattern. ☐ M.4.5.5: Identify arithmetic patterns. ☐ M.4.5.6: Demonstrate computational fluency, including quick recall, of addition multiplication facts.	☐ Identify a rule when given a pattern. Examples: multiplication and division—"Input $\times$ 3 = Output". addition and subtraction—"Input + 8 = Output". ☐ Use repeated addition to solve problems with multiple addends. ☐ Count forward in multiples from a given number. Examples: 3, 6, 9, 12; 4, 8, 12, 16. ☐ Define pair, odd and even. ☐ Recall doubles addition facts with sums to 20. ☐ Skip count by 2s.

Operations with Numbers: Base Ten

Cluster	2019 Math COS Standard	
Generalize place value understanding for multidigit whole numbers.	6. *Using models and quantitative reasoning, explain that in a multi-digit whole number, a digit in any place represents ten times what it represents in the place to its right.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.4.6.1: Use place value understanding to round whole numbers to the nearest 10 or 100. ☐ M.4.6.2: Add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. ☐ M.4.6.3: Multiply one-digit whole numbers by multiples of 10 in the range 10 - 90 (e.g., 9×80, 5×60) using strategies based on place value and properties of operations. ☐ M.4.6.4: Recall basic multiplication facts. ☐ M.4.6.5: Recall that the three digits of a three-digit number represent amounts of hundreds, tens, and ones, e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. ☐ M.4.6.6: Recognize that the numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones). ☐ M.4.6.7: Recognize that 100 can be thought of as a bundle of ten tens, called a "hundred".	☐ Model place value by multiplying vertically. ☐ Model properties of operations by multiplying horizontally. ☐ Recall basic multiplication facts. ☐ Recall multiplication as repeated addition. ☐ Represent numbers with multiple concrete models. Examples: concrete models— base ten blocks, number lines, linking cubes, straw bundles. ☐ Count to 1000 by hundreds. ☐ Count to 100 by tens.

Operations with Numbers: Base Ten

Cluster	2019 Math COS Standard	
Generalize place value understanding for multidigit whole numbers.	7. *Read and write multi-digit whole numbers using standard form, word form, and expanded form.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.4.7.1: Compare two three-digit numbers based on meanings of the hundreds, tens, and one's digits using $>$, $=$, and $<$ symbols to record the results of comparisons. ☐ M.4.7.2: Read and write numbers to 1000 using base-ten numerals, number names, and expanded form. ☐ M.4.7.3: Convert a number written in expanded notation to standard form.	☐ Define greater than, less than and equal to. ☐ Compare two two-digit numbers based on meanings of the ten's and one's digits, recording the results of comparisons with the symbols $>$, $=$, and $<$. ☐ Arrange two-digit numbers in order from greatest to least or least to greatest. ☐ Identify zero as a place holder in two-digit and three-digit numbers. ☐ Model using $>$, $=$, and $<$ symbols to record the results of comparisons of two two-digit numbers. ☐ Select numbers on a number line that are more than, less than or equal to a specified number. ☐ Match the words greater than, equal to and less than to the symbols $>$, $=$, and $<$. ☐ Determine the value of the digits in the ones and tens place. ☐ Identify sets with more, less, or equal objects. ☐ Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. ☐ Number & Operations in Base Ten. ☐ Use place value understanding and properties of operations to add and subtract.

Operations with Numbers: Base Ten

Cluster	2019 Math COS Standard	
Generalize place value understanding for multidigit whole numbers.	8. Use place value understanding to compare two multi-digit numbers using $>$, $=$, and $<$ symbols.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.4.8.1: Use place value understanding to round whole numbers to the nearest 10 or 100. ☐ M.4.8.2: Model rounding whole numbers to the nearest 100. ☐ M.4.8.3: Round whole numbers from 100 to 999 using whole numbers from 10 to 99. ☐ M.4.8.4: Model rounding whole numbers to the nearest 10. ☐ M.4.8.5: Round whole numbers from 10 to 99 using whole numbers from 1 to 9. ☐ M.4.8.6: Round whole numbers from 1 to 9 and model to show proficiency.	☐ Define rounding. ☐ Round whole numbers from 100 to 999 using whole numbers from 10 to 99. ☐ Model rounding whole numbers to the nearest 100. ☐ Round whole numbers from 10 to 99 using whole numbers from 1 to 9. ☐ Model rounding whole numbers to the nearest 10. ☐ Identify the steps in rounding two-and three-digit numbers. Example: Identify the digit that may change and the number to the right. ☐ Round whole numbers from 1 to 9 and model to show proficiency. ☐ Understand that the two digits of a two-digit number represent amounts of tens and ones. ☐ Match the number in the ones, tens, and hundreds position to a pictorial representation or manipulative of the value.

Operations with Numbers: Base Ten

Cluster	2019 Math COS Standard	
Generalize place value understanding for multidigit whole numbers.	9. Round multi-digit whole numbers to any place using place value understanding.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.4.9.1: Add and subtract within 1000. ☐ M.4.9.2: Apply signs +, -, and = to actions of joining and separating sets. ☐ M.4.9.3: Add and subtract single-digit numbers. ☐ M.4.9.4: Recall basic addition and subtraction facts.	☐ Define the commutative and associative properties of addition and subtraction. ☐ Subtract within 100 using strategies and algorithms based on the relationship between addition and subtraction. ☐ Subtract within 100 using strategies and algorithms based on place value. ☐ Add within 100 using strategies and algorithms based on the relationship between addition and subtraction. Add within 100 using strategies and algorithms based on properties of operations. ☐ Add within 100 using strategies and algorithms based on place value. ☐ Recall basic addition and subtraction facts. ☐ Define regrouping, total, sum, difference and solve. ☐ Add and subtract two two-digit numbers with and without regrouping. ☐ Determine the value of the number in the ones, tens, hundreds and thousands place using concrete models or drawings and strategies based on place value. ☐ Match the number in the ones, tens, hundreds, and thousands position to a pictorial representation or manipulative of the value.

Operations with Numbers: Base Ten

Cluster	2019 Math COS Standard	
Use place value understanding and properties of operations to perform multi-digit arithmetic with whole numbers.	10. *Use place value strategies to fluently add and subtract multi-digit whole numbers and connect strategies to the standard algorithm.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.4.10.1: Multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. ☐ M.4.10.2: Multiply single-digit numbers. ☐ M.4.10.3: Recall basic multiplication facts. ☐ M.4.10.4: Apply concepts of multiplication through the use of manipulatives, number stories, skip-counting arrays, area of a rectangle, or repeated addition.	☐ Define the commutative and associative properties of addition and subtraction. ☐ Subtract within 100 using strategies and algorithms based on the relationship between addition and subtraction. ☐ Subtract within 100 using strategies and algorithms based on properties of operations. ☐ Subtract within 100 using strategies and algorithms based on place value. ☐ Add within 100 using strategies and algorithms based on the relationship between addition and subtraction. ☐ Add within 100 using strategies and algorithms based on properties of operations. ☐ Add within 100 using strategies and algorithms based on place value. ☐ Recall basic addition and subtraction facts. ☐ Define regrouping, total, sum, difference and solve. ☐ Add and subtract two two-digit numbers with and without regrouping. ☐ Determine the value of the number in the ones, tens, hundreds, and thousands place using concrete models or drawings and strategies based on place value. ☐ Match the number in the ones, tens, hundreds, and thousands position to a pictorial representation or manipulative of the value. ☐ Model written method for recording horizontal and vertical addition and subtraction problems. ☐ Represent two- and three-digit numbers with multiple models. Examples: models—base ten blocks, number lines, linking cubes, straw bundles.

*Critical Standard

		☐ Recall single-digit addition and subtraction facts. ☐ Add and subtract within 20, e.g., by using objects or drawings to represent the problem.
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Operations with Numbers: Base Ten

Cluster	2019 Math COS Standard	
Use place value understanding and properties of operations to perform multi-digit arithmetic with whole numbers.	11. *Find the product of two factors (up to four digits by a one-digit number and two two-digit numbers), using strategies based on place value and the properties of operations. Illustrate and explain the product of two factors using equations, rectangular arrays, and area models.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.4.11.1: Divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$). ☐ M.4.11.2: Divide within 100, using strategies such as properties of operations. ☐ M.4.11.3: Multiply within 100, using strategies such as properties of operations. ☐ M.4.11.4: Multiply within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$). ☐ M.4.11.5: Recall products of two one-digit numbers. ☐ M.4.11.6: Name the first 10 multiples of each one-digit natural number. Example: 7, 14, 21, 28, 35, 42, 49, 56, 63, 70. ☐ M.4.11.7: Recall basic addition, subtraction, and multiplication facts.	☐ Apply divisibility rules for 2, 5, and 10. Example: Recognizing that 32 is divisible by 2 because the digit in the ones place is even. ☐ Apply basic multiplication facts. ☐ Understand subtraction as an unknown-addend problem. ☐ Recognize division as repeated subtraction, parts of a set, parts of a whole, or the inverse of multiplication. ☐ Name the first 10 multiples of each one-digit natural number. Example: 7, 14, 21, 28, 35, 42, 49, 56, 63, 70. ☐ Recognize multiplication as repeated addition, and division as repeated subtraction. ☐ Apply properties of operations as strategies to add and subtract. ☐ Recall basic addition and subtraction facts. ☐ Use repeated addition to solve problems with multiple addends. ☐ Count forward in multiples from a given number. Examples: 3, 6, 9, 12; 4, 8, 12, 16. ☐ Recall doubles addition facts. ☐ Model written method for composing equations.

Operations with Numbers: Base Ten

Cluster	2019 Math COS Standard	
Use place value understanding and properties of operations to perform multi-digit arithmetic with whole numbers.	12. *Use strategies based on place value, properties of operations, and/or the relationship between multiplication and division to find whole-number quotients and remainders with one-digit divisors and up to four-digit dividends. a. Illustrate and/or explain quotients using equations, rectangular arrays, and/or area models.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.4.12.1: Define fraction, numerator, and denominator. ☐ M.4.12.2: Recognize fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts. ☐ M.4.12.3: Identify the parts of a fraction a/b as the quantity formed by a parts and size $1/b$. ☐ M.4.12.4: Recognize fractions as numerals that may represent division problems. ☐ M.4.12.5: Label numerator, denominator, and fraction bar. ☐ M.4.12.6: Identify parts of a whole with two, three, or four equal parts. ☐ M.4.12.7: Recognize that equal shares of identical wholes need not have the same shape. ☐ M.4.12.8: Distinguish between equal and non-equal parts.	☐ Recognize fractions as lengths from zero to one. ☐ Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2..., and represent whole-number sums and differences within 100 on a number diagram. ☐ Identify a number line. ☐ Recognize whole numbers as lengths from zero to one. ☐ Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2..., and represent whole-number sums and differences within 100 on a number diagram. ☐ Identify a number line. ☐ Label the fractions on a pre-made number line diagram. ☐ Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2..., and represent whole-number sums and differences within 100 on a number diagram. ☐ Recognize a number line diagram with equally spaced points. ☐ Compare length using non-standard units to determine which is longer.

Operations with Numbers: Fractions

Cluster	2019 Math COS Standard	
Extend understanding of fraction equivalence and ordering. <i>Denominators are limited to 2, 3, 4, 5, 6, 8, 10, 12, and 100.</i>	13. *Using area and length fraction models, explain why one fraction is equivalent to another, taking into account that the number and size of the parts differ even though the two fractions themselves are the same size. a. Apply principles of fraction equivalence to recognize and generate equivalent fractions.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.4.13.1: Identify fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts and size $1/b$. ☐ M.4.13.2: Identify a fraction as a number on the number line; represent fractions on a number line diagram. ☐ M.4.13.3: Recognize a fraction as a number on the number line. ☐ M.4.13.4: Represent fractions on a number line diagram. ☐ M.4.13.5: Recognize fractions as numerals that may represent division problems. ☐ M.4.13.6: Label numerator, denominator, and fraction bar. ☐ M.4.13.7: Identify parts of a whole with two, three, or four equal parts. ☐ M.4.13.8: Distinguish between equal and non-equal parts. ☐ M.4.13.9: Define area, length, equivalent, fraction, numerator, and denominator.	☐ Recognize fractions as lengths from zero to one. ☐ Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2..., and represent whole-number sums and differences within 100 on a number diagram. ☐ Identify a number line. ☐ Recognize whole numbers as lengths from zero to one. ☐ Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2..., and represent whole-number sums and differences within 100 on a number diagram. ☐ Identify a number line. ☐ Label the fractions on a pre-made number line diagram. ☐ Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2..., and represent whole-number sums and differences within 100 on a number diagram. ☐ Recognize a number line diagram with equally spaced points. ☐ Compare length using non-standard units to determine which is longer.

Operations with Numbers: Fractions

Cluster	2019 Math COS Standard	
Extend understanding of fraction equivalence and ordering. <i>Denominators are limited to 2, 3, 4, 5, 6, 8, 10, 12, and 100.</i>	14. *Compare two fractions with different numerators and different denominators using concrete models, benchmarks (0, $\frac{1}{2}$, 1), common denominators, and/or common numerators, recording the comparisons with symbols $>$, $=$, or $<$, and justifying the conclusions. a. Explain that comparison of two fractions is valid only when the two fractions refer to the same whole.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.4.14.1: Identify fraction $\frac{1}{b}$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction $\frac{a}{b}$ as the quantity formed by a parts and size $\frac{1}{b}$. ☐ M.4.14.2: Identify a fraction as a number on the number line, represent fractions on a number line diagram. ☐ M.4.14.3: Recognize a fraction as a number on the number line. ☐ M.4.14.4: Represent fractions on a number line diagram. ☐ M.4.14.5: Recognize fractions as numerals that may represent division problems. ☐ M.4.14.6: Label numerator, denominator, and fraction bar. ☐ M.4.14.7: Identify parts of a whole with two, three, or four equal parts. ☐ M.4.14.8: Distinguish between equal and non-equal parts.	☐ Recognize fractions as lengths from zero to one. ☐ Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2..., and represent whole-number sums and differences within 100 on a number diagram. ☐ Identify a number line. ☐ Recognize whole numbers as lengths from zero to one. ☐ Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2..., and represent whole-number sums and differences within 100 on a number diagram. ☐ Identify a number line. ☐ Label the fractions on a pre-made number line diagram. ☐ Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2..., and represent whole-number sums and differences within 100 on a number diagram. ☐ Recognize a number line diagram with equally spaced points. ☐ Compare length using non-standard units to determine which is longer.

Operations with Numbers: Fractions

Cluster	2019 Math COS Standard	
Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.	15. *Model and justify decompositions of fractions and explain addition and subtraction of fractions as joining or separating parts referring to the same whole. a. Decompose a fraction as a sum of unit fractions and as a sum of fractions with the same denominator in more than one way using area models, length models, and equations. b. Add and subtract fractions and mixed numbers with like denominators using fraction equivalence, properties of operations, and the relationship between addition and subtraction. c. Solve word problems involving addition and subtraction of fractions and mixed numbers having like denominators, using drawings, visual fraction models, and equations to represent the problem.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.4.15.1: Recognize that a whole can be partitioned into differing equal parts (halves, fourths, eighths, etc.). ☐ M.4.15.2: Identify numerator and denominator. ☐ M.4.15.3: Recall basic addition and subtraction facts. ☐ M.4.15.4: Demonstrate an understanding of fractional parts. ☐ M.4.15.5: Recall basic addition and subtraction facts. ☐ M.4.15.6: Define mixed numbers. ☐ M.4.15.7: Recall basic addition and subtraction facts. ☐ M.4.15.8: Demonstrate an understanding of fractional parts. ☐ M.4.15.9: Solve basic word problems using whole numbers. ☐ M.4.15.10: Express parts of a whole as a fraction.	☐ Define fraction, numerator, and denominator. ☐ Identify the parts of a fraction a/b as the quantity formed by a parts and size $1/b$. ☐ Label numerator, denominator, and fraction bar. ☐ Identify parts of a whole with two, three, or four equal parts. ☐ Distinguish between equal and non-equal parts. ☐ Partition circles and rectangles into two and four equal shares; describe the shares using the words halves, fourths, and quarters; and use the phrases half of, fourth of, and quarter of. ☐ Recognize fractions as lengths from zero to one. ☐ Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2..., and represent whole-number sums and differences within 100 on a number diagram.

	☐ M.4.15.11: Write number sentences for word problems. ☐ M.4.15.12: Identify key terms in word problems. ☐ M.4.15.13: Recall basic addition and subtraction facts.	
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Operations with Numbers: Fractions

Cluster	2019 Math COS Standard	
Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.	16. *Apply and extend previous understandings of multiplication to multiply a whole number times a fraction. a. Model and explain how a non-unit fraction can be represented by a whole number times the unit fraction. b. Extend previous understanding of multiplication to multiply a whole number times any fraction less than one. c. Solve word problems involving multiplying a whole number times a fraction using visual fraction models and equations to represent the problem.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.4.16.1: Recognize fractions in their simplest forms. ☐ M.4.16.2: Express whole numbers as fractions and recognize fractions that are equivalent to whole numbers. ☐ M.4.16.3: Demonstrate an understanding of fractional parts. ☐ M.4.16.4: Apply properties of operations as strategies to multiply and divide. ☐ M.4.16.5: Recall basic multiplication facts. ☐ M.4.16.6: Define multiple. ☐ M.4.16.7: Compare two fractions with the same numerator or the same denominator by reasoning about their size. ☐ M.4.16.8: Recognize that comparisons are valid only when the two fractions refer to the same whole. ☐ M.4.16.9: Record results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.	☐ Define fraction, numerator, and denominator. ☐ Identify the parts of a fraction a/b as the quantity formed by a parts and size $1/b$. ☐ Label numerator, denominator, and fraction bar. ☐ Identify parts of a whole with two, three, or four equal parts. ☐ Distinguish between equal and non-equal parts. ☐ Partition circles and rectangles into two and four equal shares; describe the shares using the words halves, fourths, and quarters; and use the phrases half of, fourth of, and quarter of. ☐ Demonstrate conceptual understanding of adding or subtracting 10 using concrete models. ☐ Recognize the place value of ones, tens, and hundreds. ☐ Count forward and backward by 100. ☐ Count forward and backward by 10.

	☐ M.4.16.10: Name the first ten multiples of each one-digit natural number. ☐ M.4.16.11: Express whole numbers as fractions and recognize fractions that are equivalent to whole numbers. ☐ M.4.16.12: Solve simple fractions using multiplication strategies. ☐ M.4.16.13: Recognize equivalent forms of fractions. ☐ M.4.16.14: Multiply proper fractions with common denominators 2-10. ☐ M.4.16.15: Solve word problems using whole numbers. ☐ M.4.16.16: Write number sentences for word problems. ☐ M.4.16.17: Identify key terms in word problems. ☐ M.4.16.18: Multiply and divide within 100. ☐ M.4.16.19: Recall basic multiplication facts.	
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Operations with Numbers: Fractions

Cluster	2019 Math COS Standard	
Understand decimal notation for fractions and compare decimal fractions. <i>Denominators are limited to 10 and 100.</i>	17. *Express, model, and explain the equivalence between fractions with denominators of 10 and 100. a. Use fraction equivalency to add two fractions with denominators of 10 and 100.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.4.17.1: Recognize equivalent forms of fractions and decimals. ☐ M.4.17.2: Demonstrate equivalent fractions using concrete objects or pictorial representation. ☐ M.4.17.3: Recognize pictorial representations of equivalent fractions and decimals in tenths and hundredths. ☐ M.4.17.4: Define equivalency. ☐ M.4.17.5: Identify place value of decimals to the tenths and hundredths. ☐ M.4.17.6: Use place value understanding to round whole numbers to the nearest 10 or 100.	☐ Define equivalent. ☐ Recognize pictorial representations of equivalent fractions. ☐ Recognize different interpretations of fractions, including parts of a set or a collection, points on a number line, numbers that lie between two consecutive whole numbers, and lengths of segments on a ruler. ☐ Recognize that equal shares of identical wholes need not have the same shape. ☐ Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape and compose new shapes from the composite shape. ☐ Label a fraction with multiple representations. ☐ Partition circles and rectangles into two and four equal shares; and describe the shares using the words halves, fourths, and quarters; and use the phrases half of, fourth of, and quarter of. ☐ Recognize different interpretations of fractions, including parts of a set or a collection, points on a number line, numbers that lie between two consecutive whole numbers, and lengths of segments on a ruler. ☐ Label a pictorial representation.

Operations with Numbers: Fractions

Cluster	2019 Math COS Standard	
Understand decimal notation for fractions and compare decimal fractions. <i>Denominators are limited to 10 and 100.</i>	18. *Use models and decimal notation to represent fractions with denominators of 10 and 100.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.4.18.1: Compare two fractions with the same numerator or the same denominator by reasoning about their size. ☐ M.4.18.2: Recognize that comparisons are valid only when the two fractions refer to the same whole. ☐ M.4.18.3: Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model. ☐ M.4.18.4: Convert fractions to decimals. ☐ M.4.18.5: Compare two decimals to tenths. ☐ M.4.18.6: Compare whole numbers. ☐ M.4.18.7: Identify comparison symbols. Examples: $>$, $<$, and $=$.	☐ Define equivalent. ☐ Recognize pictorial representations of equivalent fractions. ☐ Recognize different interpretations of fractions, including parts of a set or a collection, points on a number line, numbers that lie between two consecutive whole numbers, and lengths of segments on a ruler. ☐ Recognize that equal shares of identical wholes need not have the same shape. ☐ Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape and compose new shapes from the composite shape. ☐ Label a fraction with multiple representations. ☐ Recognize that a whole can be partitioned into differing equal parts (halves, fourths, eighths, etc.). ☐ Partition circles and rectangles into two and four equal shares; and describe the shares using the words halves, fourths, and quarters; and use the phrases half of, fourth of, and quarter of. ☐ Recognize different interpretations of fractions, including parts of a set or a collection, points on a number line, numbers that lie between two consecutive whole numbers, and lengths of segments on a ruler. ☐ Label a pictorial representation. ☐ Recognize that a fraction is a part of a whole. M. 3.15b.4: Partition circles and rectangles into two and four equal shares; describe the shares using the words halves, fourths, and quarters; and use the phrases half of, fourth of, and quarter of. ☐ Define numerator and denominator.

		☐ Partition circles and rectangles into two, three, or four equal shares; describe the shares using the words halves, thirds, half of, a third of, etc.; and describe the whole as two halves, three thirds, or four fourths. ☐ Recognize that a whole can be partitioned into differing equal parts (halves, fourths, eighths, etc.). ☐ Identify parts of a whole. ☐ Represent a fraction with a pictorial model. ☐ Identify $<$, $>$, and $=$ signs. ☐ Recognize that equal shares of identical wholes need not have the same shape. ☐ Recognize that a whole can be partitioned into equal parts (halves, fourths, eighths, etc.). ☐ Order three objects by length; compare the lengths of two objects indirectly by using a third object. ☐ Define greater than, less than and equal to. ☐ Compare two two-digit numbers based on meanings of the ten's and one's digits, recording the results of comparisons with the symbols $>$, $=$, and $<$. ☐ Arrange two-digit numbers in order from greatest to least or least to greatest. ☐ Identify zero as a place holder in two-digit and three-digit numbers. ☐ Model using $>$, $=$, and $<$ symbols to record the results of comparisons of two two-digit numbers. ☐ Select numbers on a number line that are more than, less than or equal to a specified number. ☐ Match the words greater than, equal to and less than to the symbols $>$, $=$, and $<$. ☐ Determine the value of the digits in the ones and tens place. ☐ Identify sets with more, less, or equal objects.
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Operations with Numbers: Fractions

Cluster	2019 Math COS Standard	
Understand decimal notation for fractions and compare decimal fractions. <i>Denominators are limited to 10 and 100.</i>	19. Use visual models and reasoning to compare two decimals to hundredths (referring to the same whole), recording comparisons using symbols $>$, $=$, or $<$, and justifying the conclusions.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.4.19.1: Compare two fractions with the same numerator or the same denominator by reasoning about their size. ☐ M.4.19.2: Recognize that comparisons are valid only when the two fractions refer to the same whole. ☐ M.4.19.3: Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model. ☐ M.4.19.4: Convert fractions to decimals. ☐ M.4.19.5: Compare two decimals to tenths. ☐ M.4.19.6: Compare whole numbers. ☐ M.4.19.7: Identify comparison symbols. Examples: $>$, $<$, and $=$.	☐ Define equivalent. ☐ Recognize pictorial representations of equivalent fractions. ☐ Recognize different interpretations of fractions, including parts of a set or a collection, points on a number line, numbers that lie between two consecutive whole numbers, and lengths of segments on a ruler. ☐ Recognize that equal shares of identical wholes need not have the same shape. ☐ Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape and compose new shapes from the composite shape. ☐ Label a fraction with multiple representations. ☐ Recognize that a whole can be partitioned into differing equal parts (halves, fourths, eighths, etc.). ☐ Partition circles and rectangles into two and four equal shares; and describe the shares using the words halves, fourths, and quarters; and use the phrases half of, fourth of, and quarter of. ☐ Recognize different interpretations of fractions, including parts of a set or a collection, points on a number line, numbers that lie between two consecutive whole numbers, and lengths of segments on a ruler.

Data Analysis

Cluster	2019 Math COS Standard	
Represent and interpret data.	20. Interpret data in graphs (picture, bar, and line plots) to solve problems using numbers and operations. - Create a line plot to display a data set of measurements in fractions of a unit. - Solve problems involving addition and subtraction of fractions using information presented in line plots.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.4.20.1: Display data by making a line plot where the horizontal scale is marked off in appropriate units – whole numbers, halves, or quarters. ☐ M.4.20.2: Interpret data using graphs including bar, line, and circle graphs, and Venn diagrams. ☐ M.4.20.3: Identify the parts of a line plot. ☐ M.4.20.4: Recognize a line plot. ☐ M.4.20.5: Draw a scaled picture graph and a scaled bar graph to represent a data set.	☐ Define picture graph, bar graph, and data. ☐ Interpret the data to solve problems. ☐ Identify the parts of a graph (x-axis, y-axis, title, key, equal intervals, labels). ☐ Locate the data on a picture graph and a bar graph. ☐ Directly compare two objects, with a measurable attribute in common, to see which object has "more of" or "less of" the attribute and describe the difference. ☐ Define line plot. ☐ Identify the parts of a line plot. ☐ Measure objects to the nearest inch. ☐ Identify one-inch units on a ruler starting with 0. ☐ Express the length of an object as a whole number of length units by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. ☐ Directly compare two objects, with a measurable attribute in common, to see which object has "more of" or "less of" the attribute and describe the difference. ☐ Use addition and subtraction within 20 to solve addition and subtraction word problems with an unknown number. ☐ Describe a picture graph and a bar graph.

Measurement		
Cluster	2019 Math COS Standard	
Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.	21. Select and use an appropriate unit of measurement for a given attribute (length, mass, liquid volume, time) within one system of units: metric - km, m, cm; kg, g, l, ml; customary - lb, oz; time - hr, min, sec. a. Within one system of units, express measurements of a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.4.21.1: Define conversion. ☐ M.4.21.2: Define length, kilometers, meters, and centimeters. ☐ M.4.21.3: Define weight, kilograms, grams, pounds, ounces, liters, and milliliters. ☐ M.4.21.4: Define hour, minute, second. ☐ M.4.21.5: Measure and estimate liquid volumes and masses of objects using standard units of grams, kilograms, and liters. ☐ M.4.21.6: Identify standard units of measurement equivalents. Examples: 60 minutes equals 1 hour, 16 ounces equals 1 pound. ☐ M.4.21.7: Match measurement units to abbreviations. Examples: kilometers (km), meters (m), centimeters (cm), kilograms (kg), grams (g), pounds (lb), ounces (oz), liters (l), milliliters (ml).	☐ Define liquid volume, mass, grams, kilograms, and liters. ☐ Recognize how the standard units of measure compare to one another. ☐ Identify key terms for word problems. Examples: Difference, altogether, in all, between. ☐ Express the length of an object as a whole number of length units by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. ☐ Recall basic addition, subtraction, multiplication, and division facts. ☐ Describe measurable attributes of objects such as length or weight. Describe several measurable attributes of a single object. ☐ Compare equivalent units of time using hours and minutes. Examples: 60 minutes = one hour, 30 minutes = one half of an hour. ☐ Recognize key vocabulary and/or phrases associated with time. Examples: Quarter 'til = 15 minutes before; half past the hour = 30 minutes after the hour. ☐ Compare the lengths of time to complete everyday activities. Examples: Brushing your teeth = about 2 minutes; riding the bus = about 20 minutes. ☐ Tell and write time in hours and half-hours using analog and digital clocks. -Recognize hour, minute, and second hands on an analog clock. ☐ Count by 5's to 60.

		☐ Determine the monetary value of a set of like and unlike bills. ☐ Determine the monetary value of a set of like and unlike coins. ☐ Apply addition and subtraction strategies. ☐ Understand key words in addition and subtraction word problems involving money. Examples: adding to, taking from, putting together, taking apart, sum, difference, all together, how much more, how much is left, in all, cents, dollar, change, paid, total. ☐ Count forward from a given number by ones, fives, tens, and twenty-fives. ☐ Identify coins and bills and their value. ☐ Identify symbols for dollar (\$), cent (¢). ☐ Identify coins by name including penny, nickel, dime, and quarter. ☐ Sort pennies, nickels, dimes, and quarters. ☐ Count 10 objects. Examples: pennies and dollar bills. ☐ Tell and write time in hours and half-hours using analog and digital clocks. ☐ Recognize vocabulary terms related to time measurements. Examples: minute, hour, half hour, o'clock, morning, evening, a.m., p.m. ☐ Illustrate time to hour and half hour. Example: Given the time 3:00, illustrate long hand and short hand positions on a clock. ☐ Identify the short hand as the hour hand, and the long hand as the minute hand on an analog clock. ☐ Identify the first number as the hour, and the numbers after the colon as the minutes on a digital clock. ☐ Write numerals 0 to 59. ☐ Recognize numerals 0 to 59. ☐ Count to 60 by fives. ☐ Distinguish between analog and digital clocks. ☐ Organize, represent, and interpret data up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.
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Measurement		
Cluster	2019 Math COS Standard	
Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.	22. Use the four operations to solve measurement word problems with distance, intervals of time, liquid volume, mass of objects, and money. - Solve measurement problems involving simple fractions or decimals. - Solve measurement problems that require expressing measurements given in a larger unit in terms of a smaller unit. - Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.4.22.1: Define distance, time, elapsed time, volume, mass. ☐ M.4.22.2: Determine elapsed time to the day with calendars and to the hour with a clock. ☐ M.4.22.3: Express liquid volumes and masses of objects using standard units of grams, kilograms, and liters. ☐ M.4.22.4: Use addition, subtraction, multiplication, and division to solve one- and two-step word problems. ☐ M.4.22.5: Recognize key terms to solve word problems. ☐ M.4.22.6: Recall basic facts for addition, subtraction, multiplication, and division. ☐ M.4.22.7: Identify monetary equivalents. Examples: four quarters equal one dollar, five one-dollar bills equal five dollars.	☐ Define liquid volume, mass, grams, kilograms, and liters. ☐ Recognize how the standard units of measure compare to one another. ☐ Identify key terms for word problems. Examples: Difference, altogether, in all, between. ☐ Express the length of an object as a whole number of length units by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. ☐ Recall basic addition, subtraction, multiplication, and division facts. ☐ Describe measurable attributes of objects such as length or weight. Describe several measurable attributes of a single object. ☐ Compare equivalent units of time using hours and minutes. Examples: 60 minutes = one hour, 30 minutes = one half of an hour. ☐ Recognize key vocabulary and/or phrases associated with time. Examples: Quarter 'til = 15 minutes before; half past the hour = 30 minutes after the hour.

Measurement		
Cluster	2019 Math COS Standard	
Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.	23. Apply area and perimeter formulas for rectangles in real-world and mathematical situations.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.4.23.1: Recall the formula for area ($L \times W$). ☐ M.4.23.2: Recognize that unit squares are equal. ☐ M.4.23.3: Recall the formula for perimeter ($P = L + L + W + W$ or $P = 2L + 2W$). ☐ M.4.23.4: Recall basic addition and multiplication facts.	☐ Define perimeter. ☐ Recall the formula for perimeter ($P = L + L + W + W$ or $P = 2L + 2W$). ☐ Recall basic addition and multiplication facts. ☐ Build and draw shapes to possess defining attributes. ☐ Express the length of an object as a whole number of length units by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. ☐ Describe measurable attributes of objects such as length or weight. ☐ Define rows, columns, and total. ☐ Identify rectangle. ☐ Count to 20 by ones. ☐ Trace partitions in a rectangle.

Measurement		
Cluster	2019 Math COS Standard	
Geometric measurement: understand concepts of angle and measure angles.	24. Identify an angle as a geometric shape formed wherever two rays share a common endpoint.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.4.24.1: Define degree, angle, ray, and vertices. ☐ M.4.24.2: Recognize and draw shapes having specified attributes such as a given number of angles or a given number of equal faces. ☐ M.4.24.3: Estimate angle measures using 45°, 90°, 180°, 270°, or 360°. ☐ M.4.24.4: Identify angle, ray, and vertices. ☐ M.4.24.5: Draw shapes to possess defining attributes.	☐ Build and draw shapes to possess defining attributes. ☐ Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles). ☐ Model partitioning circles and rectangles. ☐ Decompose pictures made of simple shapes. ☐ Identify squares, circles, triangles, and rectangles. ☐ Explore shapes or figures that can be decomposed into smaller equal parts.

Measurement		
Cluster	2019 Math COS Standard	
Geometric measurement: understand concepts of angle and measure angles.	25. Use a protractor to measure angles in whole-number degrees and sketch angles of specified measure.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.4.25.1: Define symmetry. ☐ M.4.25.2: Model using a protractor to draw angles. ☐ M.4.25.3: Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen. ☐ M.4.25.4: Measure the length of an object by selecting and using appropriate tools such as a ruler. ☐ M.4.25.5: Measure length using standard and non-standard units of measurement. ☐ M.4.25.6: Plot points on grids, graphs, and maps using coordinates. ☐ M.4.25.7: Draw points, lines, line segments, and parallel and perpendicular lines, angles, and rays. ☐ M.4.25.8: Identify lines of symmetry on one-dimensional figures.	☐ Build and draw shapes to possess defining attributes. ☐ Express the length of an object as a whole number of length units by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. ☐ Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles). ☐ Identify a rectangle. ☐ Express the length of an object as a whole number of length units by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. ☐ Define halves, thirds, fourths, quarters, whole, parts (shares) and equal. ☐ Distinguish between equal and non-equal parts. ☐ Model partitioning circles and rectangles. ☐ Decompose pictures made of simple shapes. ☐ Identify squares, circles, triangles, and rectangles. ☐ Explore shapes or figures that can be decomposed into smaller equal parts.

Measurement		
Cluster	2019 Math COS Standard	
Geometric measurement: understand concepts of angle and measure angles.	26. Decompose an angle into non-overlapping parts to demonstrate that the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems on a diagram to find unknown angles in real-world or mathematical problems.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.4.26.1: Identify straight angles. ☐ M.4.26.2: Recognize angle measures such as 45°, 90°, 180°, 270°, 300°. ☐ M.4.26.3: Recall basic addition and subtraction facts. ☐ M.4.26.4: Skip count by fives and tens.	☐ Build and draw shapes to possess defining attributes. ☐ Express the length of an object as a whole number of length units by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. ☐ Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles). ☐ Identify a rectangle. ☐ Express the length of an object as a whole number of length units by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. ☐ Define halves, thirds, fourths, quarters, whole, parts (shares) and equal. ☐ Distinguish between equal and non-equal parts. ☐ Model partitioning circles and rectangles. ☐ Decompose pictures made of simple shapes. ☐ Identify squares, circles, triangles, and rectangles. ☐ Explore shapes or figures that can be decomposed into smaller equal parts.

Geometry

Cluster	2019 Math COS Standard	
Draw and identify lines and angles, and identify shapes by properties of their lines and angles.	27. *Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines, and identify these in two-dimensional figures.	
	Learning Objectives	Prior Knowledge Skills
	☐ M. 4.27.1: Define points, lines, line segments, rays, right angle, acute angle, obtuse angle, perpendicular lines, and parallel lines. ☐ M. 4.27.2: Define two-dimensional figure. ☐ M. 4.27.3: Recognize one-dimensional points, lines, and line segments. ☐ M. 4.27.4: Model shapes in the world by building shapes from components.	☐ Recall the vocabulary of shapes (labels, sides, faces, vertices, etc.). ☐ Recognize and draw shapes having specified attributes such as a given number of angles. ☐ Build and draw shapes to possess defining attributes. ☐ Sort shapes into categories. ☐ Define side, angle, face, closed, and open. ☐ Use vocabulary related to shape attributes. Examples: sides, angles, face, closed, open. ☐ Trace shapes. ☐ Sort triangles, quadrilaterals, pentagons, hexagons, and cubes. ☐ Explore triangles, quadrilaterals, pentagons, hexagons, and cubes.

Geometry

Cluster	2019 Math COS Standard	
Draw and identify lines and angles, and identify shapes by properties of their lines and angles.	28. *Identify two-dimensional figures based on the presence or absence of parallel or perpendicular lines or the presence or absence of angles of a specified size. a. Describe right triangles as a category and identify right triangles.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.4.28.1: Define right angle. ☐ M.4.28.2: Recognize that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). ☐ M.4.28.3: Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories. ☐ M.4.28.4: Recognize and draw shapes having specified attributes such as a given number of angles or a given number of equal faces. ☐ M.4.28.5: Identify triangles.	☐ Recall the vocabulary of shapes (labels, sides, faces, vertices, etc.). ☐ Recognize and draw shapes having specified attributes such as a given number of angles. ☐ Build and draw shapes to possess defining attributes. ☐ Sort shapes into categories. ☐ Define side, angle, face, closed, and open. ☐ Use vocabulary related to shape attributes. Examples: sides, angles, face, closed, open. ☐ Trace shapes. ☐ Sort triangles, quadrilaterals, pentagons, hexagons, and cubes. ☐ Explore triangles, quadrilaterals, pentagons, hexagons, and cubes.

Geometry		
Cluster	2019 Math COS Standard	
Draw and identify lines and angles and identify shapes by properties of their lines and angles.	29. Define a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. a. Identify line-symmetric figures and draw lines of symmetry.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.4.29.1: Identify line symmetric figures. ☐ M.4.29.2: Draw lines of symmetry on a one-dimensional figure. ☐ M. 4.29.3: Recognize lines of symmetry on a one-dimensional figure.	☐ Recognize a fraction as part of a whole. ☐ Decompose a large pre-made shape using smaller shapes. ☐ Compose a large pre-made shape using smaller shapes. ☐ Partition a rectangle into rows and columns of same-size squares, and count to find the total number of them. ☐ Partition circles and rectangles into two, three, or four equal shares; describe the shares using the words halves, thirds, half of, a third of, etc.; and describe the whole as two halves, three thirds, or four fourths. ☐ Define halves, thirds, fourths, quarters, whole, parts (shares) and equal. ☐ Distinguish between equal and non-equal parts. ☐ Model partitioning circles and rectangles. ☐ Decompose pictures made of simple shapes.

Grade 5

Operations and Algebraic Thinking

Cluster	2019 Math COS Standard	
Write and interpret numerical expressions.	1. Write, explain, and evaluate simple numerical expressions involving the four operations to solve up to two-step problems. Include expressions involving parentheses, brackets, or braces, using commutative, associative, and distributive properties.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.5.1.1: Define parentheses, braces, and brackets. ☐ M.5.1.2: Distinguish between non-numerical and numerical expression. ☐ M.5.1.3: Recognize expressions. ☐ M.5.1.4: Apply properties of operations as strategies to add and subtract. ☐ M.5.1.5: Represent addition and subtraction with objects, mental images, drawings, expressions, or equations.	☐ Write, explain, and evaluate numerical expressions representing two-step problems in context. ☐ Evaluate numerical expressions with grouping symbols. ☐ Translate a numerical expression into words. ☐ Write a numerical expression given a mathematical expression in words.

Operations and Algebraic Thinking

Cluster	2019 Math COS Standard	
Analyze patterns and relationships.	2. Generate two numerical patterns using two given rules and complete an input/output table for the data. a. Use data from an input/output table to identify apparent relationships between corresponding terms. b. Form ordered pairs from values in an input/output table. b. Graph ordered pairs from an input/output table on a coordinate plane.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.5.2.1: Construct repeating and growing patterns with a variety of representations. ☐ M.5.2.2: Continue an existing pattern. ☐ M.5.2.3: Identify arithmetic patterns (including patterns in the addition table or multiplication table).	☐ Generate two numerical patterns using two given rules. ☐ Complete an input/output table for data. ☐ Identify relationship between terms in an input/output table. ☐ Form ordered pairs from an input/output table. ☐ Graph ordered pairs on a coordinate plane.

Operations with Numbers: Base Ten

Cluster	2019 Math COS Standard	
Understand the place value system.	3. Using models and quantitative reasoning, explain that in a multi-digit number, including decimals, a digit in any place represents ten times what it represents in the place to its right and $\frac{1}{10}$ of what it represents in the place to its left. a. Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, using whole-number exponents to denote powers of 10. b. Explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10, using whole-number exponents to denote powers of 10.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.5.3.1: Use place value understanding to round whole numbers to the nearest 10 or 100. ☐ M.5.3.2: Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits using $>$, $=$, and $<$ symbols to record the results of comparisons. ☐ M.5.3.3: Identify that the three digits of a three-digit number represent amounts of hundreds, tens, and ones.	☐ Reason and explain the relationship between two successive place values. ☐ Explain patterns of zeros of the product when multiplying by powers of 10. ☐ Explain patterns in placement of decimals when multiplying or dividing by power of 10. ☐ Write powers of 10 using exponential notation.

Operations with Numbers: Base Ten

Cluster	2019 Math COS Standard	
Understand the place value system.	4. *Read, write, and compare decimals to thousandths. - Read and write decimals to thousandths using base-ten numerals, number names, and expanded form. - Compare two decimals to thousandths based on the meaning of the digits in each place, using $>$, $=$, and $<$ to record the results of comparisons.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.5.4.1: Recognize decimals as parts of a whole. ☐ M.5.4.2: Compare whole numbers. ☐ M.5.4.3: Write whole numbers in words and expanded form. ☐ M.5.4.4: Read whole numbers. ☐ M.5.4.5: Define expanded notation and standard form. ☐ M.5.4.6: Convert a number written in expanded to standard form. ☐ M.5.4.7: Define hundredths and thousandths. ☐ M.5.4.8: Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits using $>$, $=$, and $<$ symbols to record the results of comparisons. ☐ M.5.4.9: Identify comparison symbols. Examples: $>$, $=$, and $<$.	☐ Read and write decimal values in word form, standard form, and expanded form. ☐ Compare decimals to thousandths using $<$, $>$, or $=$. ☐ Understand rounding decimals using place value.

Operations with Numbers: Base Ten

Cluster	2019 Math COS Standard	
Understand the place value system.	5. Use place value understanding to round decimals to thousandths.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.5.5.1: Round multi-digit whole numbers to any place. ☐ M.5.5.2: Round whole numbers to the nearest 10 or 100.	☐ Use the standard algorithm to find a product.

Operations with Numbers: Base Ten

Cluster	2019 Math COS Standard	
Perform operations with multi-digit whole numbers and decimals to hundredths.	6. *Fluently multiply multi-digit whole numbers using the standard algorithm.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.5.6.1: Demonstrate steps in setting up a long multiplication problem. ☐ M.5.6.2: Multiply 2-digit numbers by 1-digit multiplier. ☐ M.5.6.3: Multiply 1-digit numbers by 1-digit multiplier. ☐ M.5.6.4: Recall basic multiplication facts. ☐ M.5.6.5: Recall repeated addition facts.	☐ Find whole number quotients and remainders using a variety of strategies based on place value and properties of operations. ☐ Illustrate and explain the calculation using equations, arrays, and area mode. ☐ Use concrete models, drawings, and strategies to add, subtract, multiply, and divide decimals. ☐ Relate strategies for operations with decimals to a written method and explain reasoning used. ☐ Solve real-world context problems involving decimals.

Operations with Numbers: Base Ten

Cluster	2019 Math COS Standard	
Perform operations with multi-digit whole numbers and decimals to hundredths.	7. *Use strategies based on place value, properties of operations, and/or the relationship between multiplication and division to find whole-number quotients and remainders with up to four-digit dividends and two-digit divisors. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.5.7.1: Construct a division equation with an example of the division algorithm. ☐ M.5.7.2: Illustrate the division algorithm using a one-digit divisor and a 2-digit dividend. ☐ M.5.7.3: Identify the place value of a division problem. ☐ M.5.7.4: Restate the inverse process of division as multiplication. ☐ M.5.7.5: Recall basic multiplication facts.	☐ Use concrete models, drawings, and strategies to add, subtract, multiply, and divide decimals. ☐ Relate strategies for operations with decimals to a written method and explain reasoning used. ☐ Solve real-world context problems involving decimals.

Operations with Numbers: Base Ten

Cluster	2019 Math COS Standard	
Perform operations with multi-digit whole numbers and decimals to hundredths.	8. *Add, subtract, multiply, and divide decimals to hundredths using strategies based on place value, properties of operations, and/or the relationships between addition/subtraction and multiplication/division; relate the strategy to a written method, and explain the reasoning used. a. Use concrete models and drawings to solve problems with decimals to hundredths. b. Solve problems in a real-world context with decimals to hundredths.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.5.8.1: Use decimal notation for fractions with denominators 10 or 100. ☐ M.5.8.2: Multiply and divide within 100, using strategies such as the relationship between multiplication and division or properties of operations. ☐ M.5.8.3: Add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. ☐ M.5.8.4: Apply properties of operations as strategies to multiply and divide. ☐ M.5.8.5: Identify that 100 can be thought of as a bundle of ten tens, called a “hundred”. ☐ M.5.8.6: Identify that the numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones). ☐ M.5.8.7: Recall basic addition, subtraction, multiplication, and division facts.	☐ Use fraction equivalence to add and subtract fractions and mixed numbers with unlike denominators. ☐ Find whole number quotients and remainders using a variety of strategies based on place value and properties of operations. ☐ Illustrate and explain the calculation using equations, arrays, and area models. ☐ Use concrete models, drawings, and strategies to add, subtract, multiply, and divide decimals. ☐ Relate strategies for operations with decimals to a written method and explain reasoning used. ☐ Solve real-world context problems involving decimals.

Operations with Numbers: Fractions

Cluster	2019 Math COS Standard	
Use equivalent fractions as a strategy to add and subtract fractions.	9. *Model and solve real-word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.5.9.1: Add and subtract mixed numbers with like denominators. ☐ M.5.9.2: Recognize that comparisons are valid only when the two fractions refer to the same whole. ☐ M.5.9.3: Record the results of comparisons with symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model. ☐ M.5.9.4: Recognize a fraction as a number on the number line; represent fractions on a number line diagram. ☐ M.5.9.5: Recognize key terms to solve word problems. ☐ M.5.9.6: Apply properties of operations for addition and subtraction. ☐ M.5.9.7: Recall basic addition and subtraction facts.	☐ Use fraction equivalence to add and subtract fractions and mixed numbers with unlike denominators.

Operations with Numbers: Fractions

Cluster	2019 Math COS Standard	
Use equivalent fractions as a strategy to add and subtract fractions.	10. *Add and subtract fractions and mixed numbers with unlike denominators, using fraction equivalence to calculate a sum or difference of fractions or mixed numbers with like denominators.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.5.10.1: Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. ☐ M.5.10.2: Explain equivalence of fractions in special cases and compare fractions by reasoning about their size. ☐ M.5.10.3: Identify two fractions as equivalent (equal) if they are the same size or the same point on a number line. ☐ M.5.10.4: Recognize and generate simple equivalent fractions. ☐ M.5.10.5: Express whole numbers as fractions and recognize fractions that are equivalent to whole numbers. ☐ M.5.10.6: Compare two fractions with the same numerator or the same denominator by reasoning about their size. ☐ M.5.10.7: Recall basic addition, subtraction, multiplication, and division facts.	☐ Use fraction equivalence to add and subtract fractions and mixed numbers with unlike denominators.

Operations with Numbers: Fractions

Cluster	2019 Math COS Standard	
Apply and extend previous understandings of multiplication and division to multiply and divide fractions.	11. Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers. - Model and interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). - Use visual fraction models, drawings, or equations to represent word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.5.11.1: Define a mixed number. ☐ M.5.11.2: Generate equivalent fractions. ☐ M.5.11.3: Recognize a fraction as a number on the number line; represent fractions on a number line diagram.	☐ Find products of a fraction times a whole number and products of a fraction times a fraction. ☐ Use area models, linear models or set models to represent products.

Operations with Numbers: Fractions

Cluster	2019 Math COS Standard	
Apply and extend previous understandings of multiplication and division to multiply and divide fractions.	12. *Apply and extend previous understandings of multiplication to find the product of a fraction times a whole number or a fraction times a fraction. a. Use a visual fraction model (area model, set model, or linear model) to show $(a/b \times q)$ and create a story context for this equation to interpret the product as a part of a partition of q into b equal parts. b. Use a visual fraction model (area model, set model, or linear model) to show $(a/b) \times (c/d)$ and create a story context for this equation to interpret the product. c. Multiply fractional side lengths to find areas of rectangles and represent fraction products as rectangular areas. d. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths to show that the area is the same as would be found by multiplying the side lengths.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.5.12.1: Define proper fraction. ☐ M.5.12.2: Multiply fractions using denominators between 2 and 5. ☐ M.5.12.3: Identify proper and improper fractions. ☐ M.5.12.4: Recall basic multiplication facts. ☐ M.5.12.5: Model changing a whole number to a fraction. ☐ M.5.12.6: Partition a rectangle into rows and columns of same size squares, and count to find the total number of them. ☐ M.5.12.7: Label the numerator and denominator of a fraction. ☐ M.5.12.8: Count the area squares for the length and width.	☐ Solve real-word problems involving multiplication of fractions and mixed numbers. ☐ Write equations to represent the word situation. ☐ Use visual fraction models to represent the problem.

*Critical Standard

	☐ M.5.12.9: Identify the width and length of a rectangle.	
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Operations with Numbers: Fractions

Cluster	2019 Math COS Standard	
Apply and extend previous understandings of multiplication and division to multiply and divide fractions.	13. Interpret multiplication as scaling (resizing). a. Compare the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. <i>Example: Use reasoning to determine which expression is greater? _____.</i> b. Explain why multiplying a given number by a fraction greater than 1 results in a product greater than the given number and relate the principle of fraction equivalence. c. Explain why multiplying a given number by a fraction less than 1 results in a product smaller than the given number and relate the principle of fraction equivalence.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.5.13.1: Define scaling. ☐ M.5.13.2: Define principle of fraction equivalence. ☐ M.5.13.3: Multiply a fraction by a whole number. ☐ M.5.13.4: Compare two fractions with the same numerator or the same denominator by reasoning about their size. ☐ M.5.13.5: Recognize that comparisons are valid only when the two fractions refer to the same whole. ☐ M.5.13.6: Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model. ☐ M.5.13.7: Express whole numbers as fractions and recognize fractions that are equivalent to whole numbers. ☐ M.5.13.8: Identify factor and product. ☐ M.5.13.9: Use comparison symbols. Examples: $>$, $=$, or $<$.	☐ Interpret multiplication as scaling. ☐ Use reasoning to compare products of multiplication expressions. ☐ Reason and explain when multiplying a given number by a fraction why the product will be greater than or less than the original number.

Operations with Numbers: Fractions		
Cluster	2019 Math COS Standard	
Apply and extend previous understandings of multiplication and division to multiply and divide fractions.	14. *Model and solve real-world problems involving multiplication of fractions and mixed numbers using visual fraction models, drawings, or equations to represent the problem.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.5.14.1: Define improper fraction, mixed number, fraction, equations, numerator, denominator. ☐ M.5.14.2: Multiply proper fractions with common denominators 2-10. ☐ M.5.14.3: Solve problems using whole numbers. ☐ M.5.14.4: Write number sentences for word problems. ☐ M.5.14.5: Identify key terms to solve multiplication word problems. Examples: times, every, at this rate, each, per, equal/equally, in all, total. ☐ M.5.14.6: Recall basic multiplication facts.	☐ Divide unit fractions by a whole number and whole numbers by unit fractions. ☐ Use visual models to illustrate quotients. ☐ Create story contexts for division. ☐ Use relationship between multiplication and division to explain quotients.

Operations with Numbers: Fractions

Cluster	2019 Math COS Standard	
Apply and extend previous understandings of multiplication and division to multiply and divide fractions.	15. *Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. - Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions and illustrate using visual fraction models, drawings, and equations to represent the problem. - Create a story context for a unit fraction divided by a whole number, and use a visual fraction model to show the quotient. - Create a story context for a whole number divided by a unit fraction, and use a visual fraction model to show the quotient.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.5.15.1: Define quotient. ☐ M.5.15.2: Multiply a fraction by a whole number. ☐ M.5.15.3: Recognize key terms to solve word problems. Examples: times, every, at this rate, each, per, equal/equally, in all, total. ☐ M.5.15.4: Recall basic multiplication and division facts. ☐ M.5.15.5: Express whole numbers as fractions. ☐ M.5.15.6: Recognize fractions that are equivalent to whole numbers. ☐ M.5.15.7: Recall basic multiplication and division facts. ☐ M.5.15.8: Solve word problems involving multiplication of a fraction by a whole number. ☐ M.5.15.9: Recognize key terms to solve word problems.	☐ Find products of a fraction times a whole number and products of a fraction times a fraction. ☐ Use area models, linear models or set models to represent products. ☐ Create a story context to represent equations $(a/b) \times q$ and $(a/b) \times (c/d)$ to interpret products. ☐ Find area of rectangles with fractional side lengths and represent products as rectangular areas. ☐ Find the area of a rectangle by tiling the area of a rectangle with unit squares.

*Critical Standard

	☐ M.5.15.10: Recall basic multiplication and division facts.	
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Data Analysis

Cluster	2019 Math COS Standard	
Represent and interpret data.	16. Make a line plot to display a data set of measurements in fractions of a unit $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$. a. Add, subtract, multiply, and divide fractions to solve problems involving information presented in line plots. <i>Note: Division is limited to unit fractions by whole numbers and whole numbers by unit fractions.</i>	
	Learning Objectives	Prior Knowledge Skills
	☐ M.5.16.1: Make a line plot to display a data set of measurements in fractions of a unit. ☐ M.5.16.2: Solve problems involving addition and subtraction of fractions by using information presented in line plots. ☐ M.5.16.3: Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. ☐ M.5.16.4: Solve one-and two-step “how many more” and “how many less” problems using information presented in scaled bar graphs. ☐ M.5.16.5: Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. ☐ M.5.16.6: Solve simple put-together, take-apart, and compare problems using information presented in a bar graph.	☐ Recall the meaning of a fraction as part of a whole. ☐ Identify the location of a fraction on a number line. ☐ Compare fractions by finding common denominators. ☐ Find an unknown value to complete a number sentence. ☐ Read a line plot and bar graph.

Measurement		
Cluster	2019 Math COS Standard	
Convert like measurement units within a given measurement system.	17. *Convert among different-sized standard measurement units within a given measurement system and use these conversions in solving multi-step, real-world problems.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.5.17.1: Identify relative sizes of measurement units within one system of units, including km, m, cm; kg, g; lb, oz; l, ml; and hr, min, sec. ☐ M.5.17.2: Express measurements in a larger unit in terms of a smaller unit. ☐ M.5.17.3: Solve two-step word problems. ☐ M.5.17.4: Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). ☐ M.5.17.5: Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. ☐ M.5.17.6: Recall basic addition, subtraction, multiplication, and division facts.	☐ Create a line plot with appropriate intervals. ☐ Represent data on a line plot. ☐ Apply strategies for solving problems involving all four operations with the fractional data. ☐ Convert measurement units. ☐ Solve multi-step word problems involving measurement conversions.

Measurement		
Cluster	2019 Math COS Standard	
Geometric measurement: Understand concepts of volume and relate volume to multiplication and to addition.	18. Identify volume as an attribute of solid figures, and measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised (non-standard) units. Pack a solid figure without gaps or overlaps using n unit cubes to demonstrate volume as n cubic units.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.5.18.1: Define volume including the formulas $V = l \times w \times h$, and $V = B \times h$. ☐ M.5.18.2: Define solid figures. ☐ M.5.18.3: Define unit cube. ☐ M.5.18.4: Recognize that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). ☐ M.5.18.5: Describe attributes of three-dimensional figures. ☐ M.5.18.6: Describe attributes of two-dimensional figures. ☐ M.5.18.7: Compare the unit size of volume/capacity in the metric system including milliliters and liters. ☐ M.5.18.8: Define cubic inches, cubic centimeters, and cubic feet. ☐ M.5.18.9: Compare the unit size of volume/capacity in the customary system including fluid ounces, cups, pints, quarts, gallons. ☐ M.5.18.10: Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).	☐ Count unit cubes to find volume. ☐ Demonstrate volume by packing a solid figure with unit cubes. ☐ Solve word problems involving volume. ☐ Use associative property of multiplication to find volume. ☐ Relate operations of multiplication and addition to finding volume. ☐ Apply formulas to find volume of right rectangular prisms. ☐ Find volume of solid figures composed of two rectangular prisms.

	☐ M.5.18.11: Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). ☐ M.5.18.12: Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. ☐ M.5.18.13: Recall basic multiplication facts. ☐ M.5.18.14: Fluently add.	
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Measurement		
Cluster	2019 Math COS Standard	
Geometric measurement: Understand concepts of volume and relate volume to multiplication and to addition.	19. *Relate volume to the operations of multiplication and addition, and solve real-world and mathematical problems involving volume. - Use the associative property of multiplication to find the volume of a right rectangular prism and relate it to packing the prism with unit cubes. Show that the volume can be determined by multiplying the three edge lengths or by multiplying the height by the area of the base. - Apply the formulas $V = l \times w \times h$ and $V = B \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real-world and mathematical problems. - Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the two parts, applying this technique to solve real-world problems.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.5.19.1: Define volume. ☐ M.5.19.2: Recognize angle measure as additive. ☐ M.5.19.3: Apply the area and perimeter formulas for rectangles in real-world and mathematical problems. ☐ M.5.19.4: Solve real-world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters. ☐ M.5.19.5: Recognize the formula for volume. ☐ M.5.19.6: Recall the attributes of three-dimensional solids. ☐ M.5.19.7: Recall basic multiplication facts.	☐ Count unit cubes to find volume. ☐ Demonstrate volume by packing a solid figure with unit cubes. ☐ Convert measurement units. ☐ Solve multi-step word problems involving measurement conversions.

	☐ M.5.19.8: Fluently add. ☐ M.5.19.9: Compare the unit size of volume/capacity in the metric system including milliliters and liters. ☐ M.5.19.10: Measure and estimate liquid volumes. ☐ M.5.19.11: Recall basic multiplication facts. ☐ M.5.19.12: Compare the unit size of volume/capacity in the metric system including milliliters and liters. ☐ M.5.19.13: Recognize the formula for volume. ☐ M.5.19.14: Recall basic multiplication facts. ☐ M.5.19.15: Describe attributes of three-dimensional figures. ☐ M.5.19.16: Describe attributes of two-dimensional figures. ☐ M.5.19.17: Identify solid figures.	
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Geometry

Cluster	2019 Math COS Standard	
Graph points on the coordinate plane to solve real-world and mathematical problems.	20. *Graph points in the first quadrant of the coordinate plane, and interpret coordinate values of points to represent real-world and mathematical problems.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.5.20.1: Define ordered pair of numbers, quadrant one, coordinate plane, and plot points. ☐ M.5.20.2: Label the horizontal axis (x). ☐ M.5.20.3: Label the vertical axis (y). ☐ M.5.20.4: Identify the x and y values in ordered pairs. ☐ M.5.20.5: Model writing ordered pairs.	☐ Graph points in the first quadrant. ☐ Interpret coordinate values in context of the problem.

Geometry

Cluster	2019 Math COS Standard	
Classify two-dimensional figures into categories based on their properties.	21. Classify triangles according to side length (isosceles, equilateral, scalene) and angle measure (acute, obtuse, right, equiangular).	
	Learning Objectives	Prior Knowledge Skills
	☐ M.5.21.1: Define isosceles, equilateral, scalene, right and equiangular triangles; obtuse, acute, and right angle; vertex/vertices. ☐ M.5.21.2: Identify a right triangle. ☐ M.5.21.3: Sort and categorize shapes. ☐ M.5.21.4: Recognize and draw shapes having specified attributes.	☐ Recall the vocabulary of shapes (labels, sides, faces, vertices, etc.). ☐ Recognize and draw shapes having specified attributes such as a given number of angles. ☐ Build and draw shapes to possess defining attributes. ☐ Sort shapes into categories. ☐ Define side, angle, face, closed, and open. ☐ Use vocabulary related to shape attributes. Examples: sides, angles, face, closed, open. ☐ Trace shapes. ☐ Sort triangles, quadrilaterals, pentagons, hexagons, and cubes. ☐ Explore triangles, quadrilaterals, pentagons, hexagons, and cubes.

Geometry

Cluster	2019 Math COS Standard	
Classify two-dimensional figures into categories based on their properties.	22. Classify quadrilaterals in a hierarchy based on properties.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.5.22.1: Define vertex/vertices and angle. ☐ M.5.22.2: Identify shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). ☐ M.5.22.3: Recognize and draw shapes having specified attributes such as a given number of angles or a given number of equal faces. ☐ M.5.22.4: Identify triangles, quadrilaterals, pentagons, hexagons, heptagons, and octagons based on the number of sides, angles, and vertices.	☐ Classify triangles according to side measures and angle measures.

Geometry

Cluster	2019 Math COS Standard	
Classify two-dimensional figures into categories based on their properties.	23. Explain that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. <i>Example: All rectangles have four right angles, and squares have four right angles, so squares are rectangles.</i>	
	Learning Objectives	Prior Knowledge Skills
	☐ M.5.23.1: Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). ☐ M.5.23.2: Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories. ☐ M.5.23.3: Recognize attributes of shapes. ☐ M.5.23.4: Recall the vocabulary of shapes (labels, sides, faces, vertices, etc.). ☐ M.5.23.5: Sort shapes into categories.	☐ Classify triangles according to side measures and angle measures. ☐ Classify quadrilaterals based on properties. ☐ Explain the relationship between shapes in categories and subcategories.

Grade 6

Proportional Reasoning

Cluster	2019 Math COS Standard	
Develop an understanding of ratio concepts and use reasoning about ratios to solve problems.	1. *Use appropriate notations $[a/b, a \text{ to } b, a:b]$ to represent a proportional relationship between quantities and use ratio language to describe the relationship between quantities.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.6.1.1: Define quantity, fraction, and ratio. ☐ M.6.1.2: Identify the units or quantities being compared. Example: Read $2/3$ as 2 out of 3. ☐ M.6.1.3: Write a ratio in appropriate notation; $[a/b, a \text{ to } b, a:b]$. ☐ M.6.1.4: Draw a model of a given ratio or fraction. ☐ M.6.1.5: Identify the numerator and denominator of a fraction.	☐ Compare two fractions with the same numerator or the same denominator by reasoning about their size. ☐ Addition and subtraction of fractions as joining and separating parts referring to the same whole. ☐ Label numerator, denominator, and fraction bar. ☐ Recognize fraction 1 as the quantity formed by 1 part when a whole is partitioned into b equal parts.

Proportional Reasoning

Cluster	2019 Math COS Standard	
Develop an understanding of ratio concepts and use reasoning about ratios to solve problems.	2. *Use unit rates to represent and describe ratio relationships.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.6.2.1: Define unit rate, proportion, and rate. ☐ M.6.2.2: Create a ratio or proportion from a given word problem. ☐ M.6.2.3: Calculate unit rate by using ratios or proportions. ☐ M.6.2.4: Write a ratio as a fraction.	☐ Recall basic multiplication facts. ☐ Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. ☐ Recognize key terms to solve word problems. Examples: times, every, at this rate, each, per, equal/equally, in all, total. ☐ Recognize a fraction as a number on the number line. ☐ Label numerator, denominator, and fraction bar.

Proportional Reasoning

Cluster	2019 Math COS Standard	
Develop an understanding of ratio concepts and use reasoning about ratios to solve problems.	3. *Use ratio and rate reasoning to solve mathematical and real-world problems (including but not limited to percent, measurement conversion, and equivalent ratios) using a variety of models, including tables of equivalent ratios, tape diagrams, double number lines, and equations.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.6.3.1: Define ratio, rate, proportion, percent, equivalent, input, output, ordered pairs, diagram, unit rate, and table. ☐ M.6.3.2: Create a ratio or proportion from a given word problem, diagram, table, or equation. ☐ M.6.3.3: Calculate unit rate or rate by using ratios or proportions with or without a calculator. ☐ M.6.3.4: Restate real world problems or mathematical problems. ☐ M.6.3.5: Construct a graph from a set of ordered pairs given in the table of equivalent ratios. ☐ M.6.3.6: Calculate missing input and/or output values in a table with or without a calculator. ☐ M.6.3.7: Draw and label a table of equivalent ratios from given information. ☐ M.6.3.8: Identify the parts of a table of equivalent ratios (input, output, etc.). ☐ M.6.3.9: Compute the unit rate, unit price, and constant speed with or without a calculator. ☐ M.6.3.10: Create a proportion or ratio from a given word problem.	☐ Recognize arithmetic patterns (including geometric patterns or patterns in the addition table or multiplication table). Examples: Continued Geometric Pattern by drawing the next three shapes. ☐ Complete the numerical pattern for the following chart when given the rule, "Input + 5 = Output". ☐ Recognize that comparisons are valid only when the two fractions refer to the same whole. ☐ Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. ☐ Recognize key terms to solve word problems. Examples: times, every, at this rate, each, per, equal/equally, in all, total. ☐ Recall basic multiplication facts. ☐ Recognize equivalent forms of fractions and decimals. ☐ Recognize a fraction as a number on the number line. ☐ Label numerator, denominator, and fraction bar.

	☐ M.6.3.11: Identify the two units being compared. ☐ M.6.3.12: Define percent. ☐ M.6.3.13: Calculate a proportion for missing information with or without a calculator. ☐ M.6.3.14: Identify a proportion from given information. ☐ M.6.3.15: Solve a proportion using part over whole equals percent over 100 with or without a calculator. ☐ M.6.3.16: Form a ratio. ☐ M.6.3.17: Convert like measurement units within a given system with or without a calculator. (Example: 120 min = 2 hrs). ☐ M.6.3.18: Know relative sizes of measurement units within one system of units, including km, m, cm; kg, g; lb, oz; l, ml; and hr, min, sec.	
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Number Systems and Operations

Cluster	2019 Math COS Standard	
Use prior knowledge of multiplication and division to divide fractions.	4. Interpret and compute quotients of fractions using visual models and equations to represent problems. a. Use quotients of fractions to analyze and solve problems.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.6.4.1: Define fraction (including numerator and denominator), reciprocal, equation, and quotient. ☐ M.6.4.2: Construct an equation from a given word problem. ☐ M.6.4.3: Discuss the process of multiplying by the reciprocal. ☐ M.6.4.4: Interpret division of fractions by multiplying by the reciprocal. ☐ M.6.4.5: Demonstrate division of fractions using a visual fraction model. ☐ M.6.4.6: Demonstrate multiplication of fractions using a visual fraction model.	☐ Express whole numbers as fractions and recognize fractions that are equivalent to whole numbers. ☐ Write number sentences for word problems. ☐ Identify key terms to solve multiplication word problems. Examples: times, every, at this rate, each, per, equal/equally, in all, total. ☐ Recall basic multiplication facts. ☐ Recognize key terms to solve word problems. Examples: times, every, at this rate, each, per, equal/equally, in all, total. ☐ Label numerator, denominator, and fraction bar. ☐ Recognize a fraction as a number on the number line.

Number Systems and Operations		
Cluster	2019 Math COS Standard	
Compute multi-digit numbers fluently and determine common factors and multiples.	5. Fluently divide multi-digit whole numbers using a standard algorithm to solve real-world and mathematical problems.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.6.5.1: Define factors and multiples. ☐ M.6.5.2: Discuss the steps for solving a division problem. ☐ M.6.5.3: Recognize division and multiplication as inverse operations. ☐ M.6.5.4: Recall basic division and multiplication facts. ☐ M.6.5.5: Solve real-world division problems with and without models or a calculator.	☐ Express whole numbers as fractions and recognize fractions that are equivalent to whole numbers. ☐ Write number sentences for word problems. ☐ Identify key terms to solve multiplication word problems. Examples: times, every, at this rate, each, per, equal/equally, in all, total. ☐ Recall basic multiplication facts. ☐ Recognize key terms to solve word problems. Examples: times, every, at this rate, each, per, equal/equally, in all, total. ☐ Label numerator, denominator, and fraction bar. ☐ Recognize a fraction as a number on the number line.

Number Systems and Operations

Cluster	2019 Math COS Standard	
Compute multi-digit numbers fluently and determine common factors and multiples.	6. Add, subtract, multiply, and divide decimals using a standard algorithm.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.6.6.1: Solve division problems involving multi-digit whole numbers and decimal numbers with or without a calculator. ☐ M.6.6.2: Solve multiplication problems involving multi-digit whole numbers and decimal numbers with or without a calculator. ☐ M.6.6.3: Recall basic multiplication and division facts. ☐ M.6.6.4: Solve addition and subtraction of multi-digit decimal numbers (emphasis on alignment). ☐ M.6.6.5: Solve addition and subtraction of multi-digit whole numbers. ☐ M.6.6.6: Recognize place value of whole numbers and decimals. ☐ M.6.6.7: Demonstrate addition, subtraction, multiplication, and division of whole numbers and decimals using manipulatives.	☐ Recall basic multiplication facts. ☐ Recall basic addition, subtraction, multiplication, and division facts. ☐ Use place value understanding to round whole numbers to the nearest 10 or 100. ☐ Identify place value of decimals to the tenths and hundredths. ☐ Recall basic addition and subtraction facts. ☐ Recall basic addition and subtraction facts. ☐ Recall basic addition, subtraction, and multiplication facts. ☐ Multiply within 100, using strategies such as the relationship between multiplication and division. ☐ Multiply within 100, using strategies such as properties of operations. ☐ Divide within 100, using strategies such as properties of operations. ☐ Divide within 100, using strategies such as the relationship between multiplication and division. ☐ Add and subtract within 1000.

Number Systems and Operations

Cluster	2019 Math COS Standard	
Compute multi-digit numbers fluently and determine common factors and multiples.	7. Use the distributive property to express the sum of two whole numbers with a common factor as a multiple of a sum of two whole numbers with no common factor.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.6.7.1: Define greatest common factor, least common multiple, and the distributive property. ☐ M.6.7.2: Design problems using greatest common factor and the distributive property. ☐ M.6.7.3: Show an understanding of how to solve a problem using the distributive property, with or without the use of a calculator.	☐ Identify factor and product. ☐ Explain why addition and subtraction strategies work, using place value and the properties of operations. ☐ Apply properties of operations as strategies to multiply and divide.

Number Systems and Operations

Cluster	2019 Math COS Standard	
Compute multi-digit numbers fluently and determine common factors and multiples.	8. Find the greatest common factor (GCF) and least common multiple (LCM) of two or more whole numbers. a. Use factors and multiples to determine prime factorization.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.6.8.1: Identify the least common multiple of a given set of numbers, with or without the use of a calculator. ☐ M.6.8.2: List multiples of any given whole number, with or without the use of a calculator. ☐ M.6.8.3: Identify the greatest common factors of a given set of numbers, with or without the use of a calculator. ☐ M.6.8.4: Define prime factorization. ☐ M.6.8.5: List common factors of given whole numbers, with or without the use of a calculator. ☐ M.6.8.6: Identify the prime factorization of a single digit number, with or without the use of a calculator. ☐ M.6.8.7: Identify the prime factorization of any two digit whole number, with or without the use of a calculator.	☐ Define Multiple. ☐ Name the first ten multiples of each one-digit natural number. ☐ Name the first 10 multiples of each one-digit natural number. Example: 7, 14, 21, 28, 35, 42, 49, 56, 63, 70. ☐ Count within 1000; skip-count by 5s, 10s, and 100s. ☐ Recall basic multiplication facts. ☐ Name the first ten multiples of each one-digit natural number. ☐ Identify all factor pairs for a whole number in the range 1-20. ☐ Apply properties of operations as strategies to multiply and divide. ☐ Define factors, prime number, and composite number.

Number Systems and Operations

Cluster	2019 Math COS Standard	
Apply knowledge of the number system to represent and use rational numbers in a variety of forms.	9. Use signed numbers to describe quantities that have opposite directions or values and to represent quantities in real-world contexts.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.6.9.1: Give examples of positive and negative numbers to represent quantities having opposite directions in real-world contexts. ☐ M.6.9.2: Discover that the opposite of the opposite of a number is the number itself. ☐ M.6.9.3: Show on a number line that numbers that are equal distance from 0 and on opposite sides of 0 have opposite signs.	☐ Locate positive numbers on a horizontal number line. ☐ Locate positive numbers on a vertical number line. Examples: thermometer, map. ☐ Recognize different interpretations of fractions, including parts of a set or a collection, points on a number line, numbers that lie between two consecutive whole numbers, and lengths of segments on a ruler. ☐ Represent fractions on a number line diagram. ☐ Recognize a fraction as a number on the number line. ☐ Identify a fraction as a number on the number line; represent fractions on a number line diagram.

Number Systems and Operations

Cluster	2019 Math COS Standard	
Apply knowledge of the number system to represent and use rational numbers in a variety of forms.	10. *Locate integers and other rational numbers on a horizontal or vertical line diagram. - Define opposites as numbers located on opposite sides of 0 and the same distance from 0 on a number line. - Use rational numbers in real-world and mathematical situations, explaining the meaning of 0 in each situation.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.6.10.1: Define integers, positive and negative numbers. ☐ M.6.10.2: Demonstrate the location of positive and negative numbers on a vertical and horizontal number line. ☐ M.6.10.3: Give examples of positive and negative numbers to represent quantities having opposite directions in real-world contexts. ☐ M.6.10.4: Discuss the measure of centering of 0 in relationship to positive and negative numbers. ☐ M.6.10.5: Discover that the opposite of the opposite of a number is the number itself. ☐ M.6.10.6: Show on a number line that numbers that are equal distance from 0 and on opposite sides of 0 have opposite signs. ☐ M.6.10.7: Define rational number. ☐ M.6.10.8: Plot pairs of integers and/or rational numbers on a coordinate plane. ☐ M.6.10.9: Arrange integers and /or rational numbers on a horizontal or vertical number line.	☐ Model writing ordered pairs. ☐ Identify the x and y values in ordered pairs. ☐ Label the vertical axis (y). ☐ Label the horizontal axis (x). ☐ Define ordered pair of numbers, quadrant one, coordinate plane, and plot points. ☐ Locate positive numbers on a vertical number line. Examples: thermometer, map. ☐ Locate positive numbers on a horizontal number line. ☐ Locate negative numbers on a horizontal number line. ☐ Label x- and y-axis and zero on a coordinate. ☐ Illustrate vertical and horizontal number lines. ☐ Specify locations on the coordinate system. ☐ Define x-axis, y-axis, and zero on a coordinate. ☐ Define ordered pair of numbers. ☐ Define parentheses, braces, and brackets.

	☐ M.6.10.10: Locate the position of integers and/or rational numbers on a horizontal or vertical number line. ☐ M.6.10.11: Identify a rational number as a point on the number line. ☐ M.6.10.12: Name the pairs of integers and /or rational numbers of a point on a coordinate plane.	
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Number Systems and Operations

Cluster	2019 Math COS Standard	
Apply knowledge of the number system to represent and use rational numbers in a variety of forms.	11. *Find the position of pairs of integers and other rational numbers on the coordinate plane. a. Identify quadrant locations of ordered pairs on the coordinate plane based on the signs of the x and y coordinates. b. Identify (a,b) and (a,-b) as reflections across the x-axis. c. Identify (a,b) and (-a,b) as reflections across the y-axis. d. Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane, including finding distances between points with the same first or second coordinate.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.6.11.1: Define quadrant, coordinate plane, coordinate axes (x-axis and y-axis), horizontal, vertical, and reflection. ☐ M.6.11.2: Demonstrate an understanding of an extended coordinate plane. ☐ M.6.11.3: Draw a four-quadrant coordinate plane. ☐ M.6.11.4: Draw and extend vertical and horizontal number lines. ☐ M.6.11.5: Interpret graphing points in all four quadrants of the coordinate plane in real-world situations. ☐ M.6.11.6: Recall how to graph points in all four quadrants of the coordinate plane. ☐ M.6.11.7: Define ordered pairs. ☐ M.6.11.8: Name the pairs of integers and/or rational numbers of a point on a coordinate plane. ☐ M.6.11.9: Demonstrate when two ordered pairs differ only by signs, the locations of	☐ Model writing ordered pairs. ☐ Identify the x and y values in ordered pairs. ☐ Label the vertical axis (y). ☐ Label the horizontal axis (x). ☐ Define ordered pair of numbers, quadrant one, coordinate plane, and plot points. ☐ Locate positive numbers on a vertical number line. Examples: thermometer, map. ☐ Locate positive numbers on a horizontal number line. ☐ Locate negative numbers on a horizontal number line. ☐ Label x- and y-axis and zero on a coordinate. ☐ Illustrate vertical and horizontal number lines. ☐ Specify locations on the coordinate system. ☐ Define x-axis, y-axis, and zero on a coordinate. ☐ Define ordered pair of numbers. ☐ Locate positive numbers on a horizontal number line. ☐ Locate negative numbers on a horizontal number line. ☐ Define symmetry. ☐ Identify lines of symmetry on one-dimensional figures.

	the points are related by reflections across one or both axes. ☐ M.6.11.10 : Identify which signs indicate the location of a point in a coordinate plane. ☐ M.6.11.11 : Recall how to plot ordered pairs on a coordinate plane. ☐ M.6.11.12 : Define reflections. ☐ M.6.11.13: Calculate the distances between points having the same first or second coordinate using absolute value.	
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Number Systems and Operations

Cluster	2019 Math COS Standard	
Apply knowledge of the number system to represent and use rational numbers in a variety of forms.	12. *Explain the meaning of absolute value and determine the absolute value of rational numbers in real-world contexts.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.6.12.1: Define absolute value and rational numbers. ☐ M.6.12.2: Recall how to order numbers. ☐ M.6.12.3: Give examples of the magnitude for a positive or negative quantity in a real-world situation using absolute value. ☐ M.6.12.4: Recognize the absolute value of a rational number is its' distance from 0 on a vertical and horizontal number line.	☐ Model writing ordered pairs. ☐ Identify the x and y values in ordered pairs. ☐ Label the vertical axis (y). ☐ Label the horizontal axis (x). ☐ Define ordered pair of numbers, quadrant one, coordinate plane, and plot points.

Number Systems and Operations

Cluster	2019 Math COS Standard	
Apply knowledge of the number system to represent and use rational numbers in a variety of forms.	13. *Compare and order rational numbers and absolute value of rational numbers with and without a number line in order to solve real-world and mathematical problems.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.6.13.1: Define rational number. ☐ M.6.13.2: Plot pairs of integers and/or rational numbers on a coordinate plane. ☐ M.6.13.3: Arrange integers and/or rational numbers on a horizontal or vertical number line. ☐ M.6.13.4: Locate the position of integers and/or rational numbers on a horizontal or vertical number line. ☐ M.6.13.5: Evaluate a statement about order using comparisons of absolute value. ☐ M.6.13.6: Recall how to order positive and negative numbers. (Use number line if needed.).	☐ Locate positive numbers on a vertical number line. Examples: thermometer, map. ☐ Locate positive numbers on a horizontal number line. ☐ Locate negative numbers on a horizontal number line. ☐ Label x- and y-axis and zero on a coordinate. ☐ Illustrate vertical and horizontal number lines. ☐ Specify locations on the coordinate system. ☐ Define x-axis, y-axis, and zero on a coordinate. ☐ Define ordered pair of numbers. ☐ Interpret data using graphs including bar, line, and circle graphs, and Venn diagrams. ☐ Display data by making a line plot where the horizontal scale is marked off in appropriate units whole numbers, halves, or quarters.

Algebra and Functions

Cluster	2019 Math COS Standard	
Apply knowledge of arithmetic to read, write, and evaluate algebraic expressions.	14. Write, evaluate, and compare expressions involving whole number exponents.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.6.14.1: Define exponent, numerical expression, algebraic expression, variable, base, power, square of a number, and cube of a number. ☐ M.6.14.2: Compute a numerical expression with exponents, with or without a calculator. ☐ M.6.14.3: Restate exponential numbers as repeated multiplication. ☐ M.6.14.4: Choose the correct value to replace each variable in the expression (Substitution). ☐ M.6.14.5: Calculate the multiplication of single or multi-digit whole numbers, with or without a calculator.	☐ Product, power of 10. ☐ Recognize decimal place value using visual representations. ☐ Recall multiplication and division facts of 10. ☐ Skip count forward and backward by 10. ☐ Recognize decimals as parts of a whole. ☐ Compare whole numbers. ☐ Read whole numbers. ☐ Write whole numbers in words and expanded form. ☐ Define expanded notation and standard form. ☐ Convert a number written in expanded to standard form. ☐ Read and write numbers to 1000 using base-ten numerals, number names, and expanded form. ☐ Recall basic multiplication facts. ☐ Recall basic addition, subtraction, multiplication, and division facts.

Algebra and Functions

Cluster	2019 Math COS Standard	
Apply knowledge of arithmetic to read, write, and evaluate algebraic expressions.	15. *Write, read, and evaluate expressions in which letters represent numbers in real-world contexts. - Interpret a variable as an unknown value for any number in a specified set, depending on the context. - Write expressions to represent verbal statements and real-world scenarios. - Identify parts of an expression using mathematical terms such as sum, term, product, factor, quotient, and coefficient. - Evaluate expressions (which may include absolute value and whole number exponents) with respect to order of operations.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.6.15.1: Define algebraic expression and variable. ☐ M.6.15.2: Convert mathematical terms to mathematical symbols and numbers. ☐ M.6.15.3: Translate verbal and numerical expression using all operations. ☐ M.6.15.4: Define coefficient, constant and term. ☐ M.6.15.5: Match mathematical terms with correct mathematical symbols. ☐ M.6.15.6: Convert mathematical terms to mathematical symbols and numbers. ☐ M.6.15.7: Calculate an expression in the correct order. with or without a calculator (Ex. exponents, mult./div. from left to right, and add/sub. from left to right). ☐ M.6.15.8: Choose the correct value to replace each variable in the algebraic expression (Substitution).	☐ Recognize key terms to solve word problems. Examples: times, every, at this rate, each, per, equal/equally, in all, total. ☐ Recognize key terms to solve word problems. Examples: times, every, at this rate, each, per, equal/equally, in all, total. ☐ Define simple expression. ☐ Recall simple equations. ☐ Recognize properties of addition and multiplication. ☐ Recall addition, subtraction, multiplication, division symbols. ☐ Define parentheses, braces, and brackets. ☐ Define numerical expression. ☐ Recognize expressions. ☐ Apply properties of operations as strategies to add and subtract. ☐ Recall properties of operations as strategies to add and subtract. ☐ Represent addition and subtraction with objects, mental images, drawings, expressions, or equations. ☐ Use addition, subtraction, multiplication, and division to solve one-and two- step word problems. ☐ Recognize key terms to solve word problems.

	☐ M.6.15.9: Calculate a numerical expression, with or without a calculator (Ex. $V=4 \times 4 \times 4$). ☐ M.6.15.10: Recognize the correct order to solve expressions with more than one operation.	☐ Apply properties of operations as strategies to multiply and divide. ☐ Apply the area and perimeter formulas for rectangles in real-world and mathematical problems. ☐ Recall the formula for area ($L \times W$). ☐ Recognize that unit squares are equal. ☐ Recall the formula for perimeter ($P= L+L+W+W$ or $P=2L + 2W$). ☐ Recall basic addition and multiplication facts.
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Algebra and Functions

Cluster	2019 Math COS Standard	
Apply knowledge of arithmetic to read, write, and evaluate algebraic expressions.	16. *Generate equivalent algebraic expressions using the properties of operations, including inverse, identity, commutative, associative, and distributive.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.6.16.1: Define equivalent, simplify, term, distributive property, associative property of addition and multiplication, and the commutative property of addition and multiplication. ☐ M.6.16.2: Simplify expressions with parentheses (Ex. $5(4 + x) = 20 + 5x$). ☐ M.6.16.3: Combine terms that are alike of a given expression. ☐ M.6.16.4: Recognize the property demonstrated in a given expression. ☐ M.6.16.5: Simplify an expression by dividing by the greatest common factor. Example: $18x + 6y = 6(3x + y)$. ☐ M.6.16.6: Determine the greatest common factor in an algebraic expression.	☐ Define parentheses, braces, and brackets. ☐ Define numerical expression. ☐ Recognize expressions. ☐ Apply properties of operations as strategies to add and subtract. ☐ Recall properties of operations as strategies to add and subtract. ☐ Represent addition and subtraction with objects, mental images, drawings, expressions, or equations. ☐ Define simple expression. ☐ Recall simple equations. ☐ Recognize properties of addition and multiplication. ☐ Recall addition, subtraction, multiplication, division symbols. ☐ Use addition, subtraction, multiplication, and division to solve one- and two-step word problems. ☐ Apply properties of operations as strategies to multiply and divide.

Algebra and Functions

Cluster	2019 Math COS Standard	
Apply knowledge of arithmetic to read, write, and evaluate algebraic expressions.	17. Determine whether two expressions are equivalent and justify the reasoning.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.6.17.1: Define equivalent expressions. ☐ M.6.17.2: Recognize equivalent expressions. ☐ M.6.17.3: Substitute for the variable to find the value of a given expression. ☐ M.6.17.4: Calculate a numerical expression. ☐ M.6.17.5: Recognize that a variable without a written coefficient is understood to have a coefficient of one. (Ex. $x = 1x$).	☐ Use comparison symbols. Examples: $>$, $=$, or $<$. ☐ Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model. ☐ Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits using $>$, $=$, and $<$ symbols to record the results of comparisons. ☐ Comparison symbols. Examples: $>$, $=$, and $<$. ☐ Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits using $>$, $=$, and $<$ symbols to record the results of comparisons. ☐ Compare two fractions with the same numerator or the same denominator by reasoning about their size. ☐ Recognize that comparisons are valid only when the two fractions refer to the same whole. ☐ Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model. ☐ Convert fractions to decimals. ☐ Compare two decimals to tenths. ☐ Compare whole numbers. ☐ Identify comparison symbols. Examples: $>$, $<$, and $=$.

Algebra and Functions

Cluster	2019 Math COS Standard	
Use equations and inequalities to represent and solve real-world or mathematical problems.	18. Determine whether a value is a solution to an equation or inequality by using substitution to conclude whether a given value makes the equation or inequality true.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.6.18.1: Define exponent, numerical expression, algebraic expression, variable, base, power, square of a number, and cube of a number. ☐ M.6.18.2: Compute a numerical expression with exponents, with or without a calculator. ☐ M.6.18.3: Restate exponential numbers as repeated multiplication. ☐ M.6.18.4: Choose the correct value to replace each variable in the expression (Substitution). ☐ M.6.18.5: Calculate the multiplication of single or multi-digit whole numbers, with or without a calculator.	☐ Use comparison symbols. Examples: $>$, $=$, or $<$. ☐ Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model. ☐ Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits using $>$, $=$, and $<$ symbols to record the results of comparisons. ☐ Comparison symbols. Examples: $>$, $=$, and $<$. ☐ Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits using $>$, $=$, and $<$ symbols to record the results of comparisons. ☐ Compare two fractions with the same numerator or the same denominator by reasoning about their size. ☐ Recognize that comparisons are valid only when the two fractions refer to the same whole. ☐ Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model. ☐ Convert fractions to decimals. ☐ Compare two decimals to tenths. ☐ Compare whole numbers. ☐ Identify comparison symbols. Examples: $>$, $<$, and $=$.

Algebra and Functions

Cluster	2019 Math COS Standard	
Use equations and inequalities to represent and solve real-world or mathematical problems.	19. *Write and solve an equation in the form of $x+p=q$ or $px=q$ for cases in which p , q , and x are all non-negative rational numbers to solve real-world and mathematical problems. a. Interpret the solution of an equation in the context of the problem.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.6.19.1: Define equation and variable. ☐ M.6.19.2: Set up an equation to represent the given situation, using correct mathematical operations and variables. ☐ M.6.19.3: Solve the equation represented by the real-world situation. ☐ M.6.19.4: Identify the unknown variable in a given situation. ☐ M.6.19.5: List given information from the problem. ☐ M.6.19.6: Explain the solution in the context of the problem.	☐ Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. ☐ Recognize key terms to solve word problems. Examples: times, every, at this rate, each, per, equal/equally, in all, total. ☐ Define simple expression. ☐ Recall simple equations. ☐ Recognize properties of addition and multiplication. ☐ Recall addition, subtraction, multiplication, division symbols. ☐ Define parentheses, braces, and brackets. ☐ Define numerical expression. ☐ Recognize expressions. ☐ Apply properties of operations as strategies to add and subtract. ☐ Recall properties of operations as strategies to add and subtract. ☐ Represent addition and subtraction with objects, mental images, drawings, expressions, or equations.

Algebra and Functions

Cluster	2019 Math COS Standard	
Use equations and inequalities to represent and solve real-world or mathematical problems.	20. Write and solve inequalities in the form of $x > c$, $x < c$, $x \geq c$, or $x \leq c$ to represent a constraint or condition in a real-world or mathematical problem. - Interpret the solution of an inequality in the context of a problem. - Represent the solutions of inequalities on a number line and explain that the solution set may contain infinitely many solutions.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.6.20.1: Define inequality and solution set of an inequality. ☐ M.6.20.2: Set up an inequality to represent the given situation, using correct mathematical operations and variable. ☐ M.6.20.3: Identify solution set for the inequality used to represent the situation. ☐ M.6.20.4: Recognize the inequality symbols; $<$, $>$, $\leq$, $\geq$, $=$, $>$, $<$, $\geq$ and $\leq$ ☐ M.6.20.5: Construct and label a number line. ☐ M.6.20.6: Graph the solution set on a number line for the inequality used to represent the situation.	☐ Use comparison symbols. Examples: $>$, $=$, or $<$. ☐ Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model. ☐ Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits using $>$, $=$, and $<$ symbols to record the results of comparisons. ☐ Comparison symbols. Examples: $>$, $=$, and $<$. ☐ Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits. ☐ Compare using $>$, $=$, and $<$ symbols to record the results of comparisons. ☐ Compare two fractions with the same numerator or the same denominator by reasoning about their size. ☐ Recognize that comparisons are valid only when the two fractions refer to the same whole. ☐ Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model. ☐ Convert fractions to decimals. ☐ Compare two decimals to tenths. ☐ Compare whole numbers. ☐ Identify comparison symbols. Examples: $>$, $<$, and $=$.

Algebra and Functions

Cluster	2019 Math COS Standard	
Identify and analyze relationships between independent and dependent variables.	21. *Identify, represent, and analyze two quantities that change in relationship to one another in real-world or mathematical situations. Use tables, graphs, and equations to represent the relationship between independent and dependent variables.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.6.21.1: Define dependent variable, independent variable, ordered pairs, input, output, and coordinate plane. ☐ M.6.21.2: Examine the graph and table to determine any relationship between the variables. ☐ M.6.21.3: Recall how to draw a number line. ☐ M.6.21.4: Draw and label a coordinate plane. ☐ M.6.21.5: Analyze the pattern represented by the values in the table and develop an equation to express the relationship. ☐ M.6.21.6: Relate the table and graph to the equation. ☐ M.6.21.7: Plot independent (input) and dependent (output) values on a coordinate plane. ☐ M.6.21.8: Create a table of independent and dependent values from the equation.	☐ Model writing ordered pairs. ☐ Identify the x and y values in ordered pairs. ☐ Label the vertical axis (y). ☐ Label the horizontal axis (x). ☐ Define ordered pair of numbers, quadrant one, coordinate plane, and plot points. ☐ Locate positive numbers on a vertical number line. Examples: thermometer, map. ☐ Locate positive numbers on a horizontal number line. ☐ Locate negative numbers on a horizontal number line. ☐ Label x- and y-axis and zero on a coordinate. ☐ Illustrate vertical and horizontal number lines. ☐ Specify locations on the coordinate system. ☐ Define x-axis, y-axis, and zero on a coordinate. ☐ Define ordered pair of numbers. ☐ Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. ☐ Recognize key terms to solve word problems. Examples: times, every, at this rate, each, per, equal/equally, in all, total. ☐ Recognize that comparisons are valid only when the two fractions refer to the same whole. ☐ Recognize arithmetic patterns (including geometric patterns or patterns in the addition table or multiplication table). ☐ Recall basic multiplication facts.

		☐ Interpret data using graphs including bar, line, and circle graphs, and Venn diagrams. ☐ Identify the parts of a line plot. ☐ Recognize a line plot. ☐ Draw a scaled picture graph and a scaled bar graph to represent a data set. ☐ Recognize a fraction as a number on the number line.
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Data Analysis, Statistics, and Probability

Cluster	2019 Math COS Standard	
Use real-world and mathematical problems to analyze data and demonstrate an understanding of statistical variability and measures of center.	22. Write examples and non-examples of statistical questions, explaining that a statistical question anticipates variability in the data related to the question.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.6.22.1: Define statistical question. ☐ M.6.22.2: Identify examples of statistical questions and non-statistical questions. ☐ M.6.22.3: Compare and contrast statistical questions and non-statistical questions.	☐ Identify different types of questions. ☐ Recognize questions that produce numerical answers.

Data Analysis, Statistics, and Probability

Cluster	2019 Math COS Standard	
Use real-world and mathematical problems to analyze data and demonstrate an understanding of statistical variability and measures of center.	23. Calculate, interpret, and compare measures of center (mean, median, mode) and variability (range and interquartile range) in real-world data sets. - Determine which measure of center best represents a real-world data set. - Interpret the measures of center and variability in the context of a problem.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.6.23.1: Define numerical data set, measure of variation, and measure of center. ☐ M.6.23.2: Relate the measure of variation, of a data set, with the concept of range. ☐ M.6.23.3: Relate the measure of the center for a numerical data set with the concept of measure of center. ☐ M.6.23.4: Define numerical data set, quantitative, measure of center, median, frequency distribution, and attribute. ☐ M.6.23.5: Compare and contrast the center and variation. ☐ M.6.23.6: Collect the data. ☐ M.6.23.7: Organize the data. ☐ M.6.23.8: Describe how attribute was measured including units of measurement. ☐ M.6.23.9: Identify the attribute used to create the numerical set.	☐ Identify a numerical data set. ☐ Calculate the range of data. ☐ Organize numbers in a ordered list. ☐ Calculate the mean, median, and mean of a data set.

Data Analysis, Statistics, and Probability

Cluster	2019 Math COS Standard	
Use real-world and mathematical problems to analyze data and demonstrate an understanding of statistical variability and measures of center.	24. Represent numerical data graphically, using dot plots, line plots, histograms, stem and leaf plots, and box plots. - Analyze the graphical representation of data by describing the center, spread, shape (including approximately symmetric or skewed), and unusual features (including gaps, peaks, clusters, and extreme values). - Use graphical representations of real-world data to describe the context from which they were collected.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.6.24.1: Define dot plots, line plot, stem and leaf plots, upper quartile, lower quartile, median, histograms, and box plots. ☐ M.6.24.2: Recall how to read a graph or table. ☐ M.6.24.3: Calculate upper quartile median, lower quartile median, overall median, greatest value, and lowest value. ☐ M.6.24.4: Create box plot using calculations. ☐ M.6.24.5: Plot data on dot plots and histograms. ☐ M.6.24.6: Construct and label the display. ☐ M.6.24.7: Recognize the different types of displays. ☐ M.6.24.8: Define distribution and skew. ☐ M.6.24.9: Describe the shape of a set of data in a given distribution. ☐ M.6.24.10: Describe the spread of a set of data in a given distribution. ☐ M.6.24.11: Describe the center of a set of data in a given distribution.	☐ Identify different types of graphs. ☐ Create a bar graph and box plot. ☐ Organize data in an ordered list.

Geometry and Measurement

Cluster	2019 Math COS Standard	
Graph polygons in the coordinate plane to solve real-world and mathematical problems.	25. Graph polygons in the coordinate plane given coordinates of the vertices to solve real-world and mathematical problems. - Determine missing vertices of a rectangle with the same x-coordinate or the same y-coordinate when graphed in the coordinate plane. - Use coordinates to find the length of a side between points having the same x-coordinate or the same y coordinate. - Calculate perimeter and area of a polygon graphed in the coordinate plane (limiting to polygons in which consecutive vertices have the same x-coordinate or the same y-coordinate).	
	Learning Objectives	Prior Knowledge Skills
	☐ M.6.25.1: Define vertices. ☐ M.6.25.2: Apply absolute value to find the length of a side joining points with the same first coordinate or the same second coordinate. ☐ M.6.25.3: Plot points on a coordinate plane., then connect points for the vertices to sketch a polygon. ☐ M.6.25.4: Identify ordered pairs. ☐ M.6.25.5: Recognize polygons. ☐ M.6.25.6: Define perimeter and area. ☐ M.6.25.7: Identify the length between vertices on a coordinate plane. ☐ M.6.25.8: Calculate the perimeter and area using the distance between the vertices.	☐ Recognize and draw shapes having specified attributes such as a given number of angles or a given number of equal faces. ☐ Identify triangles, quadrilaterals, pentagons, hexagons, heptagons, and octagons based on the number of sides, angles, and vertices. ☐ Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). ☐ Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories. ☐ Recognize attributes of shapes. ☐ Recall the vocabulary of shapes (labels, sides, faces, vertices, etc.). Sort shapes into categories. ☐ Define ordered pair of numbers, quadrant one, coordinate plane, and plot points. Label the horizontal axis (x). ☐ Label the vertical axis (y). ☐ Identify the x and y values in ordered pairs. ☐ Model writing ordered pairs. ☐ Define vertex/vertices and angle. ☐ Recall the formula for area ($L \times W$).

*Critical Standard

		☐ Recognize that unit squares are equal. ☐ Recall the formula for perimeter ($P = L + L + W + W$ or $P = 2L + 2W$). ☐ Recall basic addition and multiplication facts.
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Geometry and Measurement

Cluster	2019 Math COS Standard	
Solve real-world and mathematical problems to determine area, surface area, and volume. <i>Note: Students must select and use the appropriate unit for the attribute being measured when determining length, area, angle, time, or volume.</i>	26. Calculate the area of triangles, special quadrilaterals, and other polygons by composing and decomposing them into known shapes. a. Apply the techniques of composing and decomposing polygons to find area in the context of solving real-world and mathematical problems.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.6.26.1: Define area, special quadrilaterals, right triangles, and polygons. ☐ M.6.26.2: Analyze the area of other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes. ☐ M.6.26.3: Apply area formulas to solve real-world mathematical problems. ☐ M.6.26.4: Demonstrate how the area of a rectangle is equal to the sum of the area of two equal right triangles. ☐ M.6.26.5: Explain how to find the area for rectangles. ☐ M.6.26.6: Select manipulatives to demonstrate how to compose and decompose triangles and other shapes. ☐ M.6.26.7: Recognize and demonstrate that two right triangles make a rectangle.	☐ Apply the area and perimeter formulas for rectangles in real-world and mathematical problems. ☐ Recall the formula for area ($L \times W$). ☐ Recognize that unit squares are equal. ☐ Recall the formula for perimeter ($P = L + L + W + W$ or $P = 2L + 2W$). ☐ Recall basic addition and multiplication facts. ☐ Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. ☐ Partition a shape into 4 parts with equal area and describe the area of each part as $\frac{1}{4}$ of the area of the shape. ☐ Partition circles and rectangles into two, three, or four equal shares; describe the shares using the words halves, thirds, half of, a third of, etc.; and describe the whole as two halves, three thirds, or four fourths. ☐ Recognize that equal shares of identical wholes need not have the same shape. ☐ Demonstrate equivalent fractions using concrete objects or pictorial representations. ☐ Define right angle. ☐ Recognize that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). ☐ Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.

*Critical Standard

		☐ Recognize and draw shapes having specified attributes such as a given number of angles or a given number of equal faces. ☐ Identify triangles.
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Geometry and Measurement

Cluster	2019 Math COS Standard	
Solve real-world and mathematical problems to determine area, surface area, and volume. <i>Note: Students must select and use the appropriate unit for the attribute being measured when determining length, area, angle, time, or volume.</i>	27. Determine the surface area of three-dimensional figures by representing them with nets composed of rectangles and triangles to solve real-world and mathematical problems.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.6.27.1: Define three-dimensional figures, surface area, and nets. ☐ M.6.27.2: Identify three-dimensional figures. ☐ M.6.27.3: Draw nets to find the surface area of a given three-dimensional figure. - M.6.27.4: Recall how to calculate the area of a rectangle. ☐ M.6.27.5: Select and create a three-dimensional figure using manipulatives.	☐ Describe attributes of three-dimensional figures. ☐ Describe attributes of two-dimensional figures. ☐ Identify solid figures. ☐ Recall the formula for area ($L \times W$). ☐ Recognize that unit squares are equal.

Geometry and Measurement

Cluster	2019 Math COS Standard	
Solve real-world and mathematical problems to determine area, surface area, and volume. <i>Note: Students must select and use the appropriate unit for the attribute being measured when determining length, area, angle, time, or volume.</i>	28. Apply previous understanding of volume of right rectangular prisms to those with fractional edge lengths to solve real-world and mathematical problems. a. Use models (cubes or drawings) and the volume formulas ($V = lwh$ and $V=Bh$) to find and compare volumes of right rectangular prisms.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.6.28.1: Define volume, rectangular prism, edge, and formula. ☐ M.6.28.2: Recall how to multiply fractional numbers. - M.6.28.3: Evaluate the volumes of rectangular prisms in the context of solving real-world and mathematical problems. ☐ M.6.28.4: Use models and volume formulas ($V=lwh$ and $V=Bh$) to find volumes in the context of solving real-world and mathematical problems. ☐ M.6.28.5: Calculate the volume of a rectangular prism using fractional lengths. ☐ M.6.28.6: Test the formula $V= lwh$ and $V=Bh$ with the experimental findings. ☐ M.6.28.7: Experiment with finding the volume using a variety of sizes of rectangular prisms manipulatives.	☐ Define volume. ☐ Recognize the formula for volume. ☐ Recall the attributes of three-dimensional solids. ☐ Compare the unit size of volume/capacity in the metric system including milliliters and liters. ☐ Measure and estimate liquid volumes. ☐ Describe attributes of three-dimensional figures. ☐ Describe attributes of two-dimensional figures. ☐ Define volume including the formulas $V = l \times w \times h$, and $V = B \times h$. ☐ Define solid figures. ☐ Define unit cube. ☐ Recognize that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). ☐ Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories. ☐ Describe attributes of three-dimensional figures. ☐ Describe attributes of two-dimensional figures. ☐ Compare the unit size of volume/capacity in the metric system including milliliters and liters.

Grade 7

Proportional Reasoning

Cluster	2019 Math COS Standard	
Analyze proportional relationships and use them to solve real-world and mathematical problems.	1. *Calculate unit rates of length, area, and other quantities measured in like or different units that include ratios or fractions.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.7.1.1: Define unit rate, proportions, area, length, and ratio. ☐ M.7.1.2: Recall how to find unit rates using ratios. ☐ M.7.1.3: Recall the steps used to solve division of fraction problems.	☐ Recall addition and subtraction of fractions as joining and separating parts referring to the same whole. ☐ Identify two fractions as equivalent (equal) if they are the same size or the same point on a number line. ☐ Recognize and generate simple equivalent fractions, e.g., $\frac{1}{2} = \frac{2}{4}$, $\frac{4}{6} = \frac{2}{3}$. Explain why the fractions are equivalent, e.g., by using a visual fraction model. ☐ Generate equivalent fractions. ☐ Define quantity, fraction, and ratio. ☐ Reinterpret a fraction as a ratio. Example: Read $\frac{2}{3}$ as 2 out of 3. ☐ Write a ratio as a fraction. ☐ Create a ratio or proportion from a given word problem, diagram, table, or equation. ☐ Calculate unit rate or rate by using ratios or proportions.

Proportional Reasoning

Cluster	2019 Math COS Standard	
Analyze proportional relationships and use them to solve real-world and mathematical problems.	2. *Represent a relationship between two quantities and determine whether the two quantities are related proportionally. a. Use equivalent ratios displayed in a table or in a graph of the relationship in the coordinate plane to determine whether a relationship between two quantities is proportional. b. Identify the constant of proportionality (unit rate) and express the proportional relationship using multiple representations including tables, graphs, equations, diagrams, and verbal descriptions. b. Explain in context the meaning of a point (x,y) on the graph of a proportional relationship, with special attention to the points (0,0) and (1, r) where r is the unit rate.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.7.2.1: Define proportions and proportional relationships. ☐ M.7.2.2: Demonstrate how to write ratios as a fraction. ☐ M.7.2.3: Define equivalent ratios and origin. ☐ M.7.2.4: Locate the origin on a coordinate plane. ☐ M.7.2.5 Show how to graph on Cartesian plane. ☐ M.7.2.6: Determine if the graph is a straight line through the origin. ☐ M.7.2.7: Use a table or graph to determine whether two quantities are proportional. ☐ M.7.2.8: Define a constant and equations. ☐ M.7.2.9: Create a table from a verbal description, diagram, or a graph. ☐ M.7.2.10: Identify numeric patterns and finding the rule for that pattern. ☐ M.7.2.11: Recall how to find unit rate. ☐ M.7.2.12: Recall how to write equations to represent a proportional relationship.	☐ Recall basic addition, subtraction, multiplication, and division facts. ☐ Define ordered pair of numbers. ☐ Define x-axis, y-axis, and zero on a coordinate. ☐ Specify locations on the coordinate system. ☐ Define ordered pair of numbers, quadrant one, coordinate plane, and plot points. ☐ Label the horizontal axis (x). ☐ Label the vertical axis (y). ☐ Identify the x and y values in ordered pairs. ☐ Model writing ordered pairs. ☐ Define quantity, fraction, and ratio. ☐ Reinterpret a fraction as a ratio. Example: Read $\frac{2}{3}$ as 2 out of 3. ☐ Write a ratio as a fraction. ☐ Create a ratio or proportion from a given word problem, diagram, table, or equation.

	☐ M.7.2.13: Discuss the use of variables. ☐ M.7.2.14: Define ordered pairs. ☐ M.7.2.15: Show how to plot points on a Cartesian plane. ☐ M.7.2.16: Locate the origin on the coordinate plane.	
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Proportional Reasoning

Cluster	2019 Math COS Standard	
Analyze proportional relationships and use them to solve real-world and mathematical problems.	3. *Solve multi-step percent problems in context using proportional reasoning, including simple interest, tax, gratuities, commissions, fees, markups and markdowns, percent increase, and percent decrease.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.7.3.1: Define interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, and percent error. ☐ M.7.3.2: Apply definitions to context in real world problems. ☐ M.7.3.3: Solve proportional problems. ☐ M.7.3.4: Recall how to find percent and ratios. ☐ M.7.3.5: Recall steps for solving multi-step problems.	☐ Define percent. ☐ Calculate a proportion for missing information. ☐ Identify a proportion from given information. ☐ Solve a proportion using part over whole equals percent over 100. ☐ Define equation and variable. ☐ Set up an equation to represent the given situation, using correct mathematical operations and variables. ☐ Identify the unknown, in each situation, as the variable. ☐ Solve the equation represented by the real-world situation.

Number Sense and Operations

Cluster	2019 Math COS Standard	
Apply and extend prior knowledge of addition, subtraction, multiplication, and division to operations with rational numbers.	4. *Apply and extend knowledge of operations of whole numbers, fractions, and decimals to add, subtract, multiply, and divide rational numbers including integers, signed fractions, and decimals. - Identify and explain situations where the sum of opposite quantities is 0 and opposite quantities are defined as additive inverses. - Interpret the sum of two or more rational numbers, by using a number line and in real-world contexts. - Explain subtraction of rational numbers as addition of additive inverses. - Use a number line to demonstrate that the distance between two rational numbers on the number line is the absolute value of their difference and apply this principle in real-world contexts. - Extend strategies of multiplication to rational numbers to develop rules for multiplying signed numbers, showing that the properties of the operations are preserved. - Divide integers and explain that division by zero is undefined. Interpret the quotient of integers (with a nonzero divisor) as a rational number. - Convert a rational number to a decimal using long division, explaining that the decimal form of a rational number terminates or eventually repeats.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.7.4.1: Define rational numbers, horizontal, and vertical. ☐ M.7.4.2: Recall how to extend a horizontal number line. ☐ M.7.4.3: Recall how to extend a vertical number line. ☐ M.7.4.4: Demonstrate addition and subtraction of whole numbers using a horizontal or vertical number line. ☐ M.7.4.5: Give examples of rational numbers.	☐ Define parentheses, braces, and brackets. ☐ Recall addition and subtraction of fractions as joining and separating parts referring to the same whole. ☐ Identify two fractions as equivalent (equal) if they are the same size or the same point on a number line. ☐ Recognize and generate simple equivalent fractions, e.g., $\frac{1}{2} = \frac{2}{4}$, $\frac{4}{6} = \frac{2}{3}$. Explain why the fractions are equivalent, e.g., by using a visual fraction model. ☐ Generate equivalent fractions. ☐ Show on a number line that numbers that are equal distance from 0 and on opposite sides of 0 have opposite signs.

	☐ M.7.4.6: Define absolute value and additive inverse. ☐ M.7.4.7: Explain that the sum of a number and its opposite is zero. ☐ M.7.4.8: Locate positive, negative, and zero numbers on a number line. ☐ M.7.4.9: Recall properties of addition and subtraction. ☐ M.7.4.10: Model addition and subtraction using manipulatives. ☐ M.7.4.11: Show addition and subtraction of 2 or more rational numbers using a number line within real world context. ☐ M.7.4.12: Define absolute value and additive inverse. ☐ M.7.4.13: Show subtraction as the additive inverse. ☐ M.7.4.14: Give examples of the opposite of a given number. ☐ M.7.4.15: Show addition and subtraction using a number line. ☐ M.7.4.16: Discuss various strategies for solving real-world and mathematical problems. ☐ M.7.4.17: Identify properties of operations for addition and subtraction. ☐ M.7.4.18: Recall the steps for solving addition and subtraction of rational numbers. ☐ M.7.4.19: Identify the difference between two rational numbers on a number line. ☐ M.7.4.20: Recall the steps of solving multiplication of rational numbers. ☐ M.7.4.21: Identify the pattern for multiplying signed numbers.	☐ Define rational number ☐ Arrange integers and /or rational numbers on a horizontal or vertical number line. ☐ Locate the position of integers and/or rational numbers on a horizontal or vertical number line. ☐ Identify a rational number as a point on the number line.
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	☐ M.7.4.22: Recall the steps of solving division of rational numbers. ☐ M.7.4.23: Explain that dividing a rational number zero is undefined. ☐ M.7.4.24: Recall that a fraction can be written as a division problem. ☐ M.7.4.25: Recall the steps to divide two rational numbers. ☐ M.7.4.26: Identify whether a decimal is terminating or repeating.	
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Number Sense and Operations

Cluster	2019 Math COS Standard	
Apply and extend prior knowledge of addition, subtraction, multiplication, and division to operations with rational numbers.	5. *Solve real-world and mathematical problems involving the four operations of rational numbers, including complex fractions. Apply properties of operations as strategies where applicable.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.7.5.1: Discuss various strategies for solving real-world and mathematical problems. ☐ M.7.5.2: Recall steps for solving fractional problems. ☐ M.7.5.3: Identify properties of operations for addition and multiplication. ☐ M.7.5.4: Recall the rules for multiplication and division of rational numbers. ☐ M.7.5.5: Recall the rules for addition and subtraction of rational numbers.	☐ Recall addition and subtraction of fractions as joining and separating parts referring to the same whole. ☐ Define rational number. ☐ Arrange integers and /or rational numbers on a horizontal or vertical number line. ☐ Locate the position of integers and/or rational numbers on a horizontal or vertical number line. ☐ Identify a rational number as a point on the number line.

Algebra and Functions

Cluster	2019 Math COS Standard	
Create equivalent expressions using the properties of operations.	6. *Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.7.6.1: Define linear expression, rational, coefficient, and rational coefficient. ☐ M.7.6.2: Simplify an expression by dividing by the greatest common factor (Ex. $18x + 6y = 6(3x + y)$). ☐ M.7.6.3: Simplify expressions with parentheses (Ex. $5(4 + x) = 20 + 5x$). ☐ M.7.6.4: Recognize the property demonstrated in a given expression. ☐ M.7.6.5: Combine like terms of a given expression. ☐ M.7.6.6: Recall how to find the greatest common factor. ☐ M.7.6.7: Give examples of the properties of operations including distributive, commutative, and associative.	☐ Apply properties of operations for addition and subtraction. ☐ Define equivalent, simplify, term, distributive property, associative property of addition and multiplication, and the commutative property of addition and multiplication. ☐ Simplify expressions with parentheses (Ex. $5(4 + x) = 20 + 5x$). ☐ Combine terms that are alike of a given expression. ☐ Recognize the property demonstrated in a given expression. ☐ Simplify an expression by dividing by the greatest common factor. Example: $18x + 6y = 6(3x + y)$. ☐ Determine the greatest common factor.

Algebra and Functions

Cluster	2019 Math COS Standard	
Create equivalent expressions using the properties of operations.	7. *Generate expressions in equivalent forms based on context and explain how the quantities are related.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.7.7.1: Define expression, equivalent, and equivalent expressions. ☐ M.7.7.2: Recall mathematical terms such as sum, difference, etc. ☐ M.7.7.3: Recognize that a variable without a written coefficient is understood to have a coefficient of one. ☐ M.7.7.4: Recall how to convert mathematical terms to mathematical symbols and numbers and vice versa. ☐ M.7.7.5: Restate numerical expressions with words.	☐ Define equivalent expressions. ☐ Recognize equivalent expressions. ☐ Recognize that a variable without a written coefficient is understood to be one. ☐ Convert mathematical terms to mathematical symbols and numbers (Ex. sum; +, difference; -; product; $\bullet$, quotient; $\div$)-

Algebra and Functions

Cluster	2019 Math COS Standard	
Solve real-world and mathematical problems using numerical and algebraic expressions, equations, and inequalities.	8. *Solve multi-step real-world and mathematical problems involving rational numbers (integers, signed fractions and decimals), converting between forms as needed. Assess the reasonableness of answers using mental computation and estimation strategies.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.7.8.1: Define estimation, rational numbers, and reasonable. ☐ M.7.8.2: Recall mental calculation strategies. ☐ M.7.8.3: Recall estimation strategies. ☐ M.7.8.4: Analyze the given word problem to set up a mathematical problem. ☐ M.7.8.5: Recognize the mathematical operations of rational numbers in any form, including converting between forms. (Ex. $0.25 = 1/4 = 25\%$). ☐ M.7.8.6: Recognize the rules of operations of positive and negative numbers. ☐ M.7.8.7: Recognize properties of numbers (Distributive, Associative, Commutative). ☐ M.7.8.8: Recall problem solving methods.	☐ Represent addition and subtraction with objects, mental images, drawings, expressions, or equations. ☐ Define integers, positive and negative numbers. ☐ Define rational number. ☐ Define equivalent, simplify, term, distributive property, associative property of addition and multiplication, and the commutative property of addition and multiplication.

Algebra and Functions

Cluster	2019 Math COS Standard	
Solve real-world and mathematical problems using numerical and algebraic expressions, equations, and inequalities.	9. *Use variables to represent quantities in real-world or mathematical problems and construct algebraic expressions, equations, and inequalities to solve problems by reasoning about the quantities. - Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. - Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality, and interpret it in the context of the problem.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.7.9.1: Define equation, inequality, and variable. ☐ M.7.9.2: Set up equations and inequalities to represent the given situation, using correct mathematical operations and variables. ☐ M.7.9.3: Calculate a solution or solution set by combining like terms, isolating the variable, and/or using inverse operations. ☐ M.7.9.4: Test the found number or number set for accuracy by substitution. ☐ M.7.9.5: Recall solving one step equations and inequalities. ☐ M.7.9.6: Recognize properties of numbers (Distributive, Associative, Commutative). ☐ M.7.9.7: Define equation and variable. ☐ M.7.9.8: Set up an equation to represent the given situation, using correct mathematical operations and variables.	☐ Define inequality. ☐ Define equivalent, simplify, term, distributive property, associative property of addition and multiplication, and the commutative property of addition and multiplication. ☐ Define equation, solution of an equation, solution of an inequality, and inequality. ☐ Compare and contrast equations and inequalities. ☐ Determine if an inequality is by replacing the variable with a given number. ☐ Determine if an equation is true by replacing the variable with a given number. ☐ Simplify a numerical sentence to determine equivalence. ☐ Recognize the symbols for $=$, $>$, $<$, $\neq$ and $\approx$ ☐ Define equation and variable. ☐ Set up an equation to represent the given situation, using correct mathematical operations and variables. ☐ Identify the unknown, in a given situation, as the variable.

	☐ M.7.9.9: Calculate a solution to an equation by combining like terms, isolating the variable, and/or using inverse operations. ☐ M.7.9.10: Test the found number for accuracy by substitution. Example: Is 5 an accurate solution of $2(x + 5) = 12$? ☐ M.7.9.11: Identify the unknown, in a given situation, as the variable. ☐ M.7.9.12: List given information from the problem. ☐ M.7.9.13: Recalling one-step equations. ☐ M.7.9.14: Explain the distributive property. ☐ M.7.9.15: Define inequality and variable. ☐ M.7.9.16: Set up an inequality to represent the given situation, using correct mathematical operations and variables. ☐ M.7.9.17: Calculate a solution set to an inequality by combining like terms, isolating the variable, and/or using inverse operations. ☐ M.7.9.18: Test the solution set for accuracy. ☐ M.7.9.19: Identify the unknown, of a given situation, as the variable. ☐ M.7.9.20: List information from the problem. ☐ M.7.9.21: Recall how to graph inequalities on a number line. ☐ M.7.9.22: Recall how to solve one step inequalities.	
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Data Analysis, Statistics, and Probability

Cluster	2019 Math COS Standard	
Make inferences about a population using random sampling.	10. Examine a sample of a population to generalize information about the population. - Differentiate between a sample and a population. - Compare sampling techniques to determine whether a sample is random and thus representative of a population, explaining that random sampling tends to produce representative samples and support valid inferences. - Determine whether conclusions and generalizations can be made about a population based on a sample. - Use data from a random sample to draw inferences about a population with an unknown characteristic of interest, generating multiple samples to gauge variation and making predictions or conclusions about the population. - Informally explain situations in which statistical bias may exist.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.7.10.1: Recall how to calculate range, outlier, ratio, and proportion. ☐ M.7.10.2: Define sample, data, variation, prediction, estimation, validity, population, inference, random sampling, statistic, and generalization. ☐ M.7.10.3: Explain the validity of random sampling. ☐ M.7.10.4: Differentiate the appropriate sampling method. ☐ M.7.10.5: Analyze attributes of sample size. ☐ M.7.10.6: Compare and contrast the random sampling data to the population. ☐ M.7.10.7: Compare sample size with population to check for validity. ☐ M.7.10.8: Analyze conclusions of the sample to determine its appropriateness for the population.	☐ Define statistical question. ☐ Calculate the range, mean, median, and mode of a numerical data set. ☐ Recognize the difference between population and sample. ☐ Identify bias from real world context.

	☐ M.7.10.9: Predict an outcome of the entire population based on random samplings. ☐ M.7.10.10: Discuss real world examples of valid sampling and generalizations. ☐ M.7.10.11: Recall the nature of the attribute, how it was measured, and its unit of measure. ☐ M.7.10.12: Collect data from population randomly, choosing same size samples. (Ex. If population is your school, different random samplings should be same number of students). ☐ M.7.10.13: Define and discuss bias. ☐ M.7.10.14: Compare and contrast statistical situations to determine if statistical bias exists.	
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Data Analysis, Statistics, and Probability

Cluster	2019 Math COS Standard	
Make inferences from an informal comparison of two populations.	11. Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.7.11.1: Define measure of variability, distribution, and measure of center. ☐ M.7.11.2: Analyze the skew of the distributions and recognize the difference in measure of center and variability. ☐ M.7.11.3: Compare the measure of center and measure of variability of two distributions. ☐ M.7.11.4: Relate the measure of variation with the concept of range. ☐ M.7.11.5: Relate the measure of the center with the concept of mean. ☐ M.7.11.6: Recall how to calculate measure of center and measure of variability. ☐ M.7.11.7: Discuss how to read and interpret a graph.	☐ Describe the center of a set of data in a given distribution. ☐ Compare and contrast the center and variation. ☐ Interpret graphing points in all four quadrants of the coordinate plane in real-world situations.

Data Analysis, Statistics, and Probability

Cluster	2019 Math COS Standard	
Make inferences from an informal comparison of two populations.	12. Make informal comparative inferences about two populations using measures of center and variability and/or mean absolute deviation in context.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.7.12.1: Define measure of variability, measure of center, inference and mean absolute deviation. ☐ M.7.12.2: Recall how to calculate measure of center and measure of variability. ☐ M.7.12.3: Recall that center is related to measure of center and measure of variability is related to variation. ☐ M.7.12.4: Compare and contrast the measure of center and measure of variability of two numerical data sets. ☐ M.7.12.5: Calculate the mean absolute deviation of a data set in context.	☐ Describe the center of a set of data in a given distribution. ☐ Compare and contrast the center and variation.

Data Analysis, Statistics, and Probability

Cluster	2019 Math COS Standard	
Investigate probability models.	13. Use a number from 0 to 1 to represent the probability of a chance event occurring, explaining that larger numbers indicate greater likelihood of the event occurring, while a number near zero indicates an unlikely event.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.7.13.1: Define probability and event. ☐ M.7.13.2: Recall the order of fractions on a number line. ☐ M.7.13.3: Recall how to compare fractions with like denominators. ☐ M.7.13.4: Demonstrate how to compare fractions with different denominators. ☐ M.7.13.5: Determine the likelihood of an event occurring.	☐ Recall addition and subtraction of fractions as joining and separating parts referring to the same whole. ☐ Identify two fractions as equivalent (equal) if they are the same size or the same point on a number line. ☐ Recognize and generate simple equivalent fractions, e.g., $\frac{1}{2} = \frac{2}{4}$, $\frac{4}{6} = \frac{2}{3}$. Explain why the fractions are equivalent, e.g., by using a visual fraction model. ☐ Generate equivalent fractions.

Data Analysis, Statistics, and Probability

Cluster	2019 Math COS Standard	
Investigate probability models.	14. Define and develop a probability model, including models that may or may not be uniform, where uniform models assign equal probability to all outcomes and non-uniform models involve events that are not equally likely. a. Collect and use data to predict probabilities of events. b. Compare probabilities from a model to observed frequencies, explaining possible sources of discrepancy.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.7.14.1: Define probability of chance, probability of events, outcome, and probability of observed frequency. ☐ M.7.14.2: Compare and contrast probability of chance and probability of observed frequency. - M.7.14.3: Display all outcomes in a graphic representation (probability model-tree diagram, organized list, table, etc.). ☐ M.7.14.4: Demonstrate how to write the probability as a fraction, with likely outcomes as the numerator and possible outcomes as the denominator. ☐ M.7.14.5: Recall how to simplify fractions to lowest terms. ☐ M.7.14.6: Recognize equivalent fractions. ☐ M.7.14.7: Recall how to create a table or graphic display of data. ☐ M.7.14.8: Define probability of chance, outcome, and event. ☐ M.7.14.9: List all possible outcomes using a graphic representation (probability	☐ Recall addition and subtraction of fractions as joining and separating parts referring to the same whole. ☐ Identify two fractions as equivalent (equal) if they are the same size or the same point on a number line. ☐ Recognize and generate simple equivalent fractions, e.g., $\frac{1}{2} = \frac{2}{4}$, $\frac{4}{6} = \frac{2}{3}$. Explain why the fractions are equivalent, e.g., by using a visual fraction model. ☐ Generate equivalent fractions. ☐ Recall how to read a graph or table.

	model-tree diagram, organized list, table, etc.). ☐ M.7.14.10: Using the model, count the frequency of the desired outcome. ☐ M.7.14.11: Demonstrate how to write the probability as a fraction, with likely outcomes as the numerator and possible outcomes as the denominator. ☐ M.7.14.12: Recall how to simplify fractions to lowest terms. ☐ M.7.14.13: Recognize equivalent fractions. ☐ M.7.14.14: Recall how to create a table or graphic display of data. ☐ M.7.14.15: Analyze collected data to predict probability of events. ☐ M.7.14.16: Define probability of observed frequency, outcome, discrepancy, and event. ☐ M.7.14.17: List all actual outcomes using a graphic representation (probability model-tree diagram, organized list, table, etc.). ☐ M.7.14.18: Using the model, count the frequency of the actual outcome. ☐ M.7.14.19: Demonstrate how to write the probability as a fraction, with likely outcomes as the numerator and possible outcomes as the denominator. ☐ M.7.14.20: Recall how to simplify fractions in lowest terms. ☐ M.7.14.21: Recognize equivalent fractions. ☐ M.7.14.22: Recall how to create a table or graphic display of data.	
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Data Analysis, Statistics, and Probability

Cluster	2019 Math COS Standard	
Investigate probability models.	15. Approximate the probability of an event using data generated by a simulation (experimental probability) and compare it to the theoretical probability. a. Observe the relative frequency of an event over the long run, using simulation or technology, and use those results to predict approximate relative frequency.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.7.15.1: Define probability of chance, outcome, theoretical probability, experimental probability and event. ☐ M.7.15.2: Recognize the difference between possible outcomes and likely outcomes. ☐ M.7.15.3: Write the probability as a fraction, with likely outcomes as the numerator and possible outcomes as the denominator. ☐ M.7.15.4: Recall how to simplify fraction to lowest terms. ☐ M.7.15.5: Recognize equivalent fractions. ☐ M.7.15.6: Define relative frequency.	☐ Recall addition and subtraction of fractions as joining and separating parts referring to the same whole. ☐ Identify two fractions as equivalent (equal) if they are the same size or the same point on a number line. ☐ Recognize and generate simple equivalent fractions, e.g., $\frac{1}{2} = \frac{2}{4}$, $\frac{4}{6} = \frac{2}{3}$. Explain why the fractions are equivalent, e.g., by using a visual fraction model. ☐ Generate equivalent fractions.

Data Analysis, Statistics, and Probability

Cluster	2019 Math COS Standard	
Investigate probability models.	16. Find probabilities of simple and compound events through experimentation or simulation and by analyzing the sample space, representing the probabilities as percents, decimals, or fractions. - Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams, and determine the probability of an event by finding the fraction of outcomes in the sample space for which the compound event occurred. - Design and use a simulation to generate frequencies for compound events. - Represent events described in everyday language in terms of outcomes in the sample space which composed the event.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.7.16.1: Define simple events and compound events. ☐ M.7.16.2: Discover when to add or multiply events to find probability of compound events. ☐ M.7.16.3: Recall how to find the probability of simple events. ☐ M.7.16.4: Demonstrate adding and multiplying fractions. ☐ M.7.16.5: Recognize how to obtain a common denominator when adding fractions. ☐ M.7.16.6: Recall how to add fractions with like denominators. ☐ M.7.16.7: Define simulation, frequency, and compound events. ☐ M.7.16.8: Recall how to find the probability of compound events. ☐ M.7.16.9: Create a tree diagram including all possible outcomes.	☐ Recall addition and subtraction of fractions as joining and separating parts referring to the same whole. ☐ Identify two fractions as equivalent (equal) if they are the same size or the same point on a number line. ☐ Recognize and generate simple equivalent fractions, e.g., $1/2 = 2/4$, $4/6 = 2/3$. Explain why the fractions are equivalent, e.g., by using a visual fraction model. ☐ Generate equivalent fractions. ☐ Recall how to read a graph or table.

	☐ M.7.16.10: Choose appropriate model to display outcomes (tree diagram, organized list, or table). ☐ M.7.16.11: Identify the desired outcomes in model. ☐ M.7.16.12: Create and use a simulation to illustrate compound events. ☐ M.7.16.13: Recall when to add or multiply events to find probability of compound events. ☐ M.7.16.14: Recall how to find the probability of simple events. ☐ M.7.16.15: Demonstrate adding and multiplying fractions. ☐ M.7.16.16: Recognize how to obtain a common denominator when adding fractions. ☐ M.7.16.17: Recall how to add fractions with like denominators. ☐ M.7.16.18: Recall how to construct a table.	
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Geometry and Measurement

Cluster	2019 Math COS Standard	
Construct and describe geometric figures, analyzing relationships among them.	17. Solve problems involving scale drawings of geometric figures, including computation of actual lengths and areas from a scale drawing and reproduction of a scale drawing at a different scale.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.7.17.1: Define scale, scale drawings, length, area, and geometric figures. ☐ M.7.17.2: Locate/use scale on a map. ☐ M.7.17.3: Identify proportional relationships. ☐ M.7.17.4: Recognize numeric patterns. ☐ M.7.17.5: Recall how to solve proportions.	☐ Construct repeating and growing patterns with a variety of representations. ☐ Continue an existing pattern. ☐ Identify arithmetic patterns (including patterns in the addition table or multiplication table) and explain them using properties of operations. ☐ Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. ☐ Define unit rate, proportion, and rate. Create a ratio or proportion from a given word problem.

Geometry and Measurement

Cluster	2019 Math COS Standard	
Construct and describe geometric figures, analyzing relationships among them.	18. Construct geometric shapes (freehand, using a ruler and a protractor, and using technology), given a written description or measurement constraints with an emphasis on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.7.18.1: Demonstrate how to use a protractor to draw an angle. ☐ M.7.18.2: Construct segments of a given length using a ruler. ☐ M.7.18.3: Recognize attributes of geometric shapes.	☐ Model using a protractor to draw angles. ☐ Measure the length of an object by selecting and using appropriate tools such as a ruler. ☐ Recognize attributes of shapes. ☐ Define vertex/vertices and angle.

Geometry and Measurement

Cluster	2019 Math COS Standard	
Construct and describe geometric figures, analyzing relationships among them.	19. Describe the two-dimensional figures created by slicing three-dimensional figures into plane sections.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.7.19.1: Define two-dimensional figure, three-dimensional figure, and plane section. ☐ M.7.19.2: List attributes of three-dimensional figures. ☐ M.7.19.3: List attributes of two-dimensional figures. ☐ M.7.19.4: Describe the relationship between two- and three-dimensional figures. ☐ M.7.19.5: Recognize symmetry.	☐ Identify shapes in different categories (e.g., rhombuses, rectangles, and others) that may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). ☐ Recognize and draw shapes having specified attributes such as a given number of angles or a given number of equal faces. ☐ Identify triangles, quadrilaterals, pentagons, hexagons, heptagons, and octagons based on the number of sides, angles, and vertices. ☐ Define three-dimensional figures, surface area, and nets. ☐ Select and create a three-dimensional figure using manipulatives. ☐ Identify three-dimensional figures.

Geometry and Measurement

Cluster	2019 Math COS Standard	
Solve real-world and mathematical problems involving angle measure, circumference, area, surface area, and volume. <i>Note: Students must select and use the appropriate unit for the attribute being measured when determining length, area, angle, time, or volume.</i>	20. Explain the relationships among circumference, diameter, area, and radius of a circle to demonstrate understanding of formulas for the area and circumference of a circle. - Informally derive the formula for area of a circle. - Solve area and circumference problems in real-world and mathematical situations involving circles.	
	Learning Objectives ☐ M.7.20.1: Define diameter, radius, circumference, area of a circle, and formula. ☐ M.7.20.2: Identify and label parts of a circle. ☐ M.7.20.3: Recognize the attributes of a circle. ☐ M.7.20.4: Apply the formula of area and circumference to real world mathematical situations.	Prior Knowledge Skills ☐ Define center, radius, and diameter of a circle. ☐ Identify real-world examples of radius and diameter. Examples: bicycle wheel, pizza, pie.

Geometry and Measurement

Cluster	2019 Math COS Standard	
Solve real-world and mathematical problems involving angle measure, circumference, area, surface area, and volume. <i>Note: Students must select and use the appropriate unit for the attribute being measured when determining length, area, angle, time, or volume.</i>	21. Use facts about supplementary, complementary, vertical, and adjacent angles in multi-step problems to write and solve simple equations for an unknown angle in a figure.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.7.21.1: Define supplementary angles, complementary angles, vertical angles, adjacent angles, parallel lines, perpendicular lines, and intersecting lines. ☐ M.7.21.2: Discuss strategies for solving multi-step problems and equations. ☐ M.7.21.3: Identify all types of angles. ☐ M.7.21.4: Identify right angles and straight angles.	☐ Model using a protractor to draw angles. ☐ Draw points, lines, line segments, and parallel and perpendicular lines, angles, and rays. ☐ Define vertex/vertices and angle.

Geometry and Measurement

Cluster	2019 Math COS Standard	
Solve real-world and mathematical problems involving angle measure, circumference, area, surface area, and volume. <i>Note: Students must select and use the appropriate unit for the attribute being measured when determining length, area, angle, time, or volume.</i>	22. Solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right rectangular prisms.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.7.22.1: Define volume, surface area, triangles, quadrilaterals, polygons, cubes, and right prisms. ☐ M.7.22.2: Discuss strategies for solving real-world mathematical problems. ☐ M.7.22.3: Recall formulas for calculating volume and surface area. ☐ M.7.22.4: Identify the attributes of triangles, quadrilaterals, polygons, cubes, and right prisms.	☐ Recognize the formula for volume. ☐ Define volume, rectangular prism, edge, and formula. ☐ Evaluate the volumes of rectangular prisms in the context of solving real-world and mathematical problems. ☐ Set up $V=lwh$ and $V=Bh$ to find volumes in the context of solving real-world and mathematical problems. ☐ Discover the volume of a rectangular prism using manipulatives. ☐ Define three-dimensional figures, surface area, and nets. ☐ Evaluate how to apply using surface area of a three-dimensional figure to solve real-world and mathematical problems. ☐ Draw nets to find the surface area of a given three-dimensional figure.

Grade 8

Number Systems and Operations

Cluster	2019 Math COS Standard	
Understand that the real number system is composed of rational and irrational numbers.	1. Define the real number system as composed of rational and irrational numbers. - Explain that every number has a decimal expansion; for rational numbers, the decimal expansion repeats or terminates. - Convert a decimal expansion that repeats into a rational number.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.8.1.1: Define expanding decimals, terminating decimals, rational number, and irrational number. ☐ M.8.1.2: Identify and give examples of rational numbers. ☐ M.8.1.3: Demonstrate how to convert fractions to decimals. ☐ M.8.1.4: Recall steps for division of fractions.	☐ Define rational number. ☐ Plot pairs of integers and/or rational numbers on a coordinate plane. ☐ Arrange integers and /or rational numbers on a horizontal or vertical number line. ☐ Locate the position of integers and/or rational numbers on a horizontal or vertical number line. ☐ Identify a rational number as a point on the number line. ☐ Recognize place value of whole numbers and decimals. ☐ Give examples of rational numbers.

Number Systems and Operations

Cluster	2019 Math COS Standard	
Understand that the real number system is composed of rational and irrational numbers.	2. Locate rational approximations of irrational numbers on a number line, compare their sizes, and estimate the values of the irrational numbers.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.8.2.1: Define square root, expressions, and approximations. ☐ M.8.2.2: Identify properties of exponents. ☐ M.8.2.3: Recall how to compare numbers. ☐ M.8.2.4: Identify perfect squares and square roots. ☐ M.8.2.5: Demonstrate how to locate points on a vertical or horizontal number line. ☐ M.8.2.6: Recall how to estimate.	☐ Define equivalent, simplify, term, distributive property, associative property of addition and multiplication, and the commutative property of addition and multiplication. ☐ Simplify expressions with parentheses (Ex. $5(4 + x) = 20 + 5x$). ☐ Combine terms that are alike of a given expression. ☐ Recognize the property demonstrated in a given expression. ☐ Discuss various strategies for solving real-world and mathematical problems. -Recall steps for solving fractional problems. ☐ Identify properties of operations for addition and multiplication. ☐ Recall the rules for multiplication and division of rational numbers. ☐ Recall the rules for addition and subtraction of rational numbers. ☐ Demonstrate the location of positive and negative numbers on a vertical and horizontal number line.

Algebra and Functions

Cluster	2019 Math COS Standard	
Apply concepts of integer exponents and radicals.	3. *Develop and apply properties of integer exponents to generate equivalent numerical and algebraic expressions.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.8.3.1: Define exponent, power, coefficient, integers, equivalent, and numerical expression. ☐ M.8.3.2: Restate negative exponents as positive exponents in the form $1/x^2$. ☐ M.8.3.3: Restate zero exponents as 1 ($x^0 = 1$). ☐ M.8.3.4: Recognize to add exponents when multiplying terms with like bases (Property of product of powers). ☐ M.8.3.5: Recognize to subtract exponents when dividing terms with like bases (Property of quotient of powers). ☐ M.8.3.6: Compute a numerical expression with positive exponents. ☐ M.8.3.7: Restate exponential numbers as repeated multiplication. ☐ M.8.3.8: Compute problems with adding and subtracting integers.	☐ Define exponent, numerical expression, algebraic expression, variable, base, power, square of a number, and cube of a number. ☐ Compute a numerical expression with exponents, with or without a calculator. ☐ Restate exponential numbers as repeated multiplication. ☐ Choose the correct value to replace each variable in the expression (Substitution). ☐ Calculate the multiplication of single or multi-digit whole numbers, with or without a calculator. ☐ Define integers, positive and negative numbers. ☐ Demonstrate the location of positive and negative numbers on a vertical and horizontal number line. ☐ Give examples of positive and negative numbers to represent quantities having opposite directions in real-world contexts. ☐ Discuss the measure of centering of 0 in relationship to positive and negative numbers. ☐ Discover that the opposite of the opposite of a number is the number itself. ☐ Show on a number line that numbers that are equal distance from 0 and on opposite sides of 0 have opposite signs.

Algebra and Functions

Cluster	2019 Math COS Standard	
Apply concepts of integer exponents and radicals.	4. Use square root and cube root symbols to represent solutions to equations. - Evaluate square roots of perfect squares (less than or equal to 225) and cube roots of perfect cubes (less than or equal to 1000). - Explain that the square root of a non-perfect square is irrational.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.8.4.1: Define square root, cube root, inverse, perfect square, perfect cube, and irrational number. ☐ M.8.4.2: Recognize the inverse operation of squaring a number is square root and the inverse of cubing a number is cube root. ☐ M.8.4.3: Restate exponential numbers as repeated multiplication. ☐ M.8.4.4: Calculate the multiplication of single or multi-digit whole numbers. ☐ M.8.4.5: Recognize rational and irrational numbers.	☐ Restate exponential numbers as repeated multiplication. ☐ Define rational number.

Algebra and Functions

Cluster	2019 Math COS Standard	
Apply concepts of integer exponents and radicals.	5. Estimate and compare very large or very small numbers in scientific notation.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.8.5.1: Recognize a fraction as division of the denominator into the numerator. ☐ M.8.5.2: Demonstrate that when multiplying powers of like bases; add the exponents (Property of products of powers). ☐ M.8.5.3: Demonstrate that when dividing powers of like bases; subtract the exponents (Property of quotient of powers). ☐ M.8.5.4: Demonstrate how to convert fractions to a decimal, with or without a calculator. ☐ M.8.5.5: Recall how to write numbers in scientific notation. ☐ M.8.5.6: Recall estimation strategies.	☐ Define the parts of a division problem including divisor, dividend, and quotient. ☐ Write a division equation. ☐ Apply the signs $\div$ and $=$ to the action of separating sets. ☐ Recognize division as either repeated subtraction, parts of a set, parts of a whole, or the inverse of multiplication. ☐ Model grouping with basic division facts partitioned equally (e.g., $8/2$). ☐ Apply properties of operations as strategies to subtract. ☐ Subtract within 20. ☐ Represent equal groups using manipulatives.

Algebra and Functions

Cluster	2019 Math COS Standard	
Apply concepts of integer exponents and radicals.	6. Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. - Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities. - Interpret scientific notation that has been generated by technology.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.8.6.1: Define scientific notation. ☐ M.8.6.2: Calculate multiplication and division of scientific notation, with or without a calculator. ☐ M.8.6.3: Recall properties of exponents. ☐ M.8.6.4: Recall how to write a number using scientific notation. ☐ M.8.6.5: Restate exponents as repeated multiplication. ☐ M.8.6.6: Discuss the real-world application of scientific notation (very large or very small quantities). ☐ M.8.6.7: Demonstrate difference of scientific notation symbol between paper and calculator.	☐ Recall that exponents are repeated multiplication. ☐ Demonstrate the ability to multiply and divide a number by a power of ten. ☐ Recognize the place value changes when multiplying/dividing by powers of ten.

Algebra and Functions

Cluster	2019 Math COS Standard	
Analyze the relationship between proportional and non-proportional situations.	7. Determine whether a relationship between two variables is proportional or non-proportional.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.8.7.1: Define proportional, independent variable, dependent variable, unit rate. ☐ M.8.7.2: Recall equivalent ratios and origin on a coordinate (Cartesian) plane. ☐ M.8.7.3: Recall how to write a ratio of two quantities as a fraction. ☐ M.8.7.4: Identify the unit rate of two quantities. ☐ M.8.7.5: Recall that for a relationship to be proportional, both variables must start at zero.	☐ Define unit rate, proportion, and rate. ☐ Create a ratio or proportion from a given word problem. ☐ Calculate unit rate by using ratios or proportions. ☐ Write a ratio as a fraction. ☐ Define ratio, rate, proportion, percent, equivalent, input, output, ordered pairs, diagram, unit rate, and table. ☐ Create a ratio or proportion from a given word problem, diagram, table, or equation. ☐ Calculate unit rate or rate by using ratios or proportions with or without a calculator. ☐ Restate real world problems or mathematical problems. ☐ Construct a graph from a set of ordered pairs given in the table of equivalent ratios. ☐ Calculate missing input and/or output values in a table with or without a calculator. ☐ Draw and label a table of equivalent ratios from given information. ☐ Identify the parts of a table of equivalent ratios (input, output, etc.). ☐ Compute the unit rate, unit price, and constant speed with or without a calculator. ☐ Create a proportion or ratio from a given word problem.

Algebra and Functions

Cluster	2019 Math COS Standard	
Analyze the relationship between proportional and non-proportional situations.	8. *Graph proportional relationships. Interpret the unit rate of a proportional relationship, describing the constant of proportionality as the slope of the graph which goes through the origin and has the equation $y = mx$ where m is the slope.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.8.8.1: Define proportional relationships, unit rate, and slope. ☐ M.8.8.2: Demonstrate how to write ratios. ☐ M.8.8.3: Recall how to solve proportions using cross products. ☐ M.8.8.4: Recall how to find the unit rate. ☐ M.8.8.5: Demonstrate how to graph on a Cartesian plane. ☐ M.8.8.6: Recall that for a relationship to be proportional, the graph must pass through the origin. ☐ M.8.8.7: Identify the slope-intercept form ($y=mx+b$) of an equation where m is the slope and y is the y-intercept.	☐ Define unit rate, proportion, and rate. ☐ Create a ratio or proportion from a given word problem. ☐ Calculate unit rate by using ratios or proportions. ☐ Write a ratio as a fraction. ☐ Define ratio, rate, proportion, percent, equivalent, input, output, ordered pairs, diagram, unit rate, and table. ☐ Create a ratio or proportion from a given word problem, diagram, table, or equation. ☐ Calculate unit rate or rate by using ratios or proportions with or without a calculator. ☐ Restate real world problems or mathematical problems. ☐ Construct a graph from a set of ordered pairs given in the table of equivalent ratios. ☐ Calculate missing input and/or output values in a table with or without a calculator. ☐ Draw and label a table of equivalent ratios from given information. ☐ Identify the parts of a table of equivalent ratios (input, output, etc.).

Algebra and Functions

Cluster	2019 Math COS Standard	
Analyze the relationship between proportional and non-proportional situations.	9. Interpret $y = mx + b$ as defining a linear equation whose graph is a line with m as the slope and b as the y-intercept. - Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in a coordinate plane. - Given two distinct points in a coordinate plane, find the slope of the line containing the two points and explain why it will be the same for any two distinct points on the line. - Graph linear relationships, interpreting the slope as the rate of change of the graph and the y-intercept as the initial value. - Given that the slopes for two different sets of points are equal, demonstrate that the linear equations that include those two sets of points may have different y-intercepts.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.8.9.1: Define linear functions, nonlinear functions, slope, and y-intercept. ☐ M.8.9.2: Recall how to solve problems using the distributive property. ☐ M.8.9.3: Recognize linear equations. ☐ M.8.9.4: Identify ordered pairs. ☐ M.8.9.5: Recognize ordered pairs. ☐ M.8.9.6: Define similar triangles, intercept, slope, vertical, horizontal, and origin. ☐ M.8.9.7: Recognize similar triangles. ☐ M.8.9.8: Generate the slope of a line using given ordered pairs. ☐ M.8.9.9: Analyze the graph to determine the rate of change. ☐ M.8.9.10: Demonstrate how to plot points on a coordinate plane using ordered pairs from table.	☐ Define ordered pairs. ☐ Name the pairs of integers and/or rational numbers of a point on a coordinate plane. ☐ Demonstrate when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. ☐ Identify which signs indicate the location of a point in a coordinate plane. ☐ Recall how to plot ordered pairs on a coordinate plane.

	☐ M.8.9.11: Identify the slope-intercept form ($y=mx+b$) of an equation where m is the slope and y is the y-intercept. ☐ M.8.9.12: Graph a function given the slope-intercept form of an equation. ☐ M.8.9.13: Recognize that two sets of points with the same slope may have different y-intercepts. ☐ M.8.9.14: Graph a linear equation given the slope-intercept form of an equation.	
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Algebra and Functions

Cluster	2019 Math COS Standard	
Analyze the relationship between proportional and non-proportional situations.	10. *Compare proportional and non-proportional linear relationships represented in different ways (algebraically, graphically, numerically in tables, or by verbal descriptions) to solve real-world problems.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.8.10.1: Define proportional and nonproportional. ☐ M.8.10.2: Recall that for two relationships to be proportional they must have the same unit rate and pass through the origin on a coordinate plane. ☐ M.8.10.3: Apply the rule of proportional relationship to real world context.	☐ Define unit rate, proportion, and rate. ☐ Calculate unit rate by using ratios or proportions. ☐ Write a ratio as a fraction. ☐ Define proportions and proportional relationships.

Algebra and Functions

Cluster	2019 Math COS Standard	
Analyze and solve linear equations and systems of two linear equations.	11. *Solve multi-step linear equations in one variable, including rational number coefficients, and equations that require using the distributive property and combining like terms. - Determine whether linear equations in one variable have one solution, no solution, or infinitely many solutions of the form $x = a$, $a = a$, or $a = b$ (where a and b are different numbers). - Represent and solve real-world and mathematical problems with equations and interpret each solution in the context of the problem.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.8.11.1: Define linear equation, coefficient, distributive property, and variable. ☐ M.8.11.2: Recall how to solve equations for a missing variable. ☐ M.8.11.3: Recall properties of operation for addition and multiplication. ☐ M.8.11.4: Solve multi-step equations. ☐ M.8.11.5: Identify properties of operations. ☐ M.8.11.6a: Identify how many solutions the linear equation may or may not have. ☐ M.8.11.7: Recall how to solve equations by using substitution. ☐ M.8.11.8b: Create an equation to represent a real-world situation or mathematical problem. ☐ M.8.11.9b: Analyze the solution in context of a real-world problem.	☐ Recognize properties of numbers (Distributive, Associative, Commutative). ☐ Define equation, inequality, and variable. ☐ Set up equations and inequalities to represent the given situation, using correct mathematical operations and variables. ☐ Calculate a solution or solution set by combining like terms, isolating the variable, and/or using inverse operations. ☐ Test the found number or number set for accuracy by substitution. ☐ Recall solving one step equations and inequalities. ☐ Recognize properties of numbers (Distributive, Associative, Commutative). ☐ Define equation and variable. ☐ Set up an equation to represent the given situation, using correct mathematical operations and variables. ☐ Calculate a solution to an equation by combining like terms, isolating the variable, and/or using inverse operations. ☐ Test the found number for accuracy by substitution. ☐ Example: Is 5 an accurate solution of $2(x + 5) = 12$. ☐ Identify the unknown, in a given situation, as the variable. ☐ List given information from the problem.

Algebra and Functions

Cluster	2019 Math COS Standard	
Analyze and solve linear equations and systems of two linear equations.	12. *Solve systems of two linear equations in two variables by graphing and substitution. - Explain that the solution(s) of systems of two linear equations in two variables corresponds to points of intersection on their graphs because points of intersection satisfy both equations simultaneously. - Interpret and justify the results of systems of two linear equations in two variables (one solution, no solution, or infinitely many solutions) when applied to real-world and mathematical problems.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.8.12.1: Define variables. ☐ M.8.12.2: Recall how to estimate. ☐ M.8.12.3: Recall how to solve linear equations. ☐ M.8.12.4: Demonstrate how to graph solutions to linear equations. ☐ M.8.12.5: Recall how to graph ordered pairs on a Cartesian plane. ☐ M.8.12.6: Recall that linear equations can have one solution (intersecting), no solution (parallel lines), or infinitely many solutions (graph is simultaneous). ☐ M.8.12.7: Define simultaneous. ☐ M.8.12.8: Recall how to solve linear equations. ☐ M.8.12.9: Recall properties of operations for addition and multiplication. ☐ M.8.12.10: Discover that the intersection of two lines on a coordinate plane is the solution to both equations. ☐ M.8.12.11: Define point of intersection. ☐ M.8.12.12: Recall how to solve linear equations.	☐ Define quadrant, coordinate plane, coordinate axes (x-axis and y-axis), horizontal, vertical, and reflection. ☐ Demonstrate an understanding of an extended coordinate plane. ☐ Draw a four-quadrant coordinate plane. ☐ Draw and extend vertical and horizontal number lines. ☐ Interpret graphing points in all four quadrants of the coordinate plane in real-world situations. ☐ Recall how to graph points in all four quadrants of the coordinate plane.

	☐ M.8.12.13 : Demonstrate how to graph on the Cartesian plane. ☐ M.8.12.14 : Identify ordered pairs. ☐ M.8.12.15 : Recall how to solve linear equations in two variables by using substitution. ☐ M.8.12.16 : Create a word problem from given information. ☐ M.8.12.17 : Recall how to solve linear equations. ☐ M.8.12.18: Explain how to write an equation to solve real-world mathematical problems.	
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Algebra and Functions

Cluster	2019 Math COS Standard	
Explain, evaluate, and compare functions.	13. Determine whether a relation is a function, defining a function as a rule that assigns to each input (independent value) exactly one output (dependent value), and given a graph, table, mapping, or set of ordered pairs.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.8.13.1: Define function, ordered pairs, input, output. ☐ M.8.13.2: Demonstrate how to plot points on a Cartesian plane using ordered pairs. ☐ M.8.13.3: Recall how to complete input/output tables. ☐ M.8.13.4: Recognize numeric patterns. ☐ M.8.13.5: Given a function, create a rule.	☐ Define quadrant, coordinate plane, coordinate axes (x-axis and y-axis), horizontal, vertical, and reflection. ☐ Demonstrate an understanding of an extended coordinate plane. ☐ Draw a four-quadrant coordinate plane. ☐ Draw and extend vertical and horizontal number lines. ☐ Interpret graphing points in all four quadrants of the coordinate plane in real-world situations. ☐ Recall how to graph points in all four quadrants of the coordinate plane.

Algebra and Functions

Cluster	2019 Math COS Standard	
Explain, evaluate, and compare functions.	14. Evaluate functions defined by a rule or an equation, given values for the independent variable.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.8.14.1: Define functions, independent variables, and dependent variables. ☐ M.8.14.2: Evaluate a function rule given the independent variable.	☐ Define equation and variable. ☐ Set up an equation to represent the given situation, using correct mathematical operations and variables. ☐ Calculate a solution to an equation by combining like terms, isolating the variable, and/or using inverse operations. ☐ Test the found number for accuracy by substitution. ☐ Example: Is 5 an accurate solution of $2(x + 5) = 12$. ☐ Identify the unknown, in a given situation, as the variable. ☐ List given information from the problem. ☐ Recalling one-step equations.

Algebra and Functions

Cluster	2019 Math COS Standard	
Explain, evaluate, and compare functions.	15. *Compare properties of functions represented algebraically, graphically, numerically in tables, or by verbal descriptions. a. Distinguish between linear and non-linear functions.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.8.15.1: Define rate of change. ☐ M.8.15.2: Recognize linear and nonlinear functions. ☐ M.8.15.3: Recall how to read/interpret information from a table. ☐ M.8.15.4: Identify algebraic expressions. ☐ M.8.15.5: Recall how to name points on a Cartesian plane using ordered pairs. ☐ M.8.15.6: Compare and contrast the differences between linear and nonlinear functions.	☐ Define expression, equivalent, and equivalent expressions. ☐ Recall mathematical terms such as sum, difference, etc. ☐ Recognize that a variable without a written coefficient is understood to have a coefficient of one. ☐ Recall how to convert mathematical terms to mathematical symbols and numbers and vice versa. ☐ Restate numerical expressions with words.

Algebra and Functions

Cluster	2019 Math COS Standard	
Use functions to model relationships between quantities.	16. *Construct a function to model a linear relationship between two variables. Interpret the rate of change (slope) and initial value of the linear function from a description of a relationship or from two points in a table or graph.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.8.16.1: Define function, rate of change, and initial value. ☐ M.8.16.2: Recall how to complete an input/output function table. ☐ M.8.16.3: Recall how to find the rate of change (slope) in a linear equation. ☐ M.8.16.4: Recall how to name points from a graph (ordered pairs). ☐ M.8.16.5: Analyze real-world situations to identify the rate of change and initial value from a table, graph, or description.	☐ Solve an equation by substituting a value to find an output. ☐ Find the coordinates of an ordered pair. ☐ Recognize how the steepness of a graphed line changes vertically and horizontally.

Algebra and Functions

Cluster	2019 Math COS Standard	
Use functions to model relationships between quantities.	17. *Analyze the relationship (increasing or decreasing, linear or non-linear) between two quantities represented in a graph.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.8.17.1: Define qualitative, increase, and decrease. ☐ M.8.17.2: Distinguish the difference between linear and nonlinear functions. ☐ M.8.17.3: Recall how to plot points on a Cartesian plane. ☐ M.8.17.4: Identify parts of the Cartesian plane. ☐ M.8.17.5: Recognize ordered pairs. ☐ M.8.17.6: Compare and contrast the relationship between two quantities in a graph.	☐ Define quadrant, coordinate plane, coordinate axes (x-axis and y-axis), horizontal, vertical, and reflection. ☐ Demonstrate an understanding of an extended coordinate plane. ☐ Draw a four-quadrant coordinate plane. ☐ Draw and extend vertical and horizontal number lines. ☐ Interpret graphing points in all four quadrants of the coordinate plane in real-world situations. ☐ Recall how to graph points in all four quadrants of the coordinate plane.

Data Analysis, Statistics, and Probability

Cluster	2019 Math COS Standard	
Investigate patterns of association in bivariate data.	18. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities, describing patterns in terms of positive, negative, or no association, linear and non-linear association, clustering, and outliers.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.8.18.1: Define bivariate scatter plot, outlier, cluster, linear, nonlinear, and positive and negative association. ☐ M.8.18.2: Describe patterns found in a scatter plot. ☐ M.8.18.3: Demonstrate how to label and plot information on a scatter plot (dot plot). ☐ M.8.18.4: Distinguish the difference between positive and negative correlation. ☐ M.8.18.5: Recall how to describe the spread of the scatter plot (dot plot).	☐ Define numerical data set, measure of variation, and measure of center. ☐ Relate the measure of variation, of a data set, with the concept of range. ☐ Relate the measure of the center for a numerical data set with the concept of measure of center. ☐ Define numerical data set, quantitative, measure of center, median, frequency distribution, and attribute. ☐ Compare and contrast the center and variation. ☐ Collect the data. ☐ Organize the data. ☐ Describe how attribute was measured including units of measurement. ☐ Identify the attribute used to create the numerical set.

Data Analysis, Statistics, and Probability

Cluster	2019 Math COS Standard	
Investigate patterns of association in bivariate data.	19. Given a scatter plot that suggests a linear association, informally draw a line to fit the data, and assess the model fit by judging the closeness of the data points to the line.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.8.19.1: Define scatter plot, outlier, linear, quantitative, line of best fit, and variable. ☐ M.8.19.2: Analyze scatter plots to determine line of best fit. ☐ M.8.19.3: Explain how to draw informal inferences from data distributions. ☐ M.8.19.4: Recall how to summarize numerical data sets in relation to their context. ☐ M.8.19.5: Recognize the concept of outlier and its relationship to the data distribution. ☐ M.8.19.6: Draw an estimate for a line of best fit.	☐ Define numerical data set, measure of variation, and measure of center. ☐ Relate the measure of variation, of a data set, with the concept of range. ☐ Relate the measure of the center for a numerical data set with the concept of measure of center. ☐ Define numerical data set, quantitative, measure of center, median, frequency distribution, and attribute. ☐ Compare and contrast the center and variation. ☐ Collect the data. ☐ Organize the data. ☐ Describe how attribute was measured including units of measurement. ☐ Identify the attribute used to create the numerical set.

Data Analysis, Statistics, and Probability

Cluster	2019 Math COS Standard	
Investigate patterns of association in bivariate data.	20. Use a linear model of a real-world situation to solve problems and make predictions. Describe the rate of change and y-intercept in the context of a problem using a linear model of a real-world situation.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.8.20.1: Define slope, intercept, linear, equation, and bivariate. ☐ M.8.20.2: Recall how to determine the rate of change (slope) from a graph. ☐ M.8.20.3: Identify the parts of the slope-intercept form of an equation. ☐ M.8.20.4: Recognize how to read a graph. ☐ M.8.20.5: Recall how to write an equation in slope-intercept form. ☐ M.8.20.6: Apply the identification of the slope and the y-intercept to a real-world situation. ☐ M.8.20.7: Create a graph to model a real-world situation.	☐ Define equation and variable. ☐ Set up an equation to represent the given situation, using correct mathematical operations and variables. ☐ Calculate a solution to an equation by combining like terms, isolating the variable, and/or using inverse operations. ☐ Test the found number for accuracy by substitution. Example: Is 5 an accurate solution of $2(x + 5) = 12$? ☐ Identify the unknown, in each situation, as the variable. ☐ List given information from the problem. ☐ Recalling one-step equations.

Data Analysis, Statistics, and Probability

Cluster	2019 Math COS Standard	
Investigate patterns of association in bivariate data.	21. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects, using relative frequencies calculated for rows or columns to describe possible associations between the two variables.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.8.21.1: Define relative frequency and frequency. ☐ M.8.21.2: Design a two-way table. ☐ M.8.21.3: Analyze a two-way table containing categorical variables. ☐ M.8.21.4: Calculate relative frequency. ☐ M.8.21.5: Discuss relative frequency. ☐ M.8.21.6: Design a table. ☐ M.8.21.7: Recall how to calculate frequency. ☐ M.8.21.8: Recall how to collect data.	☐ Define numerical data set, quantitative, measure of center, median, frequency distribution, and attribute. ☐ Compare and contrast the center and variation. ☐ Collect the data. ☐ Organize the data. ☐ Describe how attribute was measured including units of measurement. ☐ Identify the attribute used to create the numerical set.

Geometry and Measurement

Cluster	2019 Math COS Standard	
Understand congruence and similarity using physical models or technology.	22. Verify experimentally the properties of rigid motions (rotations, reflections, and translations): lines are taken to lines, and line segments are taken to line segments of the same length; angles are taken to angles of the same measure; and parallel lines are taken to parallel lines. a. Given a pair of two-dimensional figures, determine if a series of rigid motions maps one figure onto the other, recognizing that if such a sequence exists the figures are congruent; describe the transformation sequence that verifies a congruence relationship.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.8.22.1: Define rotation, reflection, and translation. ☐ M.8.22.2: Recognize translations (slides), rotations (turns), and reflections (flips). ☐ M.8.22.3: Distinguish between lines and line segments. ☐ M.8.22.4: Demonstrate how to measure length. ☐ M.8.22.5: Demonstrate how to use a protractor to measure angles. ☐ M.8.22.6: Identify parallel lines. ☐ M.8.22.7: Define congruent and sequence. ☐ M.8.22.8: Compare translations to reflections. ☐ M.8.22.9: Compare reflections to rotations. ☐ M.8.22.10: Compare rotations to translations. ☐ M.8.22.11: Identify attributes of two-dimensional figures. ☐ M.8.22.12: Identify congruent figures.	☐ Define ordered pairs. ☐ Name the pairs of integers and/or rational numbers of a point on a coordinate plane. ☐ Demonstrate when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. ☐ Identify which signs indicate the location of a point in a coordinate plane. ☐ Recall how to plot ordered pairs on a coordinate plane. ☐ Define reflections. ☐ Define reflections. ☐ Calculate the distances between points having the same first or second coordinate using absolute value.

Geometry and Measurement

Cluster	2019 Math COS Standard	
Understand congruence and similarity using physical models or technology.	23. *Use coordinates to describe the effect of transformations (dilations, translations, rotations, and reflections) on two-dimensional figures.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.8.23.1: Define dilation. ☐ M.8.23.2: Recall how to find scale factor. ☐ M.8.23.3: Give examples of scale drawings. ☐ M.8.23.4: Recognize translations. ☐ M.8.23.5: Recognize reflections. ☐ M.8.23.6: Recognize rotations.	☐ Define scale, scale drawings, length, area, and geometric figures. ☐ Locate/use scale on a map. ☐ Identify proportional relationships.

Geometry and Measurement

Cluster	2019 Math COS Standard	
Understand congruence and similarity using physical models or technology.	24. Given a pair of two-dimensional figures, determine if a series of dilations and rigid motions maps one figure onto the other, recognizing that if such a sequence exists the figures are similar; describe the transformation sequence that exhibits the similarity between them.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.8.24.1: Define similar. ☐ M.8.24.2: Recognize dilations. ☐ M.8.24.3: Recognize translations. ☐ M.8.24.4: Recognize rotations. ☐ M.8.24.5: Recognize reflections. ☐ M.8.24.6: Identify similar figures. ☐ M.8.24.7: Analyze an image and its dilation to determine if the two figures are similar.	☐ Define ordered pairs. ☐ Name the pairs of integers and/or rational numbers of a point on a coordinate plane. ☐ Demonstrate when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. ☐ Identify which signs indicate the location of a point in a coordinate plane. ☐ Recall how to plot ordered pairs on a coordinate plane. ☐ Define reflections. ☐ Calculate the distances between points having the same first or second coordinate using absolute value.

Geometry and Measurement

Cluster	2019 Math COS Standard	
Analyze parallel lines cut by a transversal.	25. *Analyze and apply properties of parallel lines cut by a transversal to determine missing angle measures. a. Use informal arguments to establish that the sum of the interior angles of a triangle is 180 degrees.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.8.25.1: Define exterior angles, interior angles, vertical angles, adjacent angles, alternate interior angles, alternate exterior angles, corresponding angles, and transversal. ☐ M.8.25.2: Identify attributes of triangles. ☐ M.8.25.3: Identify exterior angles, interior angles, vertical angles, adjacent angles, alternate interior angles, alternate exterior angles, and corresponding angles. ☐ M.8.25.4: Identify a transversal. ☐ M.8.25.5: Apply properties to find missing angle measures. ☐ M.8.25.6: Discover the Angle Sum Theorem (sum of the interior angles of a triangle equal 180 degrees).	☐ Define supplementary angles, complementary angles, vertical angles, adjacent angles, parallel lines, perpendicular lines, and intersecting lines. ☐ Discuss strategies for solving multi-step problems and equations. ☐ Identify all types of angles. ☐ Identify right angles and straight angles.

Geometry and Measurement

Cluster	2019 Math COS Standard	
Understand and apply the Pythagorean Theorem.	26. Informally justify the Pythagorean Theorem and its converse.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.8.26.1: Define a right angle, Pythagorean Theorem, converse, and proof. ☐ M.8.26.2: Recognize examples of right triangles. ☐ M.8.26.3: Demonstrate how to find square roots. ☐ M.8.26.4: Solve problems with exponents.	☐ Define supplementary angles, complementary angles, vertical angles, adjacent angles, parallel lines, perpendicular lines, and intersecting lines. ☐ Discuss strategies for solving multi-step problems and equations. ☐ Identify all types of angles. ☐ Identify right angles and straight angles.

Geometry and Measurement

Cluster	2019 Math COS Standard	
Understand and apply the Pythagorean Theorem.	27. Apply the Pythagorean Theorem to find the distance between two points in a coordinate plane.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.8.27.1: Recall how to name points on a Cartesian plane using ordered pairs. ☐ M.8.27.2: Recognize ordered pairs (x, y). ☐ M.8.27.3: Solve problems using the Pythagorean Theorem, with or without a calculator. ☐ M.8.27.4: Identify right triangles. ☐ M.8.27.5: Demonstrate how to find square roots, with or without a calculator. ☐ M.8.27.6: Solve problems with exponents, with or without a calculator.	☐ Define area, special quadrilaterals, right triangles, and polygons. ☐ Analyze the area of other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes. ☐ Apply area formulas to solve real-world mathematical problems. ☐ Demonstrate how the area of a rectangle is equal to the sum of the area of two equal right triangles. ☐ Explain how to find the area for rectangles. ☐ Select manipulatives to demonstrate how to compose and decompose triangles and other shapes. ☐ Recognize and demonstrate that two right triangles make a rectangle.

Geometry and Measurement

Cluster	2019 Math COS Standard	
Understand and apply the Pythagorean Theorem.	28. *Apply the Pythagorean Theorem to determine unknown side lengths of right triangles, including real-world applications.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.8.28.1: Discuss strategies for solving real-world and mathematical problems. ☐ M.8.28.2: Solve problems using the Pythagorean Theorem, with or without a calculator. ☐ M.8.28.3: Identify right triangles. ☐ M.8.28.4: Demonstrate how to find square roots, with or without a calculator. ☐ M.8.28.5: Solve problems with exponents, with or without a calculator.	☐ Define exponent, numerical expression, algebraic expression, variable, base, power, square of a number, and cube of a number. ☐ Compute a numerical expression with exponents, with or without a calculator. ☐ Restate exponential numbers as repeated multiplication. ☐ Choose the correct value to replace each variable in the expression (Substitution).

Geometry and Measurement

Cluster	2019 Math COS Standard	
Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres. <i>Note: Students must select and use the appropriate unit for the attribute being measured when determining length, area, angle, time, or volume.</i>	29. Informally derive the formulas for the volume of cones and spheres by experimentally comparing the volumes of cones and spheres with the same radius and height to a cylinder with the same dimensions.	
	Learning Objectives ☐ M.8.29.1: Define volume. ☐ M.8.29.2: Identify cone, sphere, and cylinder. ☐ M.8.29.3: Recall the meaning of a radius and diameter. ☐ M.8.29.4: Compare and contrast cone, sphere, and cylinder. ☐ M.8.29.5: Derive the formulas for the volume of a cone, cylinder, and sphere.	Prior Knowledge Skills ☐ Define volume, surface area, triangles, quadrilaterals, polygons, cubes, and right prisms. ☐ Discuss strategies for solving real-world mathematical problems. ☐ Recall formulas for calculating volume and surface area. ☐ Identify the attributes of triangles, quadrilaterals, polygons, cubes, and right prisms.

Geometry and Measurement

Cluster	2019 Math COS Standard	
Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres. <i>Note: Students must select and use the appropriate unit for the attribute being measured when determining length, area, angle, time, or volume.</i>	30. Use formulas to calculate the volumes of three-dimensional figures (cylinders, cones, and spheres) to solve real-world problems.	
	Learning Objectives	Prior Knowledge Skills
	☐ M.8.30.1: Define formula, volume, cone, cylinders, spheres, and height. ☐ M.8.30.2: Discuss the measure of volume and give examples. ☐ M.8.30.3: Solve problems with exponents, with or without a calculator. ☐ M.8.30.4: Recall how to find circumference of a circle, with or without a calculator. ☐ M.8.30.5: Identify parts of a circle. ☐ M.8.30.6: Calculate the volume of three-dimensional figures. ☐ M.8.30.7: Solve real-world problems using the volume formulas for three-dimensional figures, with or without a calculator.	☐ Define volume, surface area, triangles, quadrilaterals, polygons, cubes, and right prisms. ☐ Discuss strategies for solving real-world mathematical problems. ☐ Recall formulas for calculating volume and surface area. ☐ Identify the attributes of triangles, quadrilaterals, polygons, cubes, and right prisms. ☐ Define diameter, radius, circumference, area of a circle, and formula. ☐ Identify and label parts of a circle. ☐ Recognize the attributes of a circle. ☐ Apply the formula of area and circumference to real world mathematical situations.

Geometry with Data Analysis



Geometry with Data Analysis

Number and Quantity

Cluster	2019 Math COS Standard	
Together, irrational numbers and rational numbers complete the real number system, representing all points on the number line, while there exist numbers beyond the real numbers called complex numbers.	1. *Extend understanding of irrational and rational numbers by rewriting expressions involving radicals, including addition, subtraction, multiplication, and division, in order to recognize geometric patterns.	
	Learning Objectives	Prior Knowledge Skills
	☐ GEO.1.1: Define rational and irrational numbers and radicals. ☐ GEO.1.2: Identify the product of a nonzero rational number and an irrational number as irrational. ☐ GEO.1.3: Identify the sum of a rational number and an irrational number is irrational. ☐ GEO.1.4: Discuss why the product of two rational numbers is rational. ☐ GEO.1.5: Describe the properties of addition and multiplication rational and irrational numbers and radicals. ☐ GEO.1.6: Apply properties of fractions to add, subtract, multiply, and divide rational numbers.	☐ Define rational number. ☐ Define rational numbers, horizontal, and vertical. ☐ Recall how to extend a horizontal number line. ☐ Recall how to extend a vertical number line. ☐ Demonstrate addition and subtraction of whole numbers using a horizontal or vertical number line. ☐ Give examples of rational numbers. ☐ Define absolute value and additive inverse. ☐ Explain that the sum of a number and its opposite is zero. ☐ Locate positive, negative, and zero numbers on a number line. ☐ Recall properties of addition and subtraction. ☐ Model addition and subtraction using manipulatives. ☐ Show addition and subtraction of 2 or more rational numbers using a number line within real-world context. ☐ Define absolute value and additive inverse. ☐ Show subtraction as the additive inverse. ☐ Give examples of the opposite of a given number. ☐ Show addition and subtraction using a number line. ☐ Discuss various strategies for solving real-world and mathematical problems. ☐ Identify properties of operations for addition and subtraction. ☐ Recall the steps for solving addition and subtraction of rational numbers.

		☐ Identify the difference between two rational numbers on a number line. ☐ Recall the steps for solving multiplication and division of fraction problems. ☐ Recall the steps for solving multiplication and division of whole number problems. ☐ Define distributive property, rational numbers, and product. ☐ Solve problems using the distributive property. ☐ Recall basic multiplication facts using manipulatives. ☐ Identify the properties of operations for multiplication. ☐ Define quotient, divisor, and integer. ☐ Recall the rules for multiplying integers. ☐ Solve real-world problems. ☐ Recall the steps of division. ☐ Discuss various strategies for solving real-world and mathematical problems. ☐ Identify properties of operations for multiplication. ☐ Define terminating decimals. ☐ Give examples of equivalent fractions and decimals. ☐ Recall the steps for dividing decimals. ☐ Recall the steps of division. ☐ Discuss various strategies for solving real-world and mathematical problems. ☐ Recall steps for solving fractional problems. ☐ Identify properties of operations for addition and multiplication. ☐ Recall the rules for multiplication and division of rational numbers. ☐ Recall the rules for addition and subtraction of rational numbers. ☐ Analyze a given word problem to set up a mathematical problem. ☐ Recognize the mathematical operations of rational numbers in any form, including converting between forms. (Ex. $0.25 = \frac{1}{4} = 25\%$) ☐ Recognize the rules of operations of positive and negative numbers.
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*Critical Standard

		☐ Recognize properties of numbers (Distributive, Associative, Commutative). ☐ Recall problem solving methods. ☐ Define expanding decimals, terminating decimals, rational number, and irrational number. ☐ Identify and give examples of rational numbers. ☐ Demonstrate how to convert fractions to decimals. ☐ Recall steps for division of fractions. ☐ Recognize rational and irrational numbers.
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Number and Quantity

Cluster	2019 Math COS Standard	
Quantitative reasoning includes, and mathematical modeling requires, attention to units of measurement.	2. Use units as a way to understand problems and to guide the solution of multi-step problems. a. Choose and interpret units consistently in formulas. b. Choose and interpret the scale and the origin in graphs and data displays. c. Define appropriate quantities for the purpose of descriptive modeling. d. Choose a level of accuracy appropriate to limitations of measurements when reporting quantities.	
	Learning Objectives	Prior Knowledge Skills
	☐ GEO.2.1: Interpret units consistently in formulas. ☐ GEO.2.2: Choose units consistently in formulas. ☐ GEO.2.3: Use units as a way to guide the solution of multi-step problems. ☐ GEO.2.4: Use units as a way to understand problems. ☐ GEO.2.5: Convert between units of measurement within the same system. ☐ GEO.2.6: Choose the scale and the origin in graphs. ☐ GEO.2.7: Interpret the scale and the origin in data displays. ☐ GEO.2.8: Define units of measurement. ☐ GEO.2.9: Identify appropriate units of measure to best describe a real-world application. ☐ GEO.2.10: Recognize the limitations for each type of measurement tool. ☐ GEO.2.11: Determine the level of precision needed for real-world measurements.	☐ Convert like measurement units within a given system. (Example: 120 min = 2 hrs). ☐ Know relative sizes of measurement units within one system of units, including km, m, cm; kg, g; lb, oz; l, ml; and hr, min, sec.

*Critical Standard

	☐ GEO.2.12: Relate how rounding effects the accuracy of the measurement.	
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Algebra and Functions

Focus 2: Connecting Algebra to Functions

Cluster	2019 Math COS Standard	
Graphs can be used to obtain exact or approximate solutions of equations, inequalities, and systems of equations and inequalities—including systems of linear equations in two variables and systems of linear and quadratic equations (given or obtained by using technology).	3. Verify that the graph of a linear equation in two variables is the set of all its solutions plotted in the coordinate plane, which forms a line.	
	Learning Objectives	Prior Knowledge Skills
	☐ GEO.5.1: Define ordered pair and coordinate plane. ☐ GEO.5.2: Create linear equations with two variables. ☐ GEO.5.3: Graph linear equations on coordinate axes with labels and scales. ☐ GEO.5.4: Identify an ordered pair and plot it on the coordinate plane.	☐ Define linear functions, nonlinear functions, slope, and y-intercept. ☐ Recall how to solve problems using the distributive property. ☐ Recognize linear equations. ☐ Identify ordered pairs. ☐ Recognize ordered pairs. ☐ Define similar triangles, intercept, slope, vertical, horizontal, and origin. ☐ Recognize similar triangles. ☐ Generate the slope of a line using given ordered pairs. ☐ Analyze the graph to determine the rate of change. ☐ Demonstrate how to plot points on a coordinate plane using ordered pairs from table. ☐ Identify the slope-intercept form ($y=mx+b$) of an equation where m is the slope and y is the y-intercept. ☐ Graph a function given the slope-intercept form of an equation. ☐ Recognize that two sets of points with the same slope may have different y-intercepts. ☐ Graph a linear equation given the slope-intercept form of an equation.

Algebra and Functions

Focus 1: Algebra

Cluster	2019 Math COS Standard	
The structure of an equation or inequality (including, but not limited to, one-variable linear and quadratic equations, inequalities, and systems of linear equations in two variables) can be purposefully analyzed (with and without technology) to determine an efficient strategy to find a solution, if one exists, and then to justify the solution.	4. Find the coordinates of the vertices of a polygon determined by a set of lines, given their equations, by setting their function rules equal and solving, or by using their graphs.	
	Learning Objectives	Prior Knowledge Skills
	☐ GEO.3.1: Define systems of equations, constraints, viable solution, and nonviable solution. ☐ GEO.3.2: Determine if a solution to a system of equations or inequalities is viable or nonviable. ☐ GEO.3.3: Create a system of equations or inequalities to represent the given constraints (linear). ☐ GEO.3.4: Create an equation or inequality to represent the given constraints (linear). ☐ GEO.3.5: Determine if there is one solution, infinite solutions, or no solutions to a system of equations or inequalities.	☐ Recall how to solve linear equations. ☐ Demonstrate how to graph solutions to linear equations. ☐ Recall how to graph ordered pairs on a Cartesian plane. ☐ Recall that linear equations can have one solution (intersecting), no solution (parallel lines), or infinitely many solutions (graph is simultaneous). ☐ Define simultaneous. ☐ Recall how to solve linear equations. ☐ Recall properties of operations for addition and multiplication. ☐ Discover that the intersection of two lines on a coordinate plane is the solution to both equations. ☐ Define point of intersection. ☐ Recall how to solve linear equations. ☐ Demonstrate how to graph on the Cartesian plane. ☐ Identify ordered pairs. ☐ Recall how to solve linear equations in two variables by using substitution. ☐ Create a word problem from given information. ☐ Recall how to solve linear equations. ☐ Explain how to write an equation to solve real-world mathematical problems.

Algebra and Functions

Focus 1: Algebra

Cluster	2019 Math COS Standard	
Expressions, equations, and inequalities can be used to analyze and make predictions, both within mathematics and as mathematics is applied in different contexts – in particular, contexts that arise in relation to linear, quadratic, and exponential situations.	5. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. <i>Example: Rearrange the formula for the area of a trapezoid to highlight one of the bases.</i>	
	Learning Objectives	Prior Knowledge Skills
	☐ GEO.4.1: Accurately rearrange equations and inequalities to produce equivalent forms for use in resolving situations of interest.	☐ Define equivalent, simplify, term, distributive property, associative property of addition and multiplication, and the commutative property of addition and multiplication.

Algebra and Functions

Focus 2: Connecting Algebra to Functions

Cluster	2019 Math COS Standard	
Graphs can be used to obtain exact or approximate solutions of equations, inequalities, and systems of equations and inequalities—including systems of linear equations in two variables and systems of linear and quadratic equations (given or obtained by using technology).	6. *Derive the equation of a circle of given center and radius using the Pythagorean Theorem. - Given the endpoints of the diameter of a circle, use the midpoint formula to find its center and then use the Pythagorean Theorem to find its equation. - Derive the distance formula from the Pythagorean Theorem.	
	Learning Objectives	Prior Knowledge Skills
	☐ GEO.6.1: Define radius, diameter, midpoint, and Pythagorean Theorem. ☐ GEO.6.2: Apply the Pythagorean Theorem to find the distance from the center to a point on the circle. ☐ GEO.6.3: Derive the equation of a circle given the center and the radius. ☐ GEO.6.4: Use the midpoint formula to find the center of a circle based on the endpoints of the diameter.	☐ Identify parts of a circle. ☐ Recall how to find circumference of a circle. ☐ Recall the meaning of a radius and diameter. ☐ Identify all types of angles. ☐ Recognize the attributes of a circle. ☐ Identify and label parts of a circle. ☐ Define diameter, radius, circumference, area of a circle, and formula.

Data Analysis, Statistics, and Probability

Focus 1: Quantitative Literacy

Cluster	2019 Math COS Standard	
Mathematical and statistical reasoning about data can be used to evaluate conclusions and assess risks.	7. Use mathematical and statistical reasoning with quantitative data, both univariate data (set of values) and bivariate data (set of pairs of values) that suggest a linear association, in order to draw conclusions and assess risk. <i>Example: Estimate the typical age at which a lung cancer patient is diagnosed, and estimate how the typical age differs depending on the number of cigarettes smoked per day.</i>	
	Learning Objectives	Prior Knowledge Skills
	☐ GEO.7.1: Define bivariate scatter plot, quantitative data, outlier, cluster, linear, nonlinear, and positive and negative association. ☐ GEO.7.2: Describe patterns found in a scatter plot. ☐ GEO.7.3: Demonstrate how to label and plot information on a scatter plot (dot plot). ☐ GEO.7.4: Distinguish the difference between positive and negative correlation. ☐ GEO.7.5: Recall how to describe the spread of the scatter plot (dot plot).	☐ Define bivariate scatter plot, outlier, cluster, linear, nonlinear, and positive and negative association. ☐ Describe patterns found in a scatter plot. ☐ Demonstrate how to label and plot information on a scatter plot (dot plot). ☐ Distinguish the difference between positive and negative correlation. ☐ Recall how to describe the spread of the scatter plot (dot plot).

Data Analysis, Statistics, and Probability

Focus 2: Visualizing and Summarizing Data

Cluster	2019 Math COS Standard	
Data arise from a context and come in two types: quantitative (continuous or discrete) and categorical. Technology can be used to “clean” and organize data, including very large data sets, into a useful and manageable structure – a first step in any analysis of data.	8. Use technology to organize data, including very large data sets, into a useful and manageable structure.	
	Learning Objectives	Prior Knowledge Skills
	☐ GEO.8.1: Solve equations for y. ☐ GEO.8.2: Demonstrate use of a graphing calculator, including using a table, making a graph, and finding successive approximations. ☐ GEO.8.3: Analyze data from tables. ☐ GEO.8.4: Summarize categorical data for two categories in two-way frequency tables. ☐ GEO.8.5: Recognize possible associations and trends in the data. ☐ GEO.8.6: Create a scatter plot and line of best fit using data from a spreadsheet. ☐ GEO.8.7: Organize numerical data in a spreadsheet. ☐ GEO.8.8: Create graphical representations from classroom-generated data to model consumer costs. ☐ GEO.8.9: Create graphical representations from classroom-generated data to predict future outcomes. ☐ GEO.8.10: Create graphical representations from equations to model consumer costs. ☐ GEO.8.11: Create graphical representations from equations to predict future outcomes.	☐ Demonstrate how to plot points on a Cartesian plane using ordered pairs. ☐ Recall how to complete input/output tables. ☐ Recognize numeric patterns. ☐ Given a function, create a rule. ☐ Define linear equation, coefficient, distributive property, and variable. ☐ Recall how to solve equations for a missing variable. ☐ Recall properties of operation for addition and multiplication. ☐ Solve multi-step equations. ☐ Identify properties of operations.

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| | <ul style="list-style-type: none">☐ GEO.8.12: Create graphical representations from tables to model consumer costs.☐ GEO.8.13: Create graphical representations from tables to predict future outcomes. | |
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Data Analysis, Statistics, and Probability

Focus 2: Visualizing and Summarizing Data

Cluster	2019 Math COS Standard	
Distributions of quantitative data (continuous or discrete) in one variable should be described in the context of the data with respect to what is typical (the shape, with appropriate measures of center and variability, including standard deviation) and what is not (outliers), and these characteristics can be used to compare two or more subgroups with respect to a variable.	9. Represent the distribution of univariate quantitative data with plots on the real number line, choosing a format (dot plot, histogram, or box plot) most appropriate to the data set, and represent the distribution of bivariate quantitative data with a scatter plot. Extend from simple cases by hand to more complex cases involving large data sets using technology.	
	Learning Objectives	Prior Knowledge Skills
	☐ GEO.9.1: Organize and display univariate quantitative data using plots on a real number line, using dot plots, histograms, or box plots that is most appropriate to the given data set. ☐ GEO.9.2: Organize and display bivariate quantitative data using a scatter plot and extend from simple cases by hand to more complex cases involving a large data set using technology.	☐ Define dot plots, line plot, stem and leaf plots, upper quartile, lower quartile, median, histograms, and box plots.

Data Analysis, Statistics, and Probability

Focus 2: Visualizing and Summarizing Data

Cluster	2019 Math COS Standard	
Distributions of quantitative data (continuous or discrete) in one variable should be described in the context of the data with respect to what is typical (the shape, with appropriate measures of center and variability, including standard deviation) and what is not (outliers), and these characteristics can be used to compare two or more subgroups with respect to a variable.	10. *Use statistics appropriate to the shape of the data distribution to compare and contrast two or more data sets, utilizing the mean and median for center and the interquartile range and standard deviation for variability. a. Explain how standard deviation develops from mean absolute deviation. b. Calculate the standard deviation for a data set, using technology where appropriate.	
	Learning Objectives	Prior Knowledge Skills
	☐ GEO.10.1: Accurately find the center (median and mean) and spread (interquartile range and standard deviation) of data sets. ☐ GEO.10.2: Present viable arguments and critique arguments of others from the comparison of the center and spread of multiple data sets. ☐ GEO.10.3: Reason how standard deviation develops from the mean absolute deviation.	☐ Define measure of variability, distribution, and measure of center. ☐ Compare the measure of center and measure of variability of two distributions. ☐ Relate the measure of variation with the concept of range. ☐ Relate the measure of the center with the concept of mean. ☐ Recall how to calculate measure of center and measure of variability.

Data Analysis, Statistics, and Probability

Focus 2: Visualizing and Summarizing Data

Cluster	2019 Math COS Standard	
Distributions of quantitative data (continuous or discrete) in one variable should be described in the context of the data with respect to what is typical (the shape, with appropriate measures of center and variability, including standard deviation) and what is not (outliers), and these characteristics can be used to compare two or more subgroups with respect to a variable.	11. Interpret differences in shape, center, and spread in the context of data sets, accounting for possible effects of extreme data points (outliers) on mean and standard deviation.	
	Learning Objectives	Prior Knowledge Skills
	☐ GEO.11.1: Identify differences in shape, center, and spread when comparing two or more data sets, ☐ GEO.11.2: Identify outliers for the mean and standard deviation. ☐ GEO.11.3: Justify why there are differences in the shape, center, and spread of data sets.	☐ Define measure of variability, distribution, and measure of center. ☐ Compare the measure of center and measure of variability of two distributions. ☐ Relate the measure of variation with the concept of range. ☐ Relate the measure of the center with the concept of mean. ☐ Recall how to calculate measure of center and measure of variability.

Data Analysis, Statistics, and Probability

Focus 2: Visualizing and Summarizing Data

Cluster	2019 Math COS Standard	
Scatter plots, including plots over time, can reveal patterns, trends, clusters, and gaps that are useful in analyzing the association between two contextual variables.	12. Represent data of two quantitative variables on a scatter plot and describe how the variables are related. a. Find a linear function for a scatter plot that suggests a linear association and informally assess its fit by plotting and analyzing residuals, including the squares of the residuals, in order to improve its fit. b. Use technology to find the least-squares line of best fit for two quantitative variables.	
	Learning Objectives	Prior Knowledge Skills
	☐ GEO.12.1: Create a scatter plot of data. ☐ GEO.12.2: Calculate the fit of the function to the data by examining residuals. ☐ GEO.12.3: Describe a function to its data when there is evidence of a linear association. ☐ GEO.12.4: Use technology to find the least-squares line of best fit for two quantitative variables.	☐ Define bivariate scatter plot, outlier, cluster, linear, nonlinear, and positive and negative association. ☐ Describe patterns found in a scatter plot. ☐ Demonstrate how to label and plot information on a scatter plot (dot plot). ☐ Distinguish the difference between positive and negative correlation.

Data Analysis, Statistics, and Probability

Focus 2: Visualizing and Summarizing Data

Cluster	2019 Math COS Standard	
Analyzing the association between two quantitative variables should involve statistical procedures, such as examining (with technology) the sum of squared deviations in fitting a linear model, analyzing residuals for patterns, generating a least-squares regression line and finding a correlation coefficient, and differentiating between correlation and causation.	13. Compute (using technology) and interpret the correlation coefficient of a linear relationship.	
	Learning Objectives	Prior Knowledge Skills
	☐ GEO.13.1: Define mean, standard deviation, population, sample, and correlation coefficient. ☐ GEO.13.2: Calculate the correlation coefficient.	☐ Define measure of variability, distribution, and measure of center. ☐ Analyze the skew of the distributions and recognize the difference in measure of center and variability. ☐ Compare the measure of center and measure of variability of two distributions. ☐ Relate the measure of variation with the concept of range. ☐ Relate the measure of the center with the concept of mean. ☐ Recall how to calculate measure of center and measure of variability. ☐ Discuss how to read and interpret a graph. ☐ Define measure of variability, measure of center, inference, and mean absolute deviation. ☐ Recall how to calculate measure of center and variability. ☐ Recall that center is related to measure of center and measure of variability is related to variation. ☐ Compare and contrast the measure of center and variability of two numerical data sets. ☐ Calculate the mean absolute deviation of a data set in context.

Data Analysis, Statistics, and Probability		
Focus 2: Visualizing and Summarizing Data		
Cluster	2019 Math COS Standard	
Analyzing the association between two quantitative variables should involve statistical procedures, such as examining (with technology) the sum of squared deviations in fitting a linear model, analyzing residuals for patterns, generating a least-squares regression line, and finding a correlation coefficient, and differentiating	14. Distinguish between correlation and causation.	
	Learning Objectives	Prior Knowledge Skills
	☐ GEO.14.1: Define correlation and causation.	☐ Define bivariate scatter plot, outlier, cluster, linear, nonlinear, and positive and negative association. ☐ Describe patterns found in a scatter plot. ☐ Demonstrate how to label and plot information on a scatter plot (dot plot). ☐ Distinguish the difference between positive and negative correlation. ☐ Recall how to describe the spread of the scatter plot (dot plot).

*Critical Standard

between correlation and causation.		
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Data Analysis, Statistics, and Probability

Focus 2: Visualizing and Summarizing Data

Cluster	2019 Math COS Standard	
Data analysis techniques can be used to develop models of contextual situations and to generate and evaluate possible solutions to real problems involving those contexts.	15. *Evaluate possible solutions to real-life problems by developing linear models of contextual situations and using them to predict unknown values. - Use the linear model to solve problems in the context of the given data. - Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the given data.	
	Learning Objectives	Prior Knowledge Skills
	☐ GEO.15.1: Define slope as a rate of change. ☐ GEO. 15.2: Understand that the y-intercept is the initial amount in the context of the data. ☐ GEO.15.3: Understand that rate of change in the context of the data is the label of the y-axis divided by the label of the x-axis. ☐ GEO.15.4: Demonstrate use of a graphing calculator, including using a table, making a graph, and finding successive approximations. ☐ GEO.15.5: Given a contextual situation, interpret and defend the solution in the context of the original problem.	☐ Demonstrate how to plot points on a coordinate plane using ordered pairs from table. ☐ Analyze the graph to determine the rate of change. ☐ Generate the slope of a line using given ordered pairs. ☐ Draw and label a coordinate plane.

Geometry and Measurement

Focus 1: Measurement

Cluster	2019 Math COS Standard	
Areas and volumes of figures can be computed by determining how the figure might be obtained from simpler figures by dissection and recombination.	16. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.	
	Learning Objectives	Prior Knowledge Skills
	☐ GEO.16.1: Define two-dimensional objects and three-dimensional objects. ☐ GEO.16.2: Identify the two-dimensional figures that result from slicing three-dimensional figures as in plane section of right rectangular prisms and right rectangular pyramids. ☐ GEO.16.3: Identify three-dimensional objects generated by rotations of two-dimensional objects (as in rotating a circle to create a sphere). ☐ GEO.16.4: Distinguish between two-dimensional and three-dimensional objects.	☐ Define three-dimensional figures and nets. ☐ Identify three-dimensional figures. ☐ Select and create a three-dimensional figure using manipulatives. ☐ Define two-dimensional figure, three-dimensional figure, and plane section. ☐ List attributes of three-dimensional figures. ☐ List attributes of two-dimensional figures. ☐ Describe the relationship between two- and three-dimensional figures. ☐ Recognize symmetry.

Geometry and Measurement

Focus 1: Measurement

Cluster	2019 Math COS Standard	
Areas and volumes of figures can be computed by determining how the figure might be obtained from simpler figures by dissection and recombination.	17. Model and solve problems using surface area and volume of solids, including composite solids and solids with portions removed. - Give an informal argument for the formulas for the surface area and volume of a sphere, cylinder, pyramid, and cone using dissection arguments, Cavalieri's Principle, and informal limit arguments. - Apply geometric concepts to find missing dimensions to solve surface area or volume problems.	
	Learning Objectives	Prior Knowledge Skills
	☐ GEO.17.1: Define Cavalieri's principle, circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone; oblique, radius, diameter, height, and base. ☐ GEO.17.2: Compare surface areas of similar figures and volumes of similar figures to determine a relationship using dissection arguments, Cavalieri's principle, and informal limit arguments. ☐ GEO.17.3: Compare the characteristics and volume of oblique and right solids. ☐ GEO.17.4: Describe the properties of a given object (cylinder, pyramid, prism, and cone). ☐ GEO.17.5: Identify the necessary characteristics of a given solid to solve for its volume and surface area (cylinder, pyramid, prism, and cone). ☐ GEO.17.6: Calculate the surface area of three-dimensional figures (cylinder, pyramid, prism, and cone).	☐ Define three-dimensional figures, surface area, and nets. ☐ Identify three-dimensional figures. ☐ Evaluate how to apply using surface area of a three-dimensional figure to solve real-world and mathematical problems. ☐ Draw nets to find the surface area of a given three-dimensional figure. ☐ Recall how to calculate the area of a rectangle and triangle. ☐ Select and create a three-dimensional figure using manipulatives. ☐ Define diameter, radius, circumference, area of a circle, and formula. ☐ Identify and label parts of a circle. ☐ Recognize the attributes of a circle. ☐ Apply the formula of area and circumference to real-world mathematical situations. ☐ Define formula, volume, cone, cylinders, spheres, and height. ☐ Discuss the measure of volume and give examples. ☐ Solve problems with exponents. ☐ Recall how to find circumference of a circle. ☐ Identify parts of a circle. ☐ Calculate the volume of three-dimensional figures.

	☐ GEO.17.7: Calculate the volume of a cylinder, pyramid, prism, and cone. ☐ GEO.17.8: Calculate the area of a circle. ☐ GEO.17.9: Calculate the circumference of a circle. ☐ GEO.17.10: Calculate the area of the base shape. ☐ GEO.17.11: Identify the relationship of geometric representations to real-life objects. ☐ GEO.17.12: Identify the base shape.	☐ Solve real-world problems using the volume formulas for three-dimensional figures.
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Geometry and Measurement

Focus 1: Measurement

Cluster	2019 Math COS Standard	
Constructing approximations of measurements with different tools, including technology, can support an understanding of measurement.	18. Given the coordinates of the vertices of a polygon, compute its perimeter and area using a variety of methods, including the distance formula and dynamic geometry software, and evaluate the accuracy of the results.	
	Learning Objectives	Prior Knowledge Skills
	☐ GEO.18.1: Define area, perimeter, regular polygons, inscribed polygons, circumscribed polygons, and vertices. ☐ GEO.18.2: Analyze the given information to develop a logical process to calculate area or perimeter. ☐ GEO.18.3: Create equations for area and perimeter based on given information. ☐ GEO.18.4: Illustrate graphically an inscribed or circumscribed polygon. ☐ GEO.18.5: Solve equations given the area and perimeter. ☐ GEO.18.6: Plot given coordinates on the Cartesian plane.	☐ Define quadrant, coordinate plane, coordinate axes (x-axis and y-axis), horizontal, vertical, and reflection. ☐ Demonstrate an understanding of an extended coordinate plane. ☐ Draw and label a 4-quadrant coordinate plane. ☐ Draw and extend vertical and horizontal number lines. ☐ Interpret graphing points in all four quadrants of the coordinate plane in real-world situations. ☐ Recall how to graph points in all four quadrants of the coordinate plane. ☐ Define area. ☐ Analyze the area of other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes. ☐ Apply area formulas to solve real-world mathematical problems. ☐ Demonstrate how the area of a rectangle is equal to the sum of the area of two equal right triangles. ☐ Explain how to find the area for rectangles. ☐ Select manipulatives to demonstrate how to compose and decompose triangles and other shapes. ☐ Recognize and demonstrate that two right triangles make a rectangle. ☐ Define vertices.

		☐ Apply absolute value to find the length of a side joining points with the same first coordinate or the same second coordinate. ☐ Plot points on a coordinate plane., then connect points for the vertices to sketch a polygon. ☐ Identify ordered pairs. ☐ Recognize polygons. ☐ Define perimeter and area. ☐ Identify the length between vertices on a coordinate plane. ☐ Calculate the perimeter and area using the distance between the vertices.
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Geometry and Measurement

Focus 1: Measurement

Cluster	2019 Math COS Standard	
When an object is the image of a known object under a similarity transformation, a length, area, or volume on the image can be computed by using proportional relationships.	19. *Derive and apply the relationships between the lengths, perimeters, areas, and volumes of similar figures in relation to their scale factor.	
	Learning Objectives	Prior Knowledge Skills
	☐ GEO.19.1: Define scale factor, similarity, and proportions. ☐ GEO.19.2: Compare two figures in terms of similarity. ☐ GEO.19.3: Create proportional equations from given information. ☐ GEO.19.4: Solve proportional equations. ☐ GEO.19.5: Prove that equivalent ratios are proportions.	☐ Define unit rate, proportion, and rate. ☐ Create a ratio or proportion from a given word problem. ☐ Calculate unit rate by using ratios or proportions. ☐ Interpret a fraction as division of the numerator by the denominator. Example: $(a/b = a \text{ divided by } b)$. ☐ Write a ratio as a fraction. ☐ Define ratio, rate, proportion, percent, equivalent, input, output, ordered pairs, diagram, unit rate, and table. ☐ Create a ratio or proportion from a given word problem, diagram, table, or equation. ☐ Calculate unit rate or rate by using ratios or proportions. ☐ Restate real-world or mathematical problems. ☐ Construct a graph from a set of ordered pairs given in the table of equivalent ratios. ☐ Compute the unit rate, unit price, and constant speed. ☐ Create a proportion or ratio from a given word problem. ☐ Identify the two units being compared. ☐ Calculate a proportion for missing information. ☐ Identify a proportion from given information. ☐ Solve a proportion using part over whole equals percent over 100. ☐ Form a ratio. ☐ Define proportions and proportional relationships. ☐ Demonstrate how to write ratios as a fraction. ☐ Solve proportional problems.

*Critical Standard

		☐ Define proportional and nonproportional. ☐ Recall that for two relationships to be proportional they must have the same unit rate and pass through the origin on a coordinate plane. ☐ Apply the rule of proportional relationship to real-world context. ☐ Recall how to solve proportions using cross products.
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Geometry and Measurement

Focus 1: Measurement

Cluster	2019 Math COS Standard	
When an object is the image of a known object under a similarity transformation, a length, area, or volume on the image can be computed by using proportional relationships.	20. Derive and apply the formula for the length of an arc and the formula for the area of a sector.	
	Learning Objectives	Prior Knowledge Skills
	☐ GEO.20.1: Define arc length, radian measure, and sector. ☐ GEO.20.2: Prove the length of the arc intercepted by an angle is proportional to the radius by similarity. ☐ GEO.20.3: Prove the formula for the area of the sector. ☐ GEO.20.4: Illustrate an arc of a circle by constructing the radii of a circle.	☐ Identify parts of a circle. ☐ Recall the meaning of a radius and diameter. ☐ Identify all types of angles. ☐ Recognize the attributes of a circle. ☐ Identify and label parts of a circle. ☐ Define diameter, radius, circumference, area of a circle, and formula.

Geometry and Measurement

Focus 2: Transformation

Cluster	2019 Math COS Standard	
Applying geometric transformations to figures provides opportunities for describing the attributes of the figures preserved by the transformation and for describing symmetries by examining when a figure can be mapped onto itself.	21. *Represent transformations and compositions of transformations in the plane (coordinate and otherwise) using tools such as tracing paper and geometry software. - Describe transformations and compositions of transformations as functions that take points in the plane as inputs and give other points as outputs, using informal and formal notation. - Compare transformations which preserve distance and angle measure to those that do not.	
	Learning Objectives	Prior Knowledge Skills
	☐ GEO.21.1: Define distance, angle, input, output, plane, translation, reflection, rotation, and dilation. ☐ GEO.21.2: Compare transformation that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). ☐ GEO.21.3: Describe transformations as functions that take points in a plane as inputs and give other points as outputs. ☐ GEO.21.4: Represent transformation in the plane. ☐ GEO.21.5: Generate an input output table. ☐ GEO.21.6: Compare the distance and angles of the figures from the pre-image to the image. - GEO.21.7: Measure distance and angle(s) of an image.	☐ Define rotation, reflection, and translation. ☐ Recognize translations (slides), rotations (turns), and reflections (flips). ☐ Distinguish between lines and line segments. ☐ Demonstrate how to measure length. ☐ Demonstrate how to use a protractor to measure angles. ☐ Identify parallel lines. ☐ Define square root, cube root, inverse, perfect square, perfect cube, and irrational number. ☐ Define square root, expressions, and approximations. ☐ Identify perfect squares and square roots. ☐ Demonstrate how to locate points on a vertical or horizontal number line. ☐ Define ordered pairs. ☐ Show how to plot points on a Cartesian plane. ☐ Locate the origin on the coordinate plane. ☐ Identify the length between vertices on a coordinate plane. ☐ Recall how to read a graph or table. ☐ Draw and label a coordinate plane.

		☐ Plot independent (input) and dependent (output) values on a coordinate plane. ☐ Plot pairs of integers and/or rational numbers on a coordinate plane. ☐ Arrange integers and/or rational numbers on a horizontal or vertical number line. ☐ Locate the position of integers and/or rational numbers on a horizontal or vertical number line. ☐ Define quadrant, coordinate plane, coordinate axes (x-axis and y-axis), horizontal, vertical, and reflection. ☐ Calculate the distances between points having the same first or second coordinate using absolute value. ☐ Define number line. ☐ Demonstrate the location of positive and negative numbers on a vertical and horizontal number line. ☐ Calculate missing input and/or output values in a table.
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Geometry and Measurement

Focus 2: Transformation

Cluster	2019 Math COS Standard	
Applying geometric transformations to figures provides opportunities for describing the attributes of the figures preserved by the transformation and for describing symmetries by examining when a figure can be mapped onto itself.	22. *Explore rotations, reflections, and translations using graph paper, tracing paper, and geometry software. - Given a geometric figure and a rotation, reflection, or translation, draw the image of the transformed figure using graph paper, tracing paper, or geometry software. - Specify a sequence of rotations, reflections, or translations that will carry a given figure onto another. - Draw figures with different types of symmetries and describe their attributes.	
	Learning Objectives	Prior Knowledge Skills
	☐ GEO.22.1: Define rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. ☐ GEO.22.2: Describe the effects of rotations, reflection, and translations on two dimensional figures using coordinates. ☐ GEO.22.3: Illustrate figures transformed by a rotation, reflection, or translation. ☐ GEO.22.4: Describe the process of transforming a given figure. - GEO.22.5: Graph a figure on a coordinate plane.	☐ Recognize dilations. ☐ Recognize translations. ☐ Recognize rotations. ☐ Recognize reflections. ☐ Define rotation, reflection, and translation. ☐ Recognize translations (slides), rotations (turns), and reflections (flips). ☐ Distinguish between lines and line segments. ☐ Identify parallel lines. ☐ Demonstrate how to locate points on a vertical or horizontal number line. ☐ Define ordered pairs. ☐ Show how to plot points on a Cartesian plane. ☐ Locate the origin on the coordinate plane. ☐ Identify the length between vertices on a coordinate plane. ☐ Recall how to read a graph or table. ☐ Draw and label a coordinate plane. ☐ Plot independent (input) and dependent (output) values on a coordinate plane.

		☐ Plot pairs of integers and/or rational numbers on a coordinate plane. ☐ Arrange integers and/or rational numbers on a horizontal or vertical number line. ☐ Locate the position of integers and/or rational numbers on a horizontal or vertical number line. ☐ Define quadrant, coordinate plane, coordinate axes (x-axis and y-axis), horizontal, vertical, and reflection. ☐ Demonstrate when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. ☐ Calculate the distances between points having the same first or second coordinate using absolute value. ☐ Define number line. ☐ Demonstrate the location of positive and negative numbers on a vertical and horizontal number line.
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Geometry and Measurement

Focus 2: Transformation

Cluster	2019 Math COS Standard	
Applying geometric transformations to figures provides opportunities for describing the attributes of the figures preserved by the transformation and for describing symmetries by examining when a figure can be mapped onto itself.	23. Develop definitions of rotation, reflection, and translation in terms of angles, circles, perpendicular lines, parallel lines, and line segments.	
	Learning Objectives	Prior Knowledge Skills
	☐ GEO.23.1: Define rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. ☐ GEO.23.2: Describe the effects of rotations, reflection, and translations on two dimensional figures using coordinates. ☐ GEO.23.3: Describe the effects of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. ☐ GEO.23.4: Describe the process of transforming a given figure. ☐ GEO.23.5: Illustrate figures transformed by a rotation, reflection, or translation. ☐ GEO.23.6: Recognize the type of transformation from a pre-image to an image.	☐ Recognize dilations. ☐ Recognize translations. ☐ Recognize rotations. ☐ Recognize reflections. ☐ Analyze an image and its dilation to determine if the two figures are similar. ☐ Define dilation. ☐ Recall how to find scale factor. ☐ Give examples of scale drawings. ☐ Recognize translations. ☐ Recognize reflections. ☐ Recognize rotations. ☐ Identify parallel lines. ☐ Compare translations to reflections. ☐ Compare reflections to rotations. ☐ Compare rotations to translations. ☐ Define diameter, radius, circumference, area of a circle, and formula. ☐ Identify and label parts of a circle. ☐ Recognize the attributes of a circle. ☐ Define rotation, reflection, and translation. ☐ Recognize translations (slides), rotations (turns), and reflections (flips). ☐ Distinguish between lines and line segments.

		☐ Identify parallel lines. ☐ Define square root, cube root, inverse, perfect square, perfect cube, and irrational number. ☐ Define square root, expressions, and approximations ☐ Demonstrate how to locate points on a vertical or horizontal number line. ☐ Define ordered pairs. ☐ Show how to plot points on a Cartesian plane. ☐ Locate the origin on the coordinate plane. ☐ Identify the length between vertices on a coordinate plane. ☐ Recall how to read a graph or table. ☐ Draw and label a coordinate plane. ☐ Plot independent (input) and dependent (output) values on a coordinate plane. ☐ Plot pairs of integers and/or rational numbers on a coordinate plane. ☐ Arrange integers and/or rational numbers on a horizontal or vertical number line. ☐ Locate the position of integers and/or rational numbers on a horizontal or vertical number line. ☐ Define quadrant, coordinate plane, coordinate axes (x-axis and y-axis), horizontal, vertical, and reflection. ☐ Demonstrate when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. ☐ Calculate the distances between points having the same first or second coordinate using absolute value. ☐ Define number line. ☐ Demonstrate the location of positive and negative numbers on a vertical and horizontal number line. ☐ Calculate missing input and/or output values in a table.
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Geometry and Measurement

Focus 2: Transformation

Cluster	2019 Math COS Standard	
Showing that two figures are congruent involves showing that there is a rigid motion (translation, rotation, reflection, or glide reflection) or, equivalently, a sequence of rigid motions that maps one figure to the other.	24. *Define congruence of two figures in terms of rigid motions (a sequence of translations, rotations, and reflections); show that two figures are congruent by finding a sequence of rigid motions that maps one figure to the other. <i>Example: $\triangle ABC$ is congruent to $\triangle XYZ$ since a reflection followed by a translation maps $\triangle ABC$ onto $\triangle XYZ$.</i>	
	Learning Objectives	Prior Knowledge Skills
	☐ GEO.24.1: Define congruence. ☐ GEO.24.2: Applying the definition of congruence determine if two figures are congruent. ☐ GEO.24.3: Illustrate a sequence of rigid motions on a coordinate plane that maps one figure to another. ☐ GEO.24.4: Illustrate a vertical and horizontal shift on a coordinate plane. Example: Rectangle PQRS has vertices P(-3,5), Q(-4,2), R (3,0), S(4,3). Translate PQRS vertically 3 units. ☐ GEO.24.5: Recognize composition of transformations. ☐ GEO.24.6: Graph a figure on a coordinate plane.	☐ Recognize dilations. ☐ Recognize translations. ☐ Recognize rotations. ☐ Recognize reflections. ☐ Analyze an image and its dilation to determine if the two figures are similar. ☐ Define dilation. ☐ Recall how to find scale factor. ☐ Give examples of scale drawings. ☐ Recognize translations. ☐ Recognize reflections. ☐ Recognize rotations. ☐ Identify parallel lines. ☐ Define congruent and sequence. ☐ Compare translations to reflections. ☐ Compare reflections to rotations. ☐ Compare rotations to translations. ☐ Identify congruent figures. ☐ Define diameter, radius, circumference, area of a circle, and formula. -Identify and label parts of a circle. ☐ Recognize the attributes of a circle.

		☐ Define rotation, reflection, and translation. ☐ Recognize translations (slides), rotations (turns), and reflections (flips). ☐ Distinguish between lines and line segments. ☐ Identify parallel lines. ☐ Define square root, cube root, inverse, perfect square, perfect cube, and irrational number. ☐ Define square root, expressions, and approximations. ☐ Demonstrate how to locate points on a vertical or horizontal number line. ☐ Define ordered pairs. ☐ Show how to plot points on a Cartesian plane. ☐ Locate the origin on the coordinate plane. ☐ Identify the length between vertices on a coordinate plane. ☐ Recall how to read a graph or table. ☐ Draw and label a coordinate plane. ☐ Plot independent (input) and dependent (output) values on a coordinate plane. ☐ Plot pairs of integers and/or rational numbers on a coordinate plane. ☐ Arrange integers and/or rational numbers on a horizontal or vertical number line. ☐ Locate the position of integers and/or rational numbers on a horizontal or vertical number line. ☐ Define quadrant, coordinate plane, coordinate axes (x-axis and y-axis), horizontal, vertical, and reflection. ☐ Demonstrate when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. ☐ Calculate the distances between points having the same first or second coordinate using absolute value. ☐ Define number line.
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*Critical Standard

		☐ Demonstrate the location of positive and negative numbers on a vertical and horizontal number line. ☐ Calculate missing input and/or output values in a table.
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Geometry and Measurement

Focus 2: Transformation

Cluster	2019 Math COS Standard	
Showing that two figures are congruent involves showing that there is a rigid motion (translation, rotation, reflection, or glide reflection) or, equivalently, a sequence of rigid motions that maps one figure to the other.	25. *Verify criteria for showing triangles are congruent using a sequence of rigid motions that map one triangle to another. a. Verify that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. b. Verify that two triangles are congruent if (but not only if) the following groups of corresponding parts are congruent: angle-side-angle (ASA), side-angle-side (SAS), side-side-side (SSS), and angle-angle side (AAS). <i>Example: Given two triangles with two pairs of congruent corresponding sides and a pair of congruent included angles, show that there must be a sequence of rigid motions will map one onto the other.</i>	
	Learning Objectives	Prior Knowledge Skills
	☐ GEO.25.1: Define congruent, corresponding, triangles, angles, and the concept of if and only if. ☐ GEO.25.2: Compare angles and sides of two triangles to determine congruency. ☐ GEO.25.3: Determine the lengths of sides and the measures of angles in triangles. ☐ GEO.25.4: Identify corresponding parts of triangles.	☐ Define congruent and sequence. -Identify congruent figures. ☐ Recognize attributes of geometric shapes. ☐ Identify the length between vertices on a coordinate plane.

Geometry and Measurement

Focus 2: Transformation

Cluster	2019 Math COS Standard	
Showing that two figures are similar involves finding a similarity transformation (dilation or composite of a dilation with a rigid motion) or, equivalently, a sequence of similarity transformations that maps one figure onto the other.	26. Verify experimentally the properties of dilations given by a center and a scale factor. - Verify that a dilation takes a line not passing through the center of the dilation to a parallel line and leaves a line passing through the center unchanged. - Verify that the dilation of a line segment is longer or shorter in the ratio given by the scale factor.	
	Learning Objectives	Prior Knowledge Skills
	☐ GEO.26.1: Define dilation and scale factor. ☐ GEO.26.2: Apply a scale factor. ☐ GEO.26.2: Illustrate when given an original figure with a line (e.g., m) through it, not through the center, a parallel line to m will be created when the dilation is performed. Example: Given a line $x=$, dilate the graph and line by 2. What happened to the line? ☐ GEO.26.3: Illustrate when given an original figure with a line (e.g., m) through its center the line will remain unchanged when the dilation is performed. ☐ GEO.26.4: Illustrate dilation. Example: Find the distance of line AB , given $A(0,0)$ and $B(2,3)$, after dilating AB by a scale factor of $1/2$. ☐ GEO.26.5: Determine the change in length of a line segment after dilation. ☐ GEO.26.6: Discuss the properties of parallel lines. ☐ GEO.26.7: Dilate a line segment.	☐ Recall how to name points on a Cartesian plane using ordered pairs. ☐ Recognize ordered pairs (x, y) . ☐ Define similar. ☐ Recognize dilations. ☐ Recognize translations. ☐ Recognize rotations. ☐ Recognize reflections. ☐ Identify similar figures. ☐ Analyze an image and its dilation to determine if the two figures are similar. ☐ Define dilation. ☐ Recall how to find scale factor. ☐ Give examples of scale drawings. ☐ Identify parts of the Cartesian plane. ☐ Recognize ordered pairs. ☐ Define function, ordered pairs, input, output. ☐ Demonstrate how to plot points on a Cartesian plane using ordered pairs.

*Critical Standard

	☐ GEO.26.8: Recognize whether a dilation is an enlargement or a reduction.	
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Geometry and Measurement

Focus 2: Transformation

Cluster	2019 Math COS Standard	
Showing that two figures are similar involves finding a similarity transformation (dilation or composite of a dilation with a rigid motion) or, equivalently, a sequence of similarity transformations that maps one figure onto the other.	27. *Given two figures, determine whether they are similar by identifying a similarity transformation (sequence of rigid motions and dilations) that maps one figure to the other.	
	Learning Objectives	Prior Knowledge Skills
	☐ GEO.27.1: Establish a sequence of similarity transformations between two similar polygons. ☐ GEO.27.2: Determine if two triangles are similar based on their corresponding parts. ☐ GEO.27.3: Develop a similarity statement for two similar polygons. ☐ GEO.27.4: Identify corresponding angles and sides based on similarity statements.	☐ Recognize dilations. ☐ Recognize translations. ☐ Recognize rotations. ☐ Recognize reflections. ☐ Define rotation, reflection, and translation. ☐ Recognize translations (slides), rotations (turns), and reflections (flips). ☐ Distinguish between lines and line segments. ☐ Identify parallel lines. ☐ Demonstrate how to locate points on a vertical or horizontal number line. ☐ Define ordered pairs. ☐ Show how to plot points on a Cartesian plane. ☐ Locate the origin on the coordinate plane. ☐ Identify the length between vertices on a coordinate plane. ☐ Recall how to read a graph or table. ☐ Draw and label a coordinate plane. ☐ Plot independent (input) and dependent (output) values on a coordinate plane. ☐ Plot pairs of integers and/or rational numbers on a coordinate plane. ☐ Arrange integers and/or rational numbers on a horizontal or vertical number line.

		☐ Locate the position of integers and/or rational numbers on a horizontal or vertical number line. ☐ Define quadrant, coordinate plane, coordinate axes (x-axis and y-axis), horizontal, vertical, and reflection. ☐ Demonstrate when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. ☐ Calculate the distances between points having the same first or second coordinate using absolute value.
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Geometry and Measurement

Focus 2: Transformation

Cluster	2019 Math COS Standard	
Showing that two figures are similar involves finding a similarity transformation (dilation or composite of a dilation with a rigid motion) or, equivalently, a sequence of similarity transformations that maps one figure onto the other.	28. *Verify criteria for showing triangles are similar using a similarity transformation (sequence of rigid motions and dilations) that maps one triangle to another. - Verify that two triangles are similar if and only if corresponding pairs of sides are proportional and corresponding pairs of angles are congruent. - Verify that two triangles are similar if (but not only if) two pairs of corresponding angles are congruent (AA), the corresponding sides are proportional (SSS), or two pairs of corresponding sides are proportional, and the pair of included angles is congruent (SAS). <i>Example: Given two triangles with two pairs of congruent corresponding sides and a pair of congruent included angles, show there must be a set of rigid motions that maps one onto the other.</i>	
	Learning Objectives	Prior Knowledge Skills
	☐ GEO.28.1: Define corresponding, similarity and proportions. ☐ GEO.28.2: Evaluate the properties of the triangles to prove congruency. ☐ GEO.28.3: Create proportional equations from given information. ☐ GEO.28.4: Evaluate the angle-side-angle (ASA), side-angle-side (SAS), and side-side-side (SSS), Theorems to prove similarity. ☐ GEO.28.5: Evaluate the AA postulate to prove similarity. ☐ GEO.28.6: Compare two figures in terms of similarity. ☐ GEO.28.7: Demonstrate that equivalent ratios are proportions. ☐ GEO.28.8: Solve proportional equations.	☐ Apply properties to find missing angle measures. ☐ Identify a transversal. ☐ Identify exterior angles, interior angles, vertical angles, adjacent angles, alternate interior angles, alternate exterior angles, and corresponding angles. ☐ Identify attributes of triangles. ☐ Define exterior angles, interior angles, vertical angles, adjacent angles, alternate interior angles, alternate exterior angles, corresponding angles, and transversal. ☐ Discover the Angle Sum Theorem (sum of the interior angles of a triangle equal 180 degrees). ☐ Identify parallel lines. ☐ Demonstrate how to use a protractor to measure angles. ☐ Demonstrate how to measure length. ☐ Distinguish between lines and line segments.

*Critical Standard

		☐ Recognize translations (slides), rotations (turns), and reflections (flips). ☐ Define rotation, reflection, and translation. ☐ Recognize attributes of geometric shapes. ☐ Identify the length between vertices on a coordinate plane.
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Geometry and Measurement

Focus 3: Geometric Arguments, Reasoning, and Proof

Cluster	2019 Math COS Standard	
Using technology to construct and explore figures with constraints provides an opportunity to explore the independence and dependence of assumptions and conjectures.	29. Find patterns and relationships in figures including lines, triangles, quadrilaterals, and circles, using technology and other tools. - Construct figures, using technology and other tools, in order to make and test conjectures about their properties. - Identify different sets of properties necessary to define and construct figures.	
	Learning Objectives	Prior Knowledge Skills
	☐ GEO.29.1: Construct a copy of a segment, angle, bisection of a segment, bisection of an angle, perpendicular line, perpendicular bisector of a line segment, and parallel lines. ☐ GEO.29.2: Describe a specific construction process. ☐ GEO.29.3: Demonstrate the proper use of a geometric construction tools.	☐ Demonstrate how to use a protractor to draw an angle. ☐ Construct segments of a given length using a ruler. ☐ Recognize attributes of geometric shapes.

Geometry and Measurement

Focus 3: Geometric Arguments, Reasoning, and Proof

Cluster	2019 Math COS Standard	
Proof is the means by which we demonstrate whether a statement is true or false mathematically, and proofs can be communicated in a variety of ways (e.g., two-column, paragraph).	30. *Develop and use precise definitions of figures such as angle, circle, perpendicular lines, parallel lines, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.	
	Learning Objectives	Prior Knowledge Skills
	☐ GEO.30.1: Define angle, circle, perpendicular line, parallel line, line segment, and distance. ☐ GEO.30.2: Describe angle, circle, perpendicular line, parallel line, line segment, and distance. ☐ GEO.30.3: Illustrate a point, line, distance along a line, and distance around a circular arc. ☐ GEO.30.4: Identify angle, circle, perpendicular line, parallel line, line segment, and distance.	☐ Define exterior angles, interior angles, vertical angles, adjacent angles, alternate interior angles, alternate exterior angles, corresponding angles, and transversal. ☐ Identify exterior angles, interior angles, vertical angles, adjacent angles, alternate interior angles, alternate exterior angles, and corresponding angles. ☐ Identify a transversal. ☐ Apply properties to find missing angle measures. ☐ Define supplementary, complementary, vertical, and adjacent angles; parallel, perpendicular, and intersecting lines. ☐ Identify all types of angles. ☐ Identify right angles and straight angles. ☐ Demonstrate how to use a protractor to draw an angle. ☐ Define vertices.

Geometry and Measurement

Focus 3: Geometric Arguments, Reasoning, and Proof

Cluster	2019 Math COS Standard	
Proof is the means by which we demonstrate whether a statement is true or false mathematically, and proofs can be communicated in a variety of ways (e.g., two-column, paragraph).	31. Justify whether conjectures are true or false in order to prove theorems and then apply those theorems in solving problems, communicating proofs in a variety of ways, including flow chart, two-column, and paragraph formats. - Investigate, prove, and apply theorems about lines and angles, including but not limited to: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; the points on the perpendicular bisector of a line segment are those equidistant from the segment's endpoints. - Investigate, prove, and apply theorems about triangles, including but not limited to: the sum of the measures of the interior angles of a triangle is 180°; the base angles of isosceles triangles are congruent; the segment joining the midpoints of two sides of a triangle is parallel to the third side and half the length; a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem using triangle similarity. - Investigate, prove, and apply theorems about parallelograms and other quadrilaterals, including but not limited to both necessary and sufficient conditions for parallelograms and other quadrilaterals, as well as relationships among kinds of quadrilaterals. <i>Example: Prove that rectangles are parallelograms with congruent diagonals.</i>	
	Learning Objectives	Prior Knowledge Skills
	☐ GEO.31.1: Define vertical angle, transversal, parallel lines, alternate interior angles, corresponding angles, perpendicular bisector, line segment, equidistant, endpoints, interior angles of a triangle, base angles of isosceles triangles, isosceles triangles, midpoint, median, intersection, opposite sides, opposite	☐ Define a right angle, Pythagorean Theorem, converse, and proof. ☐ Define exterior angles, interior angles, vertical angles, adjacent angles, alternate interior angles, alternate exterior angles, corresponding angles, and transversal. ☐ Identify attributes of triangles. ☐ Identify exterior angles, interior angles, vertical angles, adjacent angles, alternate interior angles, alternate exterior angles, and corresponding angles. ☐ Identify a transversal.

	angles, diagonals, parallelogram, bisector, and converse. ☐ GEO.31.2: Develop a process that demonstrates the logical order of properties to form a proof. ☐ GEO.31.3: Arrange statements to form a logical order. ☐ GEO.31.4: Identify measures of vertical angles, alternate interior angles, corresponding angles, measures of interior angles of a triangle, base angles of isosceles triangles, isosceles triangles, midpoint, and median. ☐ GEO.31.5: Illustrate vertical angle, transversal, parallel lines, alternate interior angles, corresponding angles, perpendicular bisector, line segment, equidistant, endpoints, interior angles of a triangle, base angles of isosceles triangles, isosceles triangles, midpoint, median, intersection, opposite sides, opposite angles, diagonals, parallelograms, bisectors, and their properties. ☐ GEO.31.6: Find the measure of the third interior angle of a triangle when given the measure of the other two interior angles.	☐ Apply properties to find missing angle measures. ☐ Discover the Angle Sum Theorem (sum of the interior angles of a triangle equal 180 degrees). -Identify parallel lines. ☐ Define supplementary angles, complementary angles, vertical angles, adjacent angles, parallel lines, perpendicular lines, and intersecting lines. ☐ Select manipulatives to demonstrate how to compose and decompose triangles and other shapes. ☐ Recognize and demonstrate that two right triangles make a rectangle. ☐ Recognize polygons.
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Geometry and Measurement

Focus 3: Geometric Arguments, Reasoning, and Proof

Cluster	2019 Math COS Standard	
Proofs of theorems can sometimes be made with transformations, coordinates, or algebra; all approaches can be useful, and in some cases, one may provide a more accessible or understandable argument than another.	32. Use coordinates to prove simple geometric theorems algebraically.	
	Learning Objectives	Prior Knowledge Skills
	☐ GEO.32.1: Apply formulas, and properties of polygons, angles, and lines to draw conclusions from the given information. ☐ GEO.32.2: Identify properties of perpendicular and parallel lines, properties of polygons. ☐ GEO.32.3: Illustrate polygons created by given coordinates on a coordinate plane. ☐ GEO.32.4: Identify distance formula, circle formula, Pythagorean Theorem, midpoint.	☐ Define quadrant, coordinate plane, coordinate axes (x-axis and y-axis), horizontal, vertical, and reflection. ☐ Demonstrate an understanding of an extended coordinate plane. ☐ Draw and label a 4-quadrant coordinate plane. ☐ Draw and extend vertical and horizontal number lines. ☐ Interpret graphing points in all four quadrants of the coordinate plane in real-world situations. ☐ Recall how to graph points in all four quadrants of the coordinate plane. ☐ Define ordered pairs. ☐ Name the pairs of integers and/or rational numbers of a point on a coordinate plane. ☐ Demonstrate when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. ☐ Identify which signs indicate the location of a point in a coordinate plane. ☐ Recall how to plot ordered pairs on a coordinate plane. ☐ Identify the length between vertices on a coordinate plane. -Calculate the perimeter and area using the distance between the vertices. -Define a right angle, Pythagorean Theorem, converse, and proof. -Recognize examples of right triangles. -Demonstrate how to find square roots. -Solve problems with exponents.

Geometry and Measurement		
Focus 3: Geometric Arguments, Reasoning, and Proof		
Cluster	2019 Math COS Standard	
Proofs of theorems can sometimes be made with transformations, coordinates, or algebra; all approaches can be useful, and in some cases, one may provide a more accessible or understandable argument than another.	33. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems. <i>Example: Find the equation of a line parallel or perpendicular to a given line that passes through a given point.</i>	
	Learning Objectives	Prior Knowledge Skills
	☐ GEO.33.1: Define slope, point slope formula, slope-intercept formula, standard form of a line, parallel lines, and perpendicular lines. ☐ GEO.33.2: Demonstrate and explain algebraically how perpendicular lines have only one common point. ☐ GEO.33.3: Demonstrate and explain algebraically how parallel lines have no common points. ☐ GEO.33.4: Write and solve equations of parallel and perpendicular lines. ☐ GEO.33.5: Illustrate graphically how perpendicular lines have only one common point. ☐ GEO.33.6: Illustrate graphically how parallel lines have no common points. ☐ GEO.33.7: Write an equation of a line in slope intercept form. ☐ GEO.33.8: Find the slope of a given line.	☐ Define slope, intercept, linear, equation, and bivariate. ☐ Recall how to determine the rate of change (slope) from a graph. ☐ Identify the parts of the slope-intercept form of an equation. ☐ Recognize how to read a graph. ☐ Recall how to write an equation in slope-intercept form. ☐ Apply the identification of the slope and the y-intercept to a real-world situation. ☐ Create a graph to model a real-word situation. ☐ Define proportional relationships, unit rate, and slope. ☐ Demonstrate how to graph on a Cartesian plane ☐ Identify the slope-intercept form ($y=mx+b$) of an equation where m is the slope and y is the y-intercept. ☐ Define linear functions, nonlinear functions, slope, and y-intercept. ☐ Recognize linear equations. ☐ Identify ordered pairs. ☐ Recognize ordered pairs. ☐ Generate the slope of a line using given ordered pairs. ☐ Identify the slope-intercept form ($y=mx+b$) of an equation where m is the slope and y is the y-intercept. ☐ Graph a function given the slope-intercept form of an equation. ☐ Recognize that two sets of points with the same slope may have different y-intercepts.

*Critical Standard

		☐ Graph a linear equation given the slope-intercept form of an equation.
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Geometry and Measurement

Focus 4: Solving Applied Problems and Modeling in Geometry

Cluster	2019 Math COS Standard	
Recognizing congruence, similarity, symmetry, measurement opportunities, and other geometric ideas, including right triangle trigonometry, in real-world contexts provides a means of building understanding of these concepts and is a powerful tool for solving problems related to the physical world in which we live.	34. *Use congruence and similarity criteria for triangles to solve problems in real-world contexts.	
	Learning Objectives	Prior Knowledge Skills
	☐ GEO.34.1: Develop an equation from given information to prove congruence or similarity. ☐ GEO.34.2: Illustrate congruence and similarity in geometric figures. ☐ GEO.34.3: Apply proportional reasoning to real-world scenarios. ☐ GEO.34.4: Solve proportions.	☐ Analyze an image and its dilation to determine if the two figures are similar. ☐ Identify similar figures. ☐ Define similar. ☐ Identify congruent figures. ☐ Identify attributes of two-dimensional figures. ☐ Compare rotations to translations. ☐ Compare reflections to rotations. ☐ Compare translations to reflections. ☐ Define congruent and sequence. ☐ Apply the rule of proportional relationship to real-world context. ☐ Recognize similar triangles. ☐ Define similar triangles, intercept, slope, vertical, horizontal, and origin. ☐ Demonstrate how to plot points on a coordinate plane using ordered pairs from table. ☐ Analyze the graph to determine the rate of change. ☐ Generate the slope of a line using given ordered pairs. ☐ Graph a function given the slope-intercept form of an equation. ☐ Identify the slope-intercept form ($y=mx+b$) of an equation where m is the slope and y is the y-intercept. ☐ Graph a linear equation given the slope-intercept form of an equation. ☐ Recognize that two sets of points with the same slope may have different y-intercepts.

		☐ Identify the slope-intercept form ($y=mx+b$) of an equation where m is the slope and y is the y-intercept. ☐ Recall that for a relationship to be proportional, the graph must pass through the origin. ☐ Demonstrate how to graph on a Cartesian plane. ☐ Recall that for a relationship to be proportional, both variables must start at zero. ☐ Identify the unit rate of two quantities. ☐ Recall how to write a ratio of two quantities as a fraction. ☐ Recall equivalent ratios and origin on a coordinate (Cartesian) plane. ☐ Define proportional, independent variable, dependent variable, and unit rate. ☐ Identify proportional relationships. ☐ Locate/use scale on a map. ☐ Define scale, scale drawings, length, area, and geometric figures. ☐ Use a table or graph to determine whether two quantities are proportional. ☐ Define equivalent ratios and origin. ☐ Define unit rate, proportions, area, length, and ratio. ☐ Recognize polygons. ☐ Restate real-world problems or mathematical problems. ☐ Calculate unit rate or rate by using ratios or proportions. ☐ Create a ratio or proportion from a given word problem, diagram, table, or equation. ☐ Define ratio, rate, proportion, percent, equivalent, input, output, ordered pairs, diagram, unit rate, and table. ☐ Form a ratio. ☐ Solve a proportion using part over whole equals percent over 100. ☐ Identify a proportion from given information. ☐ Calculate a proportion for missing information. ☐ Create a proportion or ratio from a given word problem.
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Geometry and Measurement		
Focus 4: Solving Applied Problems and Modeling in Geometry		
Cluster	2019 Math COS Standard	
Recognizing congruence, similarity, symmetry, measurement opportunities, and other geometric ideas, including right triangle trigonometry, in real-world contexts provides a means of building understanding of these concepts and is a powerful tool for solving problems related to the physical world in which we live.	35. *Discover and apply relationships in similar right triangles. - Derive and apply the constant ratios of the sides in special right triangles (45"-45"-90" and 30"-60"-90"). - Use similarity to explore and define basic trigonometric ratios, including sine ratio, cosine ratio, and tangent ratio. - Explain and use the relationship between the sine and cosine of complementary angles. - Demonstrate the converse of the Pythagorean Theorem. - Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems, including finding areas of regular polygons.	
	Learning Objectives	Prior Knowledge Skills
	☐ GEO.35.1: Define trigonometric (sine, cosine, and tangent) ratios for acute angles, complementary angles, and Pythagorean Theorem. ☐ GEO.35.2: Simplify, multiply, and divide radicals. ☐ GEO.35.3: Discuss the relationship between sine and cosine angles within a triangle. ☐ GEO.35.4: Solve equations using trigonometric ratios. ☐ GEO.35.5: Apply properties of similarity to demonstrate the trigonometric ratios of right triangles. ☐ GEO.35.6: Use Pythagorean Theorem to find the missing side of a right triangle.	☐ Demonstrate how to find square roots. ☐ Identify right triangles. ☐ Solve problems using the Pythagorean Theorem. ☐ Recognize ordered pairs (x, y). ☐ Recall how to name points on a Cartesian plane using ordered pairs. ☐ Identify right triangles. ☐ Solve problems using the Pythagorean Theorem. ☐ Discuss strategies for solving real-world and mathematical problems. ☐ Recognize examples of right triangles. ☐ Define a right angle, Pythagorean Theorem, converse, and proof. ☐ Apply properties to find missing angle measures. ☐ Identify a transversal.

	☐ GEO.35.7: Create an equation using the given information of a right triangle. ☐ GEO.35.8: Identify the parts of a right triangle. Examples: legs, hypotenuse, right angle.	☐ Identify exterior, interior, vertical, adjacent, alternate interior, alternate exterior, and corresponding angles. ☐ Identify attributes of triangles. ☐ Define exterior, interior, vertical, adjacent, alternate interior, alternate exterior, corresponding, and transversal angles. ☐ Identify right and straight angles. ☐ Identify all types of angles. ☐ Identify proportional relationships. ☐ Locate/use scale on a map. ☐ Define scale, scale drawings, length, area, and geometric figures. ☐ Recall how to find unit rates using ratios. ☐ Define unit rate, proportions, area, length, and ratio. ☐ Analyze the area of other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes. ☐ Define area, special quadrilaterals, right triangles, and polygons. ☐ Recognize and demonstrate that two right triangles make a rectangle. ☐ Select manipulatives to demonstrate how to compose and decompose triangles and other shapes. ☐ Explain how to find the area for rectangles. ☐ Demonstrate how the area of a rectangle is equal to the sum of the area of two equal right triangles. ☐ Apply area formulas to solve real-world mathematical problems. ☐ Recognize polygons. ☐ Restate real-world problems or mathematical problems. ☐ Calculate unit rate or rate by using ratios or proportions. ☐ Create a ratio or proportion from a given word problem, diagram, table, or equation. ☐ Define ratio, rate, proportion, percent, equivalent, input, output, ordered pairs, diagram, unit rate, and table. ☐ Form a ratio.
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*Critical Standard

		☐ Solve a proportion using part over whole equals percent over 100. ☐ Identify a proportion from given information. ☐ Calculate a proportion for missing information. ☐ Create a proportion or ratio from a given word problem.
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Geometry and Measurement

Focus 4: Solving Applied Problems and Modeling in Geometry

Cluster	2019 Math COS Standard	
Recognizing congruence, similarity, symmetry, measurement opportunities, and other geometric ideas, including right triangle trigonometry, in real-world contexts provides a means of building understanding of these concepts and is a powerful tool for solving problems related to the physical world in which we live.	36. Use geometric shapes, their measures, and their properties to model objects and use those models to solve problems.	
	Learning Objectives	Prior Knowledge Skills
	☐ GEO.36.1: Estimate the dimensions of a given object. ☐ GEO.36.2: Discuss the properties of a given object. ☐ GEO.36.3: Identify the relationship of geometric representations to real-life objects.	☐ Recognize attributes of geometric shapes.

Geometry and Measurement

Focus 4: Solving Applied Problems and Modeling in Geometry

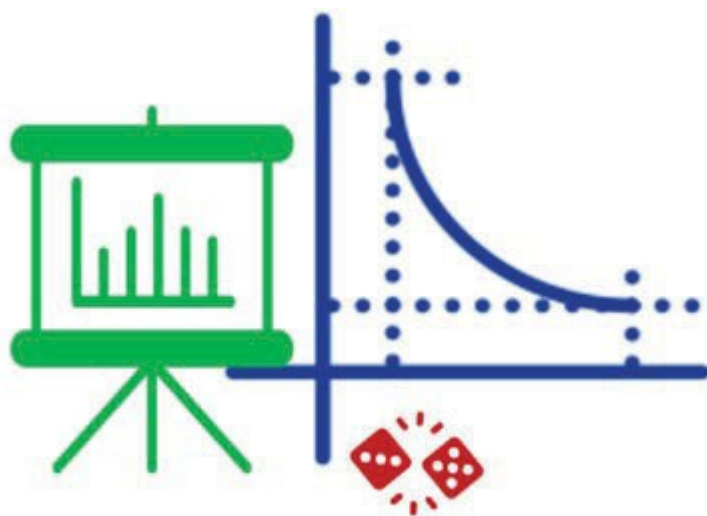
Cluster	2019 Math COS Standard	
Recognizing congruence, similarity, symmetry, measurement opportunities, and other geometric ideas, including right triangle trigonometry, in real-world contexts provides a means of building understanding of these concepts and is a powerful tool for solving problems related to the physical world in which we live.	37. Investigate and apply relationships among inscribed angles, radii, and chords, including but not limited to: the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.	
	Learning Objectives	Prior Knowledge Skills
	☐ GEO.37.1: Define inscribed angles, central angles, circumscribed angles, radius, chord, tangent, secant, and diameter. ☐ GEO.37.2: Define inscribed and circumscribed circle of a triangle. ☐ GEO.37.3: Apply knowledge of arcs, angles, and chords to solve circle related problems. ☐ GEO.37.4: Determine angle values for all angles formed in the exterior, interior and on the circle. ☐ GEO.37.5: Determine lengths of intersecting chords and secants. ☐ GEO.37.6: Discuss the relationship among inscribed angles, radii, and chords. ☐ GEO.37.7: Illustrate inscribed and circumscribed circles of a triangle and quadrilaterals inscribed in a circle. ☐ GEO.37.8: Illustrate radii, chords, diameters, tangents to curve, central, inscribed, and circumscribed angles.	☐ Identify parts of a circle. ☐ Recall how to find circumference of a circle. ☐ Recall the meaning of a radius and diameter. ☐ Identify all types of angles. ☐ Recognize the attributes of a circle. ☐ Identify and label parts of a circle. ☐ Define diameter, radius, circumference, area of a circle, and formula.

Geometry and Measurement

Focus 4: Solving Applied Problems and Modeling in Geometry

Cluster	2019 Math COS Standard	
Experiencing the mathematical modeling cycle in problems involving geometric concepts, from the simplification of the real problem through the solving of the simplified problem, the interpretation of its solution, and the checking of the solution's feasibility, introduces geometric techniques, tools, and points of view that are valuable to problem-solving.	38. Use the mathematical modeling cycle involving geometric methods to solve design problems. <i>Examples: Design an object or structure to satisfy physical constraints or minimize cost; work with typographic grid systems based on ratios; apply concepts of density based on area and volume.</i>	
	Learning Objectives	Prior Knowledge Skills
	☐ GEO.38.1: Define density, area, and volume. ☐ GEO.38.2: Illustrate a design conflict (e.g., draw a chair and a desk where the chair will not fit under the desk). ☐ GEO.38.3: Discuss the relationship between units in each modeling situation. ☐ GEO.38.4: Calculate density (D), mass (m) or volume (V) using the formula, $D = m/V$. ☐ GEO.38.5: Recognize appropriate units for various situations.	☐ Define volume. ☐ Derive the formulas for the volume of a cone, cylinder, and sphere. ☐ Calculate the volume of three-dimensional figures. ☐ Solve real-world problems using the volume formulas for three-dimensional figures.

Algebra 1 With Probability



Algebra 1 with Probability

Number and Quantity

Cluster	2019 Math COS Standard	
Together, irrational numbers and rational numbers complete the real number system, representing all points on the number line, while there exist numbers beyond the real numbers called complex numbers.	1. Explain how the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for an additional notation for radicals using rational exponents.	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGI. 1.1: Define exponent, integer, rational number, and radicals. ☐ ALGI. 1.2: Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values. ☐ ALGI. 1.3: Use notation for radicals in terms of rational exponents. ☐ ALGI. 1.4: Apply the properties of integer exponents to generate equivalent numerical expressions. ☐ ALGI. 1.5: Know the properties of integer exponents. ☐ ALGI. 1.6: Write numerical expressions involving whole-number exponents. ☐ ALGI. 1.7: Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used.	☐ Demonstrate difference of scientific notation symbol between paper and calculator. ☐ Discuss the real-world application of scientific notation (very large or very small quantities). ☐ Recall properties of exponents. ☐ Recall how to write numbers in scientific notation. ☐ Demonstrate that when dividing powers of like bases; subtract the exponents (Property of quotient of powers). ☐ Restate exponential numbers as repeated multiplication. ☐ Define exponent, integer, rational number, and radical.

Number and Quantity

Cluster	2019 Math COS Standard	
Together, irrational numbers and rational numbers complete the real number system, representing all points on the number line, while there exist numbers beyond the real numbers called complex numbers.	2. *Rewrite expressions involving radicals and rational exponents using the properties of exponents.	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGI. 2.1: Rewrite expressions involving radicals using the properties of exponents. ☐ ALGI. 2.2: Rewrite expressions involving rational exponents using the properties of exponents. ☐ ALGI. 2.3: Recognize the properties of exponents.	☐ Compute problems with adding and subtracting integers. ☐ Restate exponential numbers as repeated multiplication. ☐ Compute a numerical expression with positive exponents. ☐ Recognize to subtract exponents when dividing terms with like bases (Property of quotient of powers). ☐ Recognize to add exponents when multiplying terms with like bases (Property of product of powers). ☐ Restate zero exponents as 1 ($x^0 = 1$). ☐ Restate negative exponents as positive exponents in the form $1/x^y$. ☐ Define exponent, power, coefficient, integers, equivalent, and numerical expression. ☐ Identify perfect squares and square roots. ☐ Recall how to compare numbers. ☐ Identify properties of exponents. ☐ Define square root, expressions, and approximations. ☐ Identify and give examples of rational numbers. ☐ Recognize the mathematical operations of rational numbers in any form, including converting between forms. (Ex. $0.25=1/4=25\%$). ☐ Define rational numbers. ☐ Restate exponential numbers as repeated multiplication. ☐ Define exponent.

Number and Quantity

Cluster	2019 Math COS Standard	
Together, irrational numbers and rational numbers complete the real number system, representing all points on the number line, while there exist numbers beyond the real numbers called complex numbers.	3. Define the imaginary number i such that $i^2 = -1$.	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGI. 3.1: Define rational and irrational numbers. ☐ ALGI. 3.2: Identify the product of a nonzero rational number and an irrational number as irrational. ☐ ALGI. 3.3: Identify the sum of a rational number and an irrational number is irrational. ☐ ALGI. 3.4: Discuss why the product of two rational numbers is rational. ☐ ALGI. 3.5: Discuss why the sum of two rational numbers is rational. ☐ ALGI. 3.6: Describe the properties of addition and multiplication. ☐ ALGI. 3.7: Apply properties of fractions to add, subtract, multiply, and divide rational numbers. ☐ ALGI. 3.8: Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.	☐ Combine like terms of a given expression. ☐ Show on a number line numbers that are equal distance from 0 and on opposite sides of 0 have opposite signs. ☐ Discover that the opposite of the opposite of a number is the number itself. ☐ Give examples of positive and negative numbers to represent quantities having opposite directions in real-world contexts.

Algebra and Functions

Focus 1: Algebra

Cluster	2019 Math COS Standard	
Expressions can be rewritten in equivalent forms by using algebraic properties, including properties of addition, multiplication, and exponentiation, to make different characteristics or features visible.	4. Interpret linear, quadratic, and exponential expressions in terms of a context by viewing one or more of their parts as a single entity. <i>Example: Interpret the accrued amount of investment $P(1 + r)^t$, where P is the principal and r is the interest rate, as the product of P and a factor depending on time t.</i>	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGI.4.1: Define linear, quadratic, and exponential functions. ☐ ALGI.4.2: Classify an expression as linear, quadratic or exponential from a table. ☐ ALGI.4.3: Classify an expression as linear, quadratic or exponential from an equation. ☐ ALGI.4.4: Classify an expression as linear, quadratic or exponential from a graph. ☐ ALGI.4.5: Define terms, factors, and coefficients. ☐ ALGI.4.6: Identify factors in linear, exponential, and quadratic expressions. ☐ ALGI.4.7: Identify coefficients in linear, exponential, and quadratic expressions. ☐ ALGI.4.8: Identify terms in linear, exponential, and quadratic expressions. - ALGI.4.9: Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient). - ALGI.4.10: Recognize one or more parts of an exponential expression as a single entity. - ALGI.4.11: Recognize one or more parts of a	☐ Recognize ordered pairs. ☐ Identify ordered pairs. ☐ Recognize linear equations. ☐ Recall how to solve problems using the distributive property. ☐ Define linear functions, nonlinear functions, slope, and y-intercept.

	quadratic expression as a single entity. ALGI.4.12: Recognize one or more parts of a linear expression as a single entity.	
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Algebra and Functions

Focus 1: Algebra

Cluster	2019 Math COS Standard	
Expressions can be rewritten in equivalent forms by using algebraic properties, including properties of addition, multiplication, and exponentiation, to make different characteristics or features visible.	5. *Use the structure of an expression to identify ways to rewrite it.	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGI.5.1: Define equivalent expressions. ☐ ALGI.5.2: Rewrite an exponential expression in an alternative way. ☐ ALGI.5.3: Rewrite a quadratic expression in an alternative way. ☐ ALGI.5.4: Rewrite a linear expression in an alternative form. ☐ ALGI.5.5: Understand that rewriting an expression in different forms in a problem context can shed light on the problem. ☐ ALGI.5.6: Recall properties of exponents.	☐ Give examples of the properties of operations including distributive, commutative, and associative. ☐ Recall how to find the greatest common factor. ☐ Combine like terms of a given expression. ☐ Recognize the property demonstrated in a given expression. ☐ Simplify expressions with parentheses (Ex. $5(4 + x) = 20 + 5x$). ☐ Simplify an expression by dividing by the greatest common factor (Ex. $18x + 6y = 6(3x + y)$). ☐ Define linear expression, rational, coefficient, and rational coefficient.

Algebra and Functions

Focus 1: Algebra

Cluster	2019 Math COS Standard	
Expressions can be rewritten in equivalent forms by using algebraic properties, including properties of addition, multiplication, and exponentiation, to make different characteristics or features visible.	6. *Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. - Factor quadratic expressions with leading coefficients of one and use the factored form to reveal the zeros of the function it defines. - Use the vertex form of a quadratic expression to reveal the maximum or minimum value and the axis of symmetry of the function it defines; complete the square to find the vertex form of quadratics with a leading coefficient of one. - Use the properties of exponents to transform expressions for exponential functions. <i>Example: Identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay.</i>	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGI.6.1: Convert an expression to an alternative format. ☐ ALGI.6.2: Recognize the best format for a specific application. ☐ ALGI.6.3: Match equivalent expressions written in different forms. ☐ ALGI.6.4a: Define factor, quadratic expression and zero product property. ☐ ALGI.6.5a: Factor a quadratic expression. ☐ ALGI.6.6a: Use the zero-product property to reveal the zeros in the function. ☐ ALGI.6.7a: Solve a one-step equation. ☐ ALGI.6.8a: Solve a two-step equation. ☐ ALGI.6.9a: Determine the Greatest Common Factor (GCF).	☐ Identify how many solutions the linear equation may or may not have. ☐ Recall how to solve problems using the distributive property ☐ Explain the distributive property. ☐ Recall solving one-step equations.

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| | <ul style="list-style-type: none">☐ ALGI.6.10b: Define maximum and minimum value.☐ ALGI.6.11b: Explain the steps for completing the square.☐ ALGI.6.12b: Given -ALGI a quadratic expression in which the square has already been completed, determine the maximum or minimum values.☐ ALGI.6.13c: Define roots.☐ ALGI.6.14c: Find the equation using the distributive property.☐ ALGI.6.15c: Locate and identify the roots on a graph using the x-intercepts.☐ ALGI.6.16c: Take given roots and convert into a one-step equation set equal to zero. | |
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Algebra and Functions

Focus 1: Algebra

Cluster	2019 Math COS Standard	
Expressions can be rewritten in equivalent forms by using algebraic properties, including properties of addition, multiplication, and exponentiation, to make different characteristics or features visible.	7. *Add, subtract, and multiply polynomials, showing that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication.	
	Learning Objectives	Prior Knowledge Skills
	☐ ALG1.7.1: Combine like terms of a given expression ☐ ALG1.7.2: Define monomial, term, binomial, trinomial, and polynomial. ☐ ALG1.7.3: Multiply polynomial expressions (quadratic). ☐ ALG1.7.4: Multiply polynomial expressions (linear). ☐ ALG1.7.5: Subtract polynomial expressions. ☐ ALG1.7.6: Add polynomial expressions. ☐ ALG1.7.7: Use order of operations to evaluate and simplify algebraic and numerical expressions. ☐ ALG1.7.8: Identify the terms in a polynomial expression. ☐ ALG1.7.9: Explain the distributive property.	☐ Identify properties of exponents. ☐ Give examples of the properties of operations including distributive, commutative, and associative. ☐ Recall how to find the greatest common factor. ☐ Combine like terms of a given expression. ☐ Recognize the property demonstrated in a given expression. ☐ Simplify expressions with parentheses (Ex. $5(4 + x) = 20 + 5x$). ☐ Simplify an expression by dividing by the greatest common factor (Ex. $18x + 6y = 6(3x + y)$). ☐ Define linear expression, rational, coefficient, and rational coefficient. ☐ Combine terms that are alike of a given expression.

Algebra and Functions

Focus 1: Algebra

Cluster	2019 Math COS Standard	
Finding solutions to an equation, inequality, or system of equations or inequalities requires the checking of candidate solutions, whether generated analytically or graphically, to ensure that solutions are found and that those found are not extraneous.	8. Explain why extraneous solutions to an equation involving absolute values may arise and how to check to be sure that a candidate solution satisfies an equation.	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGI. 8.1: Define integers. ☐ ALGI. 8.2: Demonstrate the location of positive and negative numbers on a vertical and horizontal number line. ☐ ALGI. 8.3: Give examples of positive and negative numbers to represent quantities having opposite directions in real-world contexts. ☐ ALGI. 8.4: Discuss the measure of centering of 0 in relationship to positive and negative numbers. ☐ ALGI. 8.5: Substitute for the variable to find the value of a given expression. ☐ ALGI. 8.6: Explain the meaning of absolute value and determine the absolute value of rational numbers in real-world contexts. ☐ ALGI. 8.7: Compare and order rational numbers and absolute value of rational numbers with and without a number line to solve real-world and mathematical problems.	☐ Recall how to order positive and negative numbers. (Use number line if needed). ☐ Evaluate a statement about order using comparisons of absolute value. ☐ Locate the position of integers and/or rational numbers on a horizontal or vertical number line. ☐ Arrange integers and/or rational numbers on a horizontal or vertical number line. ☐ Recognize the absolute value of a rational number is its' distance from 0 on a vertical and horizontal number line.

Algebra and Functions

Focus 1: Algebra

Cluster	2019 Math COS Standard	
The structure of an equation or inequality (including, but not limited to, one-variable linear and quadratic equations, inequalities, and systems of linear equations in two variables) can be purposefully analyzed (with and without technology) to determine an efficient strategy to find a solution, if one exists, and then to justify the solution.	9. *Select an appropriate method to solve a quadratic equation in one variable. - Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Explain how the quadratic formula is derived from this form. - Solve quadratic equations by inspection (such as $x^2 = 49$), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation, and recognize that some solutions may not be real.	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGI.9.1: Define quadratic equation and zero product property. ☐ ALGI.9.2: Solve one-step equations using addition and subtraction that are set equal to zero. ☐ ALGI.9.3: Solve two-step equations using addition and subtraction that are set equal to zero. ☐ ALGI.9.4a: Define completing the square. ☐ ALGI.9.5a: Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. ☐ ALGI.9.6a: Derive the quadratic formula from the form $(x - p)^2 = q$. ☐ ALGI.9.7b: Define quadratic formula, factoring, square root, complex number, and real number.	☐ Identify perfect squares and square roots. ☐ Define square root, expressions, and approximations. ☐ Explain the distributive property. ☐ Calculate an expression in the correct order (Ex. exponents, mult./div. from left to right and add/sub. from left to right). ☐ Recalling one-step equations. ☐ List given information from the problem. ☐ Identify the unknown, in each situation, as the variable. ☐ Test the found number for accuracy by substitution. Example: Is 5 an accurate solution of $2(x + 5) = 12$? ☐ Calculate a solution to an equation by combining like terms, isolating the variable, and/or using inverse operations. ☐ Define equation and variable. ☐ Set up an equation to represent the given situation, using correct mathematical operations and variables. ☐ Recognize the correct order to solve expressions with more than one operation. ☐ Calculate a numerical expression (Ex. $V = 4 \times 4 \times 4$).

	☐ ALGI.9.8b: Solve quadratic equations by completing the square. ☐ ALGI.9.9b: Solve quadratic equations by the quadratic formula. ☐ ALGI.9.10b: Solve quadratic equations by factoring. ☐ ALGI.9.11b: Solve quadratic equations by taking square roots. ☐ ALGI.9.12b: Recognize when the quadratic formula gives complex solutions. ☐ ALGI.9.13b: Write complex solutions as $a \pm bi$ for real numbers a and b.	☐ Choose the correct value to replace each variable in the algebraic expression (Substitution).
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Algebra and Functions

Focus 1: Algebra

Cluster	2019 Math COS Standard	
The structure of an equation or inequality (including, but not limited to, one-variable linear and quadratic equations, inequalities, and systems of linear equations in two variables) can be purposefully analyzed (with and without technology) to determine an efficient strategy to find a solution, if one exists, and then to justify the solution.	10. Select an appropriate method to solve a system of two linear equations in two variables. a. Solve a system of two equations in two variables by using linear combinations; contrast situations in which use of linear combinations is more efficient with those in which substitution is more efficient. b. Contrast solutions to a system of two linear equations in two variables produced by algebraic methods with graphical and tabular methods.	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGI.10.1: Solve a system of equations using three methods (Substitution, Elimination, and Graphing). ☐ ALGI.10.2: Distinguish the similarities and differences between the three methods of solving systems of equations.	☐ Solve a system of equation by graphing. ☐ Solve a system of equation by elimination. ☐ Solve a system of equation by substitution. ☐ Understand the meaning of the solution to a system of equations. ☐ Graph a linear equation.

Algebra and Functions

Focus 1: Algebra

Cluster	2019 Math COS Standard	
Expressions, equations, and inequalities can be used to analyze and make predictions, both within mathematics and as mathematics is applied in different contexts – in particular, contexts that arise in relation to linear, quadratic, and exponential situations.	11. Create equations and inequalities in one variable and use them to solve problems in context, either exactly or approximately. Extend from contexts arising from linear functions to those involving quadratic, exponential, and absolute value functions.	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGI.11.1: Solve the equation represented by the real-world situation. ☐ ALGI.11.2: Set up an equation to represent the given situation, using correct mathematical operations and variables. ☐ ALGI.11.3: Given a contextual situation, interpret and defend the solution in the context of the original problem. ☐ ALGI.11.4: Define equation, expression, variable, equality and inequality. ☐ ALGI.11.5: Create inequalities with one variable (Exponential, Quadratic, Linear). ☐ ALGI.11.6: Create equalities with one variable (Exponential, Quadratic, Linear). ☐ ALGI.11.7: Solve two-step equations and inequalities. ☐ ALGI.11.8: Solve one-step equations and inequalities using the four basic operations. ☐ ALGI.11.9: Compare and contrast equations and inequalities. ☐ ALGI.11.10: Recognize inequality symbols including greater than, less than, greater than equal to and less than equal to.	☐ Test the found number or number set for accuracy by substitution. ☐ Set up equations and inequalities to represent the given situation, using correct mathematical operations and variables. ☐ Define equation, inequality, and variable. ☐ Convert mathematical terms to mathematical symbols and numbers.

Algebra and Functions		
Focus 1: Algebra		
Cluster	2019 Math COS Standard	
Expressions, equations, and inequalities can be used to analyze and make predictions, both within mathematics and as mathematics is applied in different contexts – in particular, contexts that arise in relation to linear, quadratic, and exponential situations.	12. *Create equations in two or more variables to represent relationships between quantities in context; graph equations on coordinate axes with labels and scales and use them to make predictions. Limit to contexts arising from linear, quadratic, exponential, absolute value, and linear piecewise functions.	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGI.12.1: Solve the equations represented by real-world situations. ☐ ALGI.12.2: Set up an equation to represent the given situation, using correct mathematical operations and variables. ☐ ALGI.12.3: Given a contextual situation, interpret and defend the solution in the context of the original problem. ☐ ALGI.12.4: Explain how to draw informal inferences from data distributions. ☐ ALGI.12.5: Define ordered pair and coordinate plane. ☐ ALGI.12.6: Create equations with two variables (exponential, quadratic and linear). ☐ ALGI.12.7: Graph equations on coordinate axes with labels and scales (exponential, quadratic, and linear). ☐ ALGI.12.8: Identify an ordered pair and plot it on the coordinate plane.	☐ Demonstrate how to plot points on a coordinate plane using ordered pairs from a table. ☐ Plot independent (input) and dependent (output) values on a coordinate plane. ☐ Draw and label a coordinate plane. ☐ Define dependent variable, independent variable, ordered pairs, input, output, and coordinate plane.

Algebra and Functions

Focus 1: Algebra

Cluster	2019 Math COS Standard	
Expressions, equations, and inequalities can be used to analyze and make predictions, both within mathematics and as mathematics is applied in different contexts – in particular, contexts that arise in relation to linear, quadratic, and exponential situations.	13. *Represent constraints by equations and/or inequalities, and solve systems of equations and/or inequalities, interpreting solutions as viable or nonviable options in a modeling context. Limit to contexts arising from linear, quadratic, exponential, absolute value, and linear piecewise functions.	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGI.13.1: Define systems of equations, constraints, viable solution, and nonviable solution. ☐ ALGI.13.2: Create a system of equations or inequalities to represent the given constraints (linear). ☐ ALGI.13.3: Create an equation or inequality to represent the given constraints (linear). ☐ ALGI.13.4: Determine if a solution to a system of equations or inequalities is viable or nonviable. ☐ ALGI.13.5: Determine if there is one solution, infinite solutions, or no solutions to a system of equations or inequalities.	☐ Recall how to draw a number line. ☐ Recognize the symbols for =, >, <, ≤ and ≥. ☐ Substitute for the variable to find the value of a given expression. ☐ Choose the correct value to replace each variable in the algebraic expression (Substitution). ☐ Convert mathematical terms to mathematical symbols and numbers. ☐ Recall how to order positive and negative numbers. (Use number line if needed). ☐ Locate the position of integers and/or rational numbers on a horizontal or vertical number line.

Algebra and Functions

Focus 2: Connecting Algebra to Functions

Cluster	2019 Math COS Standard	
Functions shift the emphasis from a point by-point relationship between two variables (input/output) to considering an entire set of ordered pairs (where each first element is paired with exactly one second element) as an entity with its own features and characteristics.	14. Given a relation defined by an equation in two variables, identify the graph of the relation as the set of all its solutions plotted in the coordinate plane. <i>Note: The graph of a relation often forms a curve (which could be a line).</i>	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGI.14.1: Understand that the graph of an equation is the solution of an equation. ☐ ALGI.14.2: Graph a linear equation and use the graph to determine the solution set. ☐ ALGI.14.3: Use a given graph to determine the solution set. ☐ ALGI.14.4: Plot given points from a table.	☐ Identify the slope-intercept form ($y=mx+b$) of an equation where m is the slope and y is the y-intercept. ☐ Graph a function given the slope-intercept form of an equation. ☐ Demonstrate how to plot points on a coordinate plane using ordered pairs from table. ☐ Recall how to plot ordered pairs on a coordinate plane. ☐ Name the pairs of integers and/or rational numbers of a point on a coordinate plane.

Algebra and Functions

Focus 2: Connecting Algebra to Functions

Cluster	2019 Math COS Standard	
Functions shift the emphasis from a point by-point relationship between two variables (input/output) to considering an entire set of ordered pairs (where each first element is paired with exactly one second element) as an entity with its own features and characteristics.	15. *Define a function as a mapping from one set (called the domain) to another set (called the range) that assigns to each element of the domain exactly one element of the range. a. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. <i>Note: If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x.</i> b. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. Limit to linear, quadratic, exponential, and absolute value functions.	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGI.15.1: Define domain, range, relation, function, table of values, input, and output. ☐ ALGI.15.2: Understand the graph of a function is the set of ordered pairs consisting of an input and the corresponding output. ☐ ALGI.15.3: Understand that a function is a rule that assigns to each input exactly one output. ☐ ALGI.15.4: Identify the equation of a function, given its graph. ☐ ALGI.15.5: Find the range of a function given its domain. ☐ ALGI.15.6: Recognize that $f(x)$ and y are the same. ☐ ALGI.15.7: Recall how to complete input/output tables. ☐ ALGI.15.8: Recall how to read/interpret information from a table.	☐ Analyze the graph to determine the rate of change. ☐ Generate the slope of a line using given ordered pairs. ☐ Define linear functions, nonlinear functions, slope, and y-intercept. ☐ Identify ordered pairs. ☐ Plot points on a coordinate plane., then connect points for the vertices to sketch a polygon.

	☐ ALGI.15.9: Define function notation. ☐ ALGI.15.10: Translate a simple word problem into function notation. ☐ ALGI.15.11: Evaluate function when given x-value.	
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Algebra and Functions

Focus 2: Connecting Algebra to Functions

Cluster	2019 Math COS Standard	
Functions shift the emphasis from a point by-point relationship between two variables (input/output) to considering an entire set of ordered pairs (where each first element is paired with exactly one second element) as an entity with its own features and characteristics.	16. Compare and contrast relations and functions represented by equations, graphs, or tables that show related values; determine whether a relation is a function. Explain that a function f is a special kind of relation defined by the equation $y = f(x)$.	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGI.16.1: Define functions, relations (ordered pairs), input, output. ☐ ALGI.16.2: Recall how to complete input/output tables. ☐ ALGI.16.3: Recall how to read/interpret information from a table. ☐ ALGI.16.4: Identify algebraic expressions. ☐ ALGI.16.5 Recall how to name points from a graph (ordered pairs). ☐ ALGI.16.6: Recall how to name points on a Cartesian plane using ordered pairs.	☐ Recall how to read a graph or table. ☐ Define dependent variable, independent variable, ordered pairs, input, output, and coordinate plane. ☐ Recall how to plot ordered pairs on a coordinate plane. ☐ Name the pairs of integers and/or rational numbers of a point on a coordinate plane.

Algebra and Functions

Focus 2: Connecting Algebra to Functions

Cluster	2019 Math COS Standard	
Functions shift the emphasis from a point by-point relationship between two variables (input/output) to considering an entire set of ordered pairs (where each first element is paired with exactly one second element) as an entity with its own features and characteristics.	17. Combine different types of standard functions to write, evaluate, and interpret functions in context. Limit to linear, quadratic, exponential, and absolute value functions. a. Use arithmetic operations to combine different types of standard functions to write and evaluate functions. <i>Example: Given two functions, one representing flow rate of water and the other representing evaporation of that water, combine the two functions to determine the amount of water in a container at a given time.</i> b. Use function composition to combine different types of standard functions to write and evaluate functions. <i>Example: Given the following relationships, determine what the expression $S(T(t))$ represents.</i>	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGI.17.1: Define functions, relations (ordered pairs), input, output. ☐ ALGI.17.2: Recall how to complete input/output tables. ☐ ALGI.17.3: Recall how to read/interpret information from a table. ☐ ALGI.17.4: Identify algebraic expressions. ☐ ALGI.17.5: Recall how to name points from a graph (ordered pairs). ☐ ALGI.17.6: Recall how to name points on a Cartesian plane using ordered pairs. ☐ ALGI.17.7a: Identify, represent, and analyze two quantities that change in relationship to one another in real-world or mathematical situations.	☐ Explain the distributive property. ☐ Give examples of the properties of operations including distributive. ☐ Combine like terms of a given expression. ☐ Recognize the correct order to solve expressions with more than one operation. ☐ Calculate a numerical expression (Ex. $V=(4 \times 4 \times 4)$). ☐ Choose the correct value to replace each variable in the algebraic expression (Substitution). ☐ Calculate an expression in the correct order (Ex. exponents, mult./div. from left to right, and add/sub. from left to right).

	☐ ALGI.17.8a: Set up an equation to represent the given situation, using correct mathematical operations and variables. ☐ ALGI.17.9b: Add, subtract, and multiply polynomials, showing that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication.	
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Algebra and Functions

Focus 2: Connecting Algebra to Functions

Cluster	2019 Math COS Standard	
Graphs can be used to obtain exact or approximate solutions of equations, inequalities, and systems of equations and inequalities – including systems of linear equations in two variables and systems of linear and quadratic equations (given or obtained by using technology).	18. Solve systems consisting of linear and/or quadratic equations in two variables graphically, using technology where appropriate.	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGI.18.1: Use the substitution method to replace a variable in the quadratic equation. ☐ ALGI.18.2: Solve for the variables in a system of equations. (Algebraically). ☐ ALGI.18.3: Graph a quadratic equation. ☐ ALGI.18.4: Graph a linear equation. ☐ ALGI.18.5: Identify the point(s) of intersection when given graphs. ☐ ALGI.18.6: Use digital tools to defend solutions to authentic problems. ☐ ALGI.18.7: Use digital tools to formulate solutions to authentic problems (Ex: electronic graphing tools, probes, spreadsheets).	☐ Given a function, create a rule. ☐ Recognize numeric patterns. ☐ Recall how to complete input/output tables. ☐ Demonstrate how to plot points on a Cartesian plane using ordered pairs. ☐ Define function, ordered pairs, input, and output. ☐ Recall that linear equations can have one solution (intersecting), no solution (parallel lines), or infinitely many solutions (graph is simultaneous). ☐ Graph a function given the slope-intercept form of an equation. ☐ Identify the slope-intercept form ($y=mx+b$) of an equation where m is the slope and y is the y-intercept. ☐ Demonstrate how to plot points on a coordinate plane using ordered pairs from table. ☐ Analyze the graph to determine the rate of change. ☐ Generate the slope of a line using given ordered pairs. ☐ Show how to plot points on a Cartesian plane. ☐ Define ordered pairs. ☐ Show how to graph on Cartesian plane. ☐ Substitute for the variable to find the value of a given expression. ☐ Recall how to plot ordered pairs on a coordinate plane. ☐ Identify which signs indicate the location of a point in a coordinate plane.

		☐ Demonstrate when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. ☐ Name the pairs of integers and/or rational numbers of a point on a coordinate plane. ☐ Define ordered pairs. ☐ Show on a number line that numbers that are equal distance from 0 and on opposite sides of 0 have opposite signs. ☐ Discover that the opposite of the opposite of a number is the number itself. ☐ Give examples of positive and negative numbers to represent quantities having opposite directions in real-world contexts. ☐ Identify the parts of a table of equivalent ratios (input, output, etc.).
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Algebra and Functions

Focus 2: Connecting Algebra to Functions

Cluster	2019 Math COS Standard	
Graphs can be used to obtain exact or approximate solutions of equations, inequalities, and systems of equations and inequalities – including systems of linear equations in two variables and systems of linear and quadratic equations (given or obtained by using technology).	19. Explain why the x-coordinates of the points where the graphs of the equations $y=f(x)$ and $y=g(x)$ intersect are the solutions of the equation $f(x) = g(x)$. a. Find the approximate solutions of an equation graphically, using tables of values, or finding successive approximations, using technology where appropriate. <i>Note: Include cases where $f(x)$ is a linear, quadratic, exponential, or absolute value function and $g(x)$ is constant or linear.</i>	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGI.19.1: Define function, function notation, linear, polynomial, rational, absolute value, exponential, and logarithmic functions, and transitive property. ☐ ALGI.19.2: Explain, using the transitive property, why the x-coordinates of the points of the graphs are solutions to the equations. ☐ ALGI.19.3: Find solutions to the equations $y = f(x)$ and $y = g(x)$ using the graphing calculator. ☐ ALGI.19.4: Solve equations for y. ☐ ALGI.19.5: Demonstrate use of a graphing calculator, including using a table, making a graph, and finding successive approximations.	☐ Test the formula $V = lwh$ and $V = Bh$ with the experimental findings. ☐ Apply area formulas to solve real-world mathematical problems. ☐ Define algebraic expression and variable.

Algebra and Functions

Focus 2: Connecting Algebra to Functions

Cluster	2019 Math COS Standard	
Graphs can be used to obtain exact or approximate solutions of equations, inequalities, and systems of equations and inequalities – including systems of linear equations in two variables and systems of linear and quadratic equations (given or obtained by using technology).	20. *Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes, using technology where appropriate.	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGI.20.1: Define the half-plane as the shaded region. ☐ ALGI.20.2: Determine the intersecting shaded region is the solution to the system. ☐ ALGI.20.3: Graph the lines of the systems and shade the appropriate region. ☐ ALGI.20.4: Determine the shaded region is the solution to the inequality. ☐ ALGI.20.5: Graph an inequality and shade the appropriate region. ☐ ALGI.20.6: Determine whether a line should be solid or dotted, depending on the inequality symbol. ☐ ALGI.20.7: Recognize inequality symbols $>$, $<$.	☐ Define function, ordered pairs, input, output. ☐ Identify the slope-intercept form ($y=mx+b$) of an equation where m is the slope and y is the y-intercept. ☐ Demonstrate how to plot points on a coordinate plane using ordered pairs from table. ☐ Generate the slope of a line using given ordered pairs. ☐ Recall how to graph inequalities on a number line. ☐ Show how to graph on Cartesian plane. ☐ Show how to plot points on a Cartesian plane. ☐ Define ordered pairs. ☐ Graph the solution set on a number line for the inequality used to represent the situation. ☐ Recall how to plot ordered pairs on a coordinate plane. ☐ Identify which signs indicate the location of a point in a coordinate plane. ☐ Demonstrate when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. ☐ Name the pairs of integers and/or rational numbers of a point on a coordinate plane. ☐ Define ordered pairs.

Algebra and Functions

Focus 3: Functions

Cluster	2019 Math COS Standard	
Functions can be described by using a variety of representations: mapping diagrams, function notation (e.g., $f(x) = x^2$), recursive definitions, tables, and graphs.	21. *Compare properties of two functions, each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). Extend from linear to quadratic, exponential, absolute value, and general piecewise.	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGI.21.1: Define function, function notation, (linear, polynomial, rational, absolute value, exponential, piecewise, and logarithmic) functions, and transitive property. ☐ ALGI.21.2: Explain, using the transitive property, why the x-coordinates of the points of the graphs are solutions to the equations. ☐ ALGI.21.3: Find solutions to the equations $y = f(x)$ and $y = g(x)$ using the graphing calculator. ☐ ALGI.21.4: Solve equations for y. ☐ ALGI.21.5: Demonstrate use of a graphing calculator, including using a table, making a graph, and finding successive approximations.	☐ Identify the slope-intercept form ($y=mx+b$) of an equation where m is the slope and y is the y-intercept. ☐ Graph a function given the slope-intercept form of an equation. ☐ Demonstrate how to plot points on a coordinate plane using ordered pairs from a table. ☐ Calculate a solution or solution set by combining like terms, isolating the variable, and/or using inverse operations. ☐ Recall how to plot ordered pairs on a coordinate plane. ☐ Name the pairs of integers and/or rational numbers of a point on a coordinate plane.

Algebra and Functions

Focus 3: Functions

Cluster	2019 Math COS Standard	
Functions can be described by using a variety of representations: mapping diagrams, function notation (e.g., $f(x) = x^2$), recursive definitions, tables, and graphs.	22. Define sequences as functions, including recursive definitions, whose domain is a subset of the integers. a. Write explicit and recursive formulas for arithmetic and geometric sequences and connect them to linear and exponential functions. <i>Example: A sequence with constant growth will be a linear function, while a sequence with proportional growth will be an exponential function.</i>	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGI.22.1: Define proportions and proportional relationships. ☐ ALGI.22.2: Write equations to represent a proportional relationship. ☐ ALGI.22.3: Discuss the use of variables in proportional relationships. ☐ ALGI.22.4: Define sequences and recursively defined sequences. ☐ ALGI.22.5: Recognize that sequences are functions whose domain is the set of all positive integers and zero.	☐ Recall that a proportion is the comparison of two ratios. ☐ Identify the appropriate equation from a proportion. ☐ Solve an equation to find an unknown quantity. ☐ Identify patterns in number sequences.

Algebra and Functions

Focus 3: Functions

Cluster	2019 Math COS Standard	
Functions that are members of the same family have distinguishing attributes (structure) common to all functions within that family.	23. *Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k \cdot f(x)$, $f(k \cdot x)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and explain the effects on the graph, using technology as appropriate. Limit to linear, quadratic, exponential, absolute value, and linear piecewise functions.	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGI.23.1: Define dilation, rotation, reflection, translation, congruent and sequence. ☐ ALGI.23.2: Identify rotations. ☐ ALGI.23.3: Identify reflections. ☐ ALGI.23.4: Identify translations. ☐ ALGI.23.5: Use digital tools to formulate solutions to authentic problems (Ex: electronic graphing tools, probes, spreadsheets).	☐ Identify congruent figures. ☐ Compare rotations to translations. ☐ Compare reflections to rotations. ☐ Compare translations to reflections. ☐ Recognize translations (slides), rotations (turns), and reflections (flips).

Algebra and Functions

Focus 3: Functions

Cluster	2019 Math COS Standard	
Functions that are members of the same family have distinguishing attributes (structure) common to all functions within that family.	24. Distinguish between situations that can be modeled with linear functions and those that can be modeled with exponential functions. - Show that linear functions grow by equal differences over equal intervals, while exponential functions grow by equal factors over equal intervals. - Define linear functions to represent situations in which one quantity changes at a constant rate per unit interval relative to another. - Define exponential functions to represent situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGI.24.1: Define linear function and exponential function. ☐ ALGI.24.2: Distinguish between graphs of a line and an exponential function. ☐ ALGI.24.3: Identify the graph of an exponential function. ☐ ALGI.24.4: Identify the graph of a line. ☐ ALGI.24.5: Plot points on a coordinate plane from a given table of values. ☐ ALGI.24.6a: Divide each y-value in a table of values by its successive y-value to determine if the quotients are the same, to prove an exponential function. ☐ ALGI.24.7a: Subtract each y-value in a table of values by its successive y-value to determine if the differences are the same, to prove a linear function. ☐ ALGI.24.8a: Apply rules for adding, subtracting, multiplying, and dividing integers.	☐ Recognize ordered pairs. ☐ Identify ordered pairs. ☐ Recognize linear equations. ☐ Recall how to solve problems using the distributive property. ☐ Define linear and nonlinear functions, slope, and y-intercept. ☐ Analyze the graph to determine the rate of change.

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| | <ul style="list-style-type: none">☐ ALGI.24.9b: Define constant rate of change as slope.☐ ALGI.24.10b: Subtract each y-value in a table of values by its successive y-value to determine if the differences are the same, to prove a linear function.☐ ALGI.24.11b: Recognize the calculated difference is the constant rate of change.☐ ALGI.24.12b: Apply rules for adding, subtracting, multiplying, and dividing integers.☐ ALGI.24.13c: Define exponential growth and decay.☐ ALGI.24.14c: Divide each y-value in a table of values by its successive y-value to determine if the quotients are the same, to prove an exponential function.☐ ALGI.24.15c: Apply the rules of multiplication and division of integers. | |
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Algebra and Functions

Focus 3: Functions

Cluster	2019 Math COS Standard	
Functions that are members of the same family have distinguishing attributes (structure) common to all functions within that family.	25. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGI.25.1: Define linear function and exponential function. ☐ ALGI.25.2: Define arithmetic sequence, geometric sequence, and input-output pairs. ☐ ALGI.25.3: Define sequences and recursively defined sequences. ☐ ALGI.25.4: Recognize that sequences are functions whose domain is the set of all positive integers and zero. ☐ ALGI.25.5: Given a chart, write an equation of a line. ☐ ALGI.25.6: Given a graph, write an equation of a line. ☐ ALGI.25.7: Given two ordered pairs, write an equation of a line.	☐ Given a function, create a rule. ☐ Recognize numeric patterns. ☐ Recall how to complete input/output tables. ☐ Demonstrate how to plot points on a Cartesian plane using ordered pairs. ☐ Define function, ordered pairs, input, output. ☐ Graph a linear equation given the slope-intercept form of an equation. ☐ Graph a function given the slope-intercept form of an equation. ☐ Identify the slope-intercept form ($y=mx+b$) of an equation where m is the slope and y is the y-intercept. ☐ Generate the slope of a line using given ordered pairs. ☐ Recall the rules for multiplying integers. ☐ Define quotient, divisor, and integer. ☐ Solve addition and subtraction of multi-digit whole numbers. ☐ Solve addition and subtraction of multi-digit decimal numbers (emphasis on alignment). ☐ Recall basic multiplication and division facts. ☐ Solve multiplication problems involving multi-digit whole numbers and decimal numbers. ☐ Solve division problems involving multi-digit whole numbers and decimal numbers.

Algebra and Functions

Focus 3: Functions

Cluster	2019 Math COS Standard	
Functions that are members of the same family have distinguishing attributes (structure) common to all functions within that family.	26. Use graphs and tables to show that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically.	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGI.26.1: Define a polynomial function and parabola. ☐ ALGI.26.4: Compare the y-values by looking at the same x-value in a variety of tables or graphs. ☐ ALGI.26.3: Identify the graph of an exponential function. ☐ ALGI.26.4: Identify the graph of a line. ☐ ALGI.26.5: Plot points on a coordinate plane from a given table of values. ☐ ALGI.26.6: Identify the graph of a quadratic function.	☐ Create a graph to model a real-word situation. ☐ Compare and contrast the relationship between two quantities in a graph. ☐ Compare and contrast the differences between linear and nonlinear functions.

Algebra and Functions

Focus 3: Functions

Cluster	2019 Math COS Standard	
Functions that are members of the same family have distinguishing attributes (structure) common to all functions within that family.	27. Interpret the parameters of functions in terms of a context. Extend from linear functions, written in the form $mx + b$, to exponential functions, written in the form ab^x. <i>Example: If the function $V(t) = 19885(0.75)^t$ describes the value of a car after it has been owned for t years, 19885 represents the purchase price of the car when $t = 0$, and 0.75 represents the annual rate at which its value decreases.</i>	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGI.27.1: Recall the formula of an exponential function. ☐ ALGI.27.2: Recall the slope-intercept form of a linear function. ☐ ALGI.27.3: Define b as growth or decay factor in the context of an exponential problem. ☐ ALGI.27.4: Define k as the initial amount in the context of an exponential problem. ☐ ALGI.27.5: Define m as the rate of change in the context of a linear problem. ☐ ALGI.27.6: Define b as the initial amount in the context of a linear problem.	☐ Solve problems with exponents. ☐ Discuss strategies for solving real-world and mathematical problems. ☐ Recognize ordered pairs. ☐ Identify parts of the Cartesian plane. ☐ Recall how to plot points on a Cartesian plane. ☐ Distinguish the difference between linear and nonlinear functions. ☐ Define qualitative, increase, and decrease. ☐ Recall how to name points from a graph (ordered pairs). ☐ Recall how to find the rate of change (slope) in a linear equation. ☐ Recall how to complete an input/output function table. ☐ Analyze real-world situations to identify the rate of change and initial value from a table, graph, or description. ☐ Define function, rate of change, and initial value.

Algebra and Functions

Focus 3: Functions

Cluster	2019 Math COS Standard	
Functions can be represented graphically and key features of the graphs, including zeros, intercepts, and, when relevant, rate of change and maximum/minimum values, can be associated with and interpreted in terms of the equivalent symbolic representation.	28. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. <i>Note: Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; maximums and minimums; symmetries; and end behavior.</i> Extend from relationships that can be represented by linear functions to quadratic, exponential, absolute value, and linear piecewise functions.	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGI.28.1: Define intercepts, intervals, relative maxima, relative minima, symmetry, end behavior, and periodicity. ☐ ALGI.28.2: For a function that models a relationship between two quantities, find the periodicity. ☐ ALGI.28.3: For a function that models a relationship between two quantities, find the end behavior. ☐ ALGI.28.4: For a function that models a relationship between two quantities, find the symmetry. ☐ ALGI.28.5: For a function that models a relationship between two quantities, find the intervals where the function is increasing, decreasing, positive, or negative. ☐ ALGI.28.6: For a function that models a relationship between two quantities, find the relative maxima and minima.	☐ Identify parts of the Cartesian plane. ☐ Graph a function given the slope-intercept form of an equation. ☐ Demonstrate how to plot points on a coordinate plane using ordered pairs from table. ☐ Recall how to plot ordered pairs on a coordinate plane. ☐ Name the pairs of integers and/or rational numbers of a point on a coordinate plane.

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| | <ul style="list-style-type: none">□ ALGI.28.7: For a function that models a relationship between two quantities, find the x and y intercepts. | |
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Algebra and Functions

Focus 3: Functions

Cluster	2019 Math COS Standard	
Functions can be represented graphically and key features of the graphs, including zeros, intercepts, and, when relevant, rate of change and maximum/minimum values, can be associated with and interpreted in terms of the equivalent symbolic representation.	29. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. Limit to linear, quadratic, exponential, and absolute value functions.	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGI.29.1: Identify equivalent ratios. ☐ ALGI.29.2: Define average rate of change as slope. ☐ ALGI.29.3: Estimate the rate of change from a graph (rise/run). ☐ ALGI.29.4: Interpret the average rate of change. ☐ ALGI.29.5: Calculate the average rate of change. ☐ ALGI.29.6: Compute the slope of a line given two ordered pairs. ☐ ALGI.29.7: Identify the slope, given slope-intercept form.	☐ Apply the identification of the slope and the y-intercept to a real-world situation. ☐ Recall how to write an equation in slope-intercept form. ☐ Recall how to solve equations by using substitution. ☐ Identify how many solutions the linear equation may or may not have. ☐ Calculate an expression in the correct order (Ex. exponents, mult./div. from left to right, and add/sub. from left to right). ☐ Define ratio, rate, proportion, percent, equivalent, input, output, ordered pairs, diagram, unit rate, and table.

Algebra and Functions

Focus 3: Functions

Cluster	2019 Math COS Standard	
Functions can be represented graphically and key features of the graphs, including zeros, intercepts, and, when relevant, rate of change and maximum/minimum values, can be associated with and interpreted in terms of the equivalent symbolic representation.	30. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. - Graph linear and quadratic functions and show intercepts, maxima, and minima. - Graph piecewise-defined functions, including step functions and absolute value functions. - Graph exponential functions, showing intercepts and end behavior.	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGI.30.1: Define piecewise-defined functions and step functions. ☐ ALGI.30.2: Graph functions expressed symbolically by hand in simple cases. ☐ ALGI.30.3: Graph functions expressed symbolically using technology for more complicated case. ☐ ALGI.30.4a: Graph quadratic functions showing maxima and minima. ☐ ALGI.30.5a: Graph quadratic functions showing intercepts. ☐ ALGI.30.6a: Graph linear functions showing intercepts. ☐ ALGI.30.7b: Define square root, cube root, and absolute value function. ☐ ALGI.30.8b: Graph piecewise-defined functions. ☐ ALGI.30.9b: Graph step functions. ☐ ALGI.30.10b: Graph cube root functions. ☐ ALGI.30.11b: Graph square root functions.	☐ Demonstrate how to plot points on a coordinate plane using ordered pairs from a table. ☐ Graph a function given the slope-intercept form of an equation. ☐ Recognize the absolute value of a rational number is its' distance from 0 on a vertical and horizontal number line. ☐ Define absolute value and rational numbers. ☐ Recall how to plot ordered pairs on a coordinate plane. ☐ Name the pairs of integers and/or rational numbers of a point on a coordinate plane.

	☐ ALGI.30.12b: Graph absolute value functions. ☐ ALGI.30.13c Identify exponential numbers as repeated multiplication. ☐ ALGI.30.14c Rewrite exponential numbers as repeated multiplication.	
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Algebra and Functions

Focus 3: Functions

Cluster	2019 Math COS Standard	
Functions model a wide variety of real situations and can help students understand the processes of making and changing assumptions, assigning variables, and finding solutions to contextual problems.	31. Use the mathematical modeling cycle to solve real-world problems involving linear, quadratic, exponential, absolute value, and linear piecewise functions.	
	Learning Objectives	Prior Knowledge Skills
	<i>Note: One does not need to move through the modeling cycle in the same order, aspects of the cycle may be repeated.</i> The Mathematical Modeling Cycle: ☐ Define the problem. ☐ Make assumptions/Define variables. ☐ Do the math/Get solutions. ☐ Assess the model and solutions. ☐ Iterate to refine and extend model. ☐ Implement and report results.	<i>Note: One does not need to move through the modeling cycle in the same order, aspects of the cycle may be repeated.</i> The Mathematical Modeling Cycle: ☐ Define the problem. ☐ Make assumptions/Define variables. ☐ Do the math/Get solutions. ☐ Assess the model and solutions. ☐ Iterate to refine and extend model. ☐ Implement and report results.

Data Analysis, Statistics, and Probability

Focus 1: Quantitative Literacy

Cluster	2019 Math COS Standard	
Mathematical and statistical reasoning about data can be used to evaluate conclusions and assess risks.	32. *Use mathematical and statistical reasoning with bivariate categorical data in order to draw conclusions and assess risk. <i>Example: In a clinical trial comparing the effectiveness of flu shots A and B, 21 subjects in treatment group A avoided getting the flu while 29 contracted it. In group B, 12 avoided the flu while 13 contracted it. Discuss which flu shot appears to be more effective in reducing the chances of contracting the flu. Possible answer: Even though more people in group A avoided the flu than in group B, the proportion of people avoiding the flu in group B is greater than the proportion in group A, which suggests that treatment B may be more effective in lowering the risk of getting the flu.</i>	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGI.32.1: Define proportional relationships, unit rate, and slope. ☐ ALGI.32.2: Define probability of chance, outcomes and events. ☐ ALGI.32.3: Define bivariate scatter plot, outlier, cluster, linear, nonlinear, and positive and negative association. ☐ ALGI.32.4: Define relative frequency, bivariate, and frequency. ☐ ALGI.32.5: Calculate frequency as it pertains to the data for a two-way table.	☐ Analyze scatter plots to determine line of best fit. ☐ Define scatter plot, outlier, linear, quantitative, line of best fit, and variable.

Data Analysis, Statistics, and Probability

Focus 1: Quantitative Literacy

Cluster	2019 Math COS Standard	
Making and defending informed, databased decisions is a characteristic of a quantitatively literate person.	33. Design and carry out an investigation to determine whether there appears to be an association between two categorical variables, and write a persuasive argument based on the results of the investigation. <i>Example: Investigate whether there appears to be an association between successfully completing a task in a given length of time and listening to music while attempting the task. Randomly assign some students to listen to music while attempting to complete the task and others to complete the task without listening to music. Discuss whether students should listen to music while studying, based on that analysis.</i>	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGI.33.1: Write arguments to support claims with clear reasons and relevant evidence. ☐ ALGI.33.2: Write a persuasive argument to justify the solution. ☐ ALGI.33.3: Introduce claim(s) and organize the reasons and evidence clearly. ☐ AGLI.33.4: Support claim(s) with clear reasons and relevant evidence, using credible sources and demonstrating an understanding of the topic or text. ☐ ALGI.33.5: Summarize numerical data sets in relation to their context. ☐ ALGI.33.6: Infer information from data distributions.	☐ Define numerical data set, quantitative, measure of center, median, frequency distribution, and attribute. ☐ Analyze a two-way table containing categorical variables. ☐ Design a two-way table. ☐ Define relative frequency and frequency.

Data Analysis, Statistics, and Probability

Focus 2: Visualizing and Summarizing Data

Cluster	2019 Math COS Standard	
Data arise from a context and come in two types: quantitative (continuous or discrete) and categorical. Technology can be used to “clean” and organize data, including very large data sets, into a useful and manageable structure a first step in any analysis of data.	34. Distinguish between quantitative and categorical data and between the techniques that may be used for analyzing data of these two types. <i>Example: The color of cars is categorical and so is summarized by frequency and proportion for each color category, while the mileage on each car’s odometer is quantitative and can be summarized by the mean.</i>	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGI.34.1: Define categorical and quantitative data. ☐ ALGI.34.2: Calculate frequency as it pertains to the data for a two-way table, graph or given context within the problem. ☐ ALGI.34.3: Investigate to determine whether there is an association between two categorical variables.	☐ Define numerical data set, quantitative, measure of center, median, frequency distribution, and attribute. ☐ Analyze a two-way table containing categorical variables. ☐ Design a two-way table. ☐ Define relative frequency and frequency.

Data Analysis, Statistics, and Probability

Focus 2: Visualizing and Summarizing Data

Cluster	2019 Math COS Standard	
The association between two categorical variables is typically represented by using two-way tables and segmented bar graphs.	35. *Analyze the possible association between two categorical variables. - Summarize categorical data for two categories in two-way frequency tables and represent using segmented bar graphs. - Interpret relative frequencies in the context of categorical data (including joint, marginal, and conditional relative frequencies). - Identify possible associations and trends in categorical data.	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGI.35.1: Define categorical data, two-way frequency table, relative frequency, joint frequency, marginal frequency, and conditional relative frequency. ☐ ALGI.35.2: Recognize possible associations and trends in the data. ☐ ALGI.35.3: Interpret conditional relative frequencies in the context of the data. ☐ ALGI.35.4: Interpret marginal frequencies in the context of the data. ☐ ALGI.35.5: Analyze data from tables.	☐ Organize the data. ☐ Collect the data. ☐ Recall how to collect data. ☐ Recall how to calculate frequency. ☐ Analyze a two-way table containing categorical variables.

Data Analysis, Statistics, and Probability

Focus 2: Visualizing and Summarizing Data

Cluster	2019 Math COS Standard	
Data analysis techniques can be used to develop models of contextual situations and to generate and evaluate possible solutions to real problems involving those contexts.	36. *Generate a two-way categorical table in order to find and evaluate solutions to real-world problems. a. Aggregate data from several groups to find an overall association between two categorical variables. b. Recognize and explore situations where the association between two categorical variables is reversed when a third variable is considered (Simpson's Paradox). <i>Example: In a certain city, Hospital 1 has a higher fatality rate than Hospital 2. But when considering mildly-injured patients and severely-injured patients as separate groups, Hospital 1 has a lower fatality rate among both groups than Hospital 2, since Hospital 1 is a Level 1 Trauma Center. Thus, Hospital 1 receives most of the severely injured patients who are less likely to survive overall but have a better chance of surviving in Hospital 1 than they would in Hospital 2.</i>	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGI.36.1: Define categorical and quantitative data. ☐ ALGI.36.2: Calculate frequency as it pertains to the data for a two-way table, graph or given context within the problem. ☐ ALGI.36.3: Put the data into a two-way categorical table and analyze the data for relationships. ☐ ALGI.36.4: Investigate to determine whether there is an association between two categorical variables. ☐ ALGI.36.5: Recognize possible associations and trends in the data. ☐ ALGI.36.6: Summarize categorical data for two categories in two-way frequency tables. ☐ ALGI.36.7: Analyze data from tables.	☐ Identify different types of data. ☐ Organize data in an ordered list. ☐ Compare and contrast data using their measures of central tendency. ☐ Read and interpret tables.

Data Analysis, Statistics, and Probability

Focus 3: Statistical Inference (Note: There are no Algebra I with Probability standards in Focus 3)

Focus 4: Probability

Cluster	2019 Math COS Standard	
Two events are independent if the occurrence of one event does not affect the probability of the other event. Determining whether two events are independent can be used for finding and understanding probabilities.	37. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGI.37.1: Define sample, validity, population, inference, random sampling, statistic, and generalization. ☐ ALGI.37.2: Identify the nature of the attribute, how it was measured, and its unit of measure. ☐ ALGI.37.3: Discuss real world examples of valid sampling and generalizations. ☐ ALGI.37.4: Compare sample size with population to check for validity. ☐ ALGI.37.5: Analyze attributes of sample size. ☐ ALGI.37.6: Differentiate between appropriate sampling methods. ☐ ALGI.37.7: Explain the validity of random sampling. ☐ ALGI.37.8: Given a contextual situation, interpret and defend the solution in the context of the original problem.	☐ Collect data from population randomly, choosing same size samples. (Ex. If population is your school, different random samplings should be same number of students) ☐ Recall the nature of the attribute, how it was measured, and its unit of measure. ☐ Discuss real world examples of valid sampling and generalizations. ☐ Compare and contrast the random sampling data to the population. ☐ Analyze attributes of sample size. ☐ Differentiate the appropriate sampling method. ☐ Explain the validity of random sampling.

Data Analysis, Statistics, and Probability

Focus 3: Statistical Inference (Note: There are no Algebra I with Probability standards in Focus 3)

Focus 4: Probability

Cluster	2019 Math COS Standard	
Two events are independent if the occurrence of one event does not affect the probability of the other event. Determining whether two events are independent can be used for finding and understanding probabilities.	38. *Explain whether two events, A and B, are independent, using two-way tables or tree diagrams.	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGI.38.1: Define probability, ratio, simple event, compound event, and independent event. ☐ ALGI.38.2: Determine the probability of a compound event. ☐ ALGI.38.3: Determine the probability of an independent event. ☐ ALGI.38.4: Determine the probability of a simple event by expressing the probability as a ratio, percent, or decimal. ☐ ALGI.38.5: Identify the probability of an event that is certain as 1 or impossible as 0. ☐ ALGI.38.6: Solve word problems involving probability. ☐ ALGI.38.7: Use proportional relationships to solve multi-step ratio and percent problems. ☐ ALGI.38.8: Recognize and represent proportional relationships as ratios between two quantities.	☐ Demonstrate how to write the probability as a fraction, with likely outcomes as the numerator and possible outcomes as the denominator. ☐ Using the model, count the frequency of the actual outcome. ☐ List all actual outcomes using a graphic representation (probability model-tree diagram, organized list, table, etc.). ☐ Define probability of observed frequency, outcome, discrepancy, and event.

Data Analysis, Statistics, and Probability

Focus 3: Statistical Inference (Note: There are no Algebra I with Probability standards in Focus 3)

Focus 4: Probability

Cluster	2019 Math COS Standard	
Conditional probabilities – that is, those probabilities that are “conditioned” by some known information – can be computed from data organized in contingency tables. Conditions or assumptions may affect the computation of a probability.	39. Compute the conditional probability of event A given event B, using two-way tables or tree diagrams.	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGI.39.1: Define likelihood, probability, and event. ☐ ALGI.39.2: Construct a graphic representation of all outcomes (probability model-tree diagram, organized list, table, etc.). ☐ ALGI.39.3: Compare and contrast probability of chance and probability of observed frequency. ☐ ALGI.39.4: Write the probability as a fraction, with likely outcomes as the numerator and possible outcomes as the denominator.	☐ Calculate the probability of a single event. ☐ Calculate the number of outcomes by listing all possible outcomes.

Data Analysis, Statistics, and Probability

Focus 3: Statistical Inference (Note: There are no Algebra I with Probability standards in Focus 3)

Focus 4: Probability

Cluster	2019 Math COS Standard	
Conditional probabilities – that is, those probabilities that are “conditioned” by some known information – can be computed from data organized in contingency tables. Conditions or assumptions may affect the computation of a probability.	40. *Recognize and describe the concepts of conditional probability and independence in everyday situations and explain them using everyday language. <i>Example: Contrast the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.</i>	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGI.40.1: Define probability using everyday language. ☐ ALGI.40.2: Compare and contrast probability of chance and probability of observed frequency. ☐ ALGI.40.3: Explain the difference between possible outcomes and likely outcomes. ☐ ALGI.40.4: Cite evidence to support analysis of what the data says explicitly as well as inferences drawn from the data.	☐ Demonstrate how to write the probability as a fraction, with likely outcomes as the numerator and possible outcomes as the denominator. ☐ Display all outcomes in a graphic representation (probability model-tree diagram, organized list, table, etc.). ☐ Compare and contrast probability of chance and probability of observed frequency. ☐ Define probability of chance, probability of events, outcome, and probability of observed frequency.

Data Analysis, Statistics, and Probability

Focus 3: Statistical Inference (Note: There are no Algebra I with Probability standards in Focus 3)

Focus 4: Probability

Cluster	2019 Math COS Standard	
Conditional probabilities – that is, those probabilities that are “conditioned” by some known information – can be computed from data organized in contingency tables. Conditions or assumptions may affect the computation of a probability.	41. Explain why the conditional probability of A given B is the fraction of B's outcomes that also belong to A and interpret the answer in context. <i>Example: the probability of drawing a king from a deck of cards, given that it is a face card, is $\frac{4/52}{12/52}$ which is $\frac{1}{3}$.</i>	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGI.41.1: Define likelihood, probability, and event. ☐ ALGI.41.2: Construct a graphic representation of all outcomes (probability model-tree diagram, organized list, table, etc.). ☐ ALGI.41.3: Compare and contrast probability of chance and probability of observed frequency. ☐ ALGI.41.4: Write the probability as a fraction, with likely outcomes as the numerator and possible outcomes as the denominator. ☐ ALGI.41.5: Explain the difference between possible outcomes and likely outcomes. ☐ ALGI.41.6: Cite evidence to support analysis of what the data says explicitly as well as inferences drawn from the data.	☐ Demonstrate how to write the probability as a fraction, with likely outcomes as the numerator and possible outcomes as the denominator. ☐ Display all outcomes in a graphic representation (probability model-tree diagram, organized list, table, etc.). ☐ Compare and contrast probability of chance and probability of observed frequency. ☐ Define probability of chance, probability of events, outcome, and probability of observed frequency.

Algebra 2 With Statistics



Algebra 2 with Statistics

Number and Quantity

Cluster	2019 Math COS Standard	
Together, irrational numbers and rational numbers complete the real number system, representing all points on the number line, while there exist numbers beyond the real numbers called complex numbers.	1. *Identify numbers written in the form $a + bi$, where a and b are real numbers and $i^2 = -1$ as complex numbers. Add, subtract, and multiply complex numbers using the commutative, associative, and distributive properties.	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGII.1.1: Define real and complex numbers, commutative, associative, and distributive properties. ☐ ALGII.1.2: Apply commutative, associative, and distributive properties to using multiplication with complex numbers. ☐ ALGII.1.3: Apply commutative, associative, and distributive properties to using addition and subtraction with complex numbers. ☐ ALGII.1.4: Use commutative, associative, and distributive properties. ☐ ALGII.1.5: Identify imaginary numbers.	☐ Review laws of integers. ☐ Apply commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. ☐ Review commutative, associative, and distributive properties. ☐ Recall solving one step equations and inequalities. ☐ Calculate a solution or solution set by combining like terms, isolating the variable, and/or using inverse operations.

Number and Quantity		
Cluster	2019 Math COS Standard	
Matrices are a useful way to represent information.	2. Use matrices to represent and manipulate data.	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGII.2.1: Define matrix. ☐ ALGII.2.2: Organize data into a matrix using rows and columns.	☐ Identify a row. ☐ Identify a column. ☐ Add complex numbers. ☐ Subtract complex numbers.

Number and Quantity		
Cluster	2019 Math COS Standard	
Matrices are a useful way to represent information.	3. Multiply matrices by scalars to produce new matrices.	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGII.3.1: Define scalar. ☐ ALGII.3.2: Multiply a matrix by a scalar.	☐ Basic multiplication facts. ☐ Identify a matrix. ☐ Multiply each element by a given scalar.

Number and Quantity		
Cluster	2019 Math COS Standard	
Matrices are a useful way to represent information.	4. *Add, subtract, and multiply matrices of appropriate dimensions.	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGII.4.1: Multiply matrices of appropriate dimensions. ☐ ALGII.4.2: Subtract matrices of appropriate dimensions. ☐ ALGII.4.3: Add matrices of appropriate dimensions.	☐ Recognize rows. ☐ Recognize columns. ☐ Recognize the distributive property.

Number and Quantity		
Cluster	2019 Math COS Standard	
Matrices are a useful way to represent information.	5. Describe the roles that zero and identity matrices play in matrix addition and multiplication, recognizing that they are similar to the roles of 0 and 1 in the real numbers. - Find the additive and multiplicative inverses of square matrices, using technology as appropriate. - Explain the role of the determinant in determining if a square matrix has a multiplicative inverse.	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGII.5.1: Define zero matrix and identity matrix. ☐ ALGII.5.2: Multiply an identity matrix by any other matrix will result in the other matrix. ☐ ALGII.5.3: Know that identity matrices have a diagonal of 1's, starting at the top left-hand corner and going down. All other entries are zeroes. ☐ ALGII.5.4: Recognize that when the zero matrix is added to any other matrix, the sum is the other matrix.	☐ Cross multiply. ☐ Basic subtraction.

Algebra and Functions		
Focus 1: Algebra		
Cluster	2019 Math COS Standard	
Expressions can be rewritten in equivalent forms by using algebraic properties, including properties of addition, multiplication, and exponentiation, to make different characteristics or features visible.	6. *Factor polynomials using common factoring techniques and use the factored form of a polynomial to reveal the zeros of the function it defines.	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGII.6.1: Define factor, monomial, binomial, trinomial, polynomial, quadratic expression and zero product property. ☐ ALGII.6.2: Factor a quadratic expression (Greatest Common Factor, completing the square, difference of two squares, perfect square trinomials, and quadratic formula). ☐ ALGII.6.3: Use the zero-product property to reveal the zeros in the function. ☐ ALGII.6.4: Solve a two-step equation. ☐ ALGII.6.5: Solve a one-step equation. ☐ ALGII 6.6: Determine the Greatest Common Factor (GCF).	☐ Combine like terms of a given expression. ☐ Define monomial, term, binomial, trinomial and polynomial. ☐ Multiply polynomial expressions (quadratic). ☐ Multiply polynomial expressions (linear). ☐ Subtract polynomial expressions. ☐ Add polynomial expressions. ☐ Use order of operations to evaluate and simplify algebraic and numerical expressions. ☐ Identify the terms in a polynomial expression. ☐ Explain the distributive property. ☐ Factor simple trinomials where $a=1$. ☐ Find the zeros of a simple binomial. ☐ Use a graphing calculator to find the zeros of simple polynomials.

Algebra and Functions		
Focus 1: Algebra		
Cluster	2019 Math COS Standard	
Expressions can be rewritten in equivalent forms by using algebraic properties, including properties of addition, multiplication, and exponentiation, to make different characteristics or features visible.	7. Prove polynomial identities and use them to describe numerical relationships.	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGII.7.1: Define polynomial identities. ☐ ALGII.7.2: Identify the polynomial identities used to manipulate numerical relationships.	☐ Define integers. ☐ Demonstrate the location of positive and negative numbers on a vertical and horizontal number line. ☐ Give examples of positive and negative numbers. ☐ Discuss the measure of centering of 0 in relationship to positive and negative numbers.

Algebra and Functions		
Focus 1: Algebra		
Cluster	2019 Math COS Standard	
Finding solutions to an equation, inequality, or system of equations or inequalities requires the checking of candidate solutions, whether generated analytically or graphically, to ensure that solutions are found and that those found are not extraneous. The structure of an equation or inequality	8. Explain why extraneous solutions to an equation may arise and how to check to be sure that a candidate solution satisfies an equation. <i>Extend to radical equations.</i>	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGII.8.1: Define rational, irrational, and radical expressions and extraneous solutions. ☐ ALGII.8.2: Simplify rational and radical equations. ☐ ALGII.8.3: Apply properties of exponents. ☐ ALGII.8.4: Evaluate solutions by substituting into the original equation.	☐ Define rational numbers. ☐ Define irrational numbers. ☐ Define radical numbers. ☐ Identify perfect squares. ☐ Identify symbols for square roots.

(including, but not limited to, one-variable linear and quadratic equations, inequalities, and systems of linear equations in two variables) can be purposefully analyzed (with and without technology) to determine an efficient strategy to find a solution, if one exists, and then to justify the solution.		
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Algebra and Functions		
Focus 1: Algebra		
Cluster	2019 Math COS Standard	
Finding solutions to an equation, inequality, or system of equations or inequalities requires the checking of candidate solutions, whether generated analytically or graphically, to ensure that solutions are found and that those found are not extraneous. The structure of an equation or inequality	9. *For exponential models, express as a logarithm the solution to $ab^{ct} = d$, where a, c, and d are real numbers, and the base b is 2 or 10; evaluate the logarithm using technology to solve an exponential equation.	
	Learning Objectives ☐ ALGII.9.1: Define logarithmic and exponential function. ☐ ALGII.9.2: Recognize the inverse relationship of logarithmic function and exponential functions. ☐ ALGII.9.3: Calculate the change of base formula for logarithms. ☐ ALGII.9.4: Apply the properties of logarithms. ☐ ALGII.9.5: Discuss the appropriateness of the solution. ☐ ALGII.9.6: Recall laws of exponents.	Prior Knowledge Skills ☐ Identify a base number and exponent.

(including, but not limited to, one-variable linear and quadratic equations, inequalities, and systems of linear equations in two variables) can be purposefully analyzed (with and without technology) to determine an efficient strategy to find a solution, if one exists, and then to justify the solution.		
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Algebra and Functions		
Focus 1: Algebra		
Cluster	2019 Math COS Standard	
Expressions, equations, and inequalities can be used to analyze and make predictions, both within mathematics and as mathematics is applied in different contexts—in particular, contexts that arise in relation to linear, quadratic, and exponential situations.	10.Create equations and inequalities in one variable and use them to solve problems. Extend to equations arising from polynomial, trigonometric (sine and cosine), logarithmic, radical, and general piecewise functions.	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGII.10.1: Define equation, expression, variable, equality, and inequality. ☐ ALGII.10.2: Create inequalities with one variable. ☐ ALGII.10.3: Create equations with one variable. ☐ ALGII.10.4: Solve two-step equations and inequalities. ☐ ALGII.10.5: Solve one-step equations and inequalities. ☐ ALGII.10.6: Compare and contrast equations and inequalities. ☐ ALGII.10.7: Recognize inequality symbols including, $\leq$, and $\geq$.	☐ Recognize inequality symbols including greater than, less than, greater than equal to, and less than equal to. ☐ Demonstrate the location of positive and negative numbers on a horizontal number line. ☐ Substitute for the variable to find the value of a given expression.

Algebra and Functions		
Focus 1: Algebra		
Cluster	2019 Math COS Standard	
Expressions, equations, and inequalities can be used to analyze and make predictions, both within mathematics and as mathematics is applied in different contexts—in particular, contexts that arise in relation to linear, quadratic, and exponential situations.	11.*Solve quadratic equations with real coefficients that have complex solutions.	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGII.11.1: Solve quadratic equations with real coefficients that have simple solutions. ☐ ALGII.11.2: Review quadratic formula, completing the square, and factoring. ☐ ALGII.11.3: Review the zero-product property.	☐ Understand that all quadratic equations have two solutions: real or imaginary. ☐ Apply quadratic equations to contextual situations. ☐ Solutions to a quadratic equation must make the original equation true and this should be verified. ☐ When the quadratic equation is derived from a contextual situation, proposed solutions to the quadratic equation should be verified within the context given, as well as mathematically. ☐ Different procedures for solving quadratic equations are necessary under different conditions. If $ab=0$, then at least one of a or b must be zero ($a=0$ or $b=0$) and this is then used to produce the two solutions to the quadratic equation. -Whether the roots of a quadratic equation are real or complex is determined by the coefficients of the quadratic equation in standard form ($ax^2+bx+c=0$).

Algebra and Functions		
Focus 1: Algebra		
Cluster	2019 Math COS Standard	
Expressions, equations, and inequalities can be used to analyze and make predictions, both within mathematics and as mathematics is applied in different contexts—in particular, contexts that arise in relation to linear, quadratic, and exponential situations.	12. Solve simple equations involving exponential, radical, logarithmic, and trigonometric functions using inverse functions.	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGII.12.1: Define function, function notation, linear, polynomial, rational, radical, absolute value, exponential, and logarithmic functions, and transitive property. ☐ ALGII.12.2: Solve an equation of the form $f(x) = c$ for a simple linear function f that has an inverse. ☐ ALGII.12.3: Write an expression for the inverse of a simple linear function f of the form $f(x) = c$. ☐ ALGII.12.4: Apply the properties of multiplicative inverses. ☐ ALGII.12.5: Apply the properties of exponentials. ☐ ALGII.12.6: Apply the substitution principle. ☐ ALGII.12.7: Solve a multi-step equation.	☐ Evaluate a function for an output given the input. ☐ Recall absolute value, radicals, exponents, and linear functions. ☐ Recall how to substitute a value for a variable. ☐ Solve an equation for a missing value.

Algebra and Functions		
Focus 1: Algebra		
Cluster	2019 Math COS Standard	
Expressions, equations, and inequalities can be used to analyze and make predictions, both within mathematics and as mathematics is applied in different contexts—in particular, contexts that arise in relation to linear, quadratic, and exponential situations.	13. *Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales and use them to make predictions. Extend to polynomial, trigonometric (sine and cosine), logarithmic, reciprocal, radical, and general piecewise functions.	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGII.13.1: Define ordered pair, coordinate plane, polynomial, trigonometric (sine and cosine), logarithmic, reciprocal, radical, and general piecewise functions. ☐ ALGII.13.2: Create equations with two variables (polynomial, trigonometric (sine and cosine), logarithmic, reciprocal, radical, and general piecewise functions). ☐ ALGII.13.3: Graph equations on coordinate axes with labels and scales (polynomial, trigonometric (sine and cosine), logarithmic, reciprocal, radical, and general piecewise functions.). ☐ ALGII.13.4: Identify an ordered pair and plot it on the coordinate plane.	☐ Identify X axis. ☐ Identify Y axis. ☐ Graph points on a coordinate plane. ☐ Enter coordinates into a table.

Algebra and Functions		
Focus 2: Connecting Algebra to Functions		
Cluster	2019 Math COS Standard	
Graphs can be used to obtain exact or approximate solutions of equations, inequalities, and systems of equations and inequalities—including systems of linear equations in two variables and systems of linear and quadratic equations (given or obtained by using technology).	14. Explain why the x-coordinates of the points where the graphs of the equations $y=f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$. Find the approximate solutions of an equation graphically, using tables of values, or finding successive approximations, using technology where appropriate. Extend to cases where $f(x)$ and/or $g(x)$ are polynomial, trigonometric (sine and cosine), logarithmic, radical, and general piecewise functions.	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGII.14.1: Define function, function notation, linear, polynomial, trigonometric (sine and cosine), rational, radical, absolute value, exponential, and logarithmic functions, general piecewise functions, and transitive property. ☐ ALGII.14.2: Explain, using the transitive property, why the x-coordinates of the points of the graphs are solutions to the equations. ☐ ALGII.14.3: Find solutions to the equations $y = f(x)$ and $y = g(x)$ using graphing technology. ☐ ALGII.14.4: Solve equations for y. ☐ ALGII.14.5: Apply the properties of multiplicative inverses. ☐ ALGII.14.6: Apply the properties of exponents.	☐ Define domain, range, relation, function, table of values, and mappings. ☐ Determine the appropriate domain for a given function. ☐ Identify functions from information in tables, sets of ordered pairs, and mappings. ☐ Translate verbal phrases into a function. ☐ Graph a function on a coordinate plane. ☐ Arrange data given as ordered pairs into a table and a table of values into ordered pairs.

*Critical Standard

	☐ ALGII.14.7: Demonstrate use of a graphing technology, including using a table, making a graph, and finding successive approximations.	
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Algebra and Functions		
Focus 3: Functions		
Cluster	2019 Math COS Standard	
Functions can be described by using a variety of representations: mapping diagrams, function notation (e.g., $f(x) = x^2$), recursive definitions, tables, and graphs.	15. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). Extend to polynomial, trigonometric (sine and cosine), logarithmic, radical, and general piecewise functions.	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGII.15.1: Compare properties of two functions each represented in a different way. ☐ ALGII.15.2: Identify properties of functions defined algebraically. ☐ ALGII.15.3: Identify properties of functions defined by verbal description. ☐ ALGII.15.4: Identify properties of functions defined graphically. ☐ ALGII.15.5: Identify properties of functions defined numerically in tables.	☐ Compare properties of two functions each represented in a different way. ☐ Identify properties of functions defined algebraically. ☐ Identify properties of functions defined by verbal description. ☐ Identify properties of functions defined graphically. ☐ Identify properties of functions defined numerically in tables.

Algebra and Functions		
Focus 3: Functions		
Cluster	2019 Math COS Standard	
Functions that are members of the same family have distinguishing attributes (structure) common to all functions within that family.	16. *Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k \cdot f(x)$, $f(k \cdot x)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Extend to polynomial, trigonometric (sine and cosine), logarithmic, reciprocal, radical, and general piecewise functions.	
	Learning Objectives ☐ ALGII.16.1: Recognize even and odd functions from algebraic expressions for them. ☐ ALGII.16.2: Recognize even and odd functions from their graphs. ☐ ALGII.16.3: Experiment with various cases of functions and illustrate an explanation of the effects on the graph using technology. ☐ ALGII.16.4: Find the value of k given the graphs of $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k. ☐ ALGII.16.5: Identify the effect on the graph of replacing $f(x)$ by $k f(x)$ and $f(kx)$ for specific values of k. ☐ ALGII.16.6: Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$ and $f(x + k)$ for specific values of k.	Prior Knowledge Skills ☐ Recognize even and odd functions from algebraic expressions for them. ☐ Recognize even and odd functions from their graphs. ☐ Identify the effect on the graph of replacing $f(x)$ by $k f(x)$ and $f(kx)$ for specific values of k.

Algebra and Functions		
Focus 3: Functions		
Cluster	2019 Math COS Standard	
Functions can be represented graphically, and key features of the graphs, including zeros, intercepts, and, when relevant, rate of change and maximum/minimum values, can be associated with and interpreted in terms of the equivalent symbolic representation.	17. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. <i>Note: Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; maximums and minimums; symmetries (including even and odd); end behavior; and periodicity.</i> Extend to polynomial, trigonometric (sine and cosine), logarithmic, reciprocal, radical, and general piecewise functions.	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGII.17.1: Define intercepts, intervals, relative maxima, relative minima, symmetry, end behavior, and periodicity. ☐ ALGII.17.2: For a function that models a relationship between two quantities, find the periodicity. ☐ ALGII.17.3: For a function that models a relationship between two quantities, find the end behavior. ☐ ALGII.17.4: For a function that models a relationship between two quantities, find the symmetry. ☐ ALGII.17.5: For a function that models a relationship between two quantities, find the intervals where the function is increasing, decreasing, positive, or negative.	☐ Compare properties of two functions each represented in a different way. ☐ Identify properties of functions defined algebraically. ☐ Identify properties of functions defined by verbal description. ☐ Identify properties of functions defined graphically. ☐ Identify properties of functions defined numerically in tables. ☐ Define standard function types as quadratic and linear.

	☐ ALGII.17.6: For a function that models a relationship between two quantities, find the relative maxima and minima. ☐ ALGII.17.7: For a function that models a relationship between two quantities, find the x and y intercepts.	
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Algebra and Functions		
Focus 3: Functions		
Cluster	2019 Math COS Standard	
Functions can be represented graphically, and key features of the graphs, including zeros, intercepts, and, when relevant, rate of change and maximum/minimum values, can be associated with and interpreted in terms of the equivalent symbolic representation.	18. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. Extend to polynomial, trigonometric (sine and cosine), logarithmic, reciprocal, radical, and general piecewise functions.	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGII.18.1: Define domain, range, relation, function, table of values, and mappings. ☐ ALGII.18.2: Determine the appropriate domain for a given function. ☐ ALGII.18.3: Identify functions from information in tables, sets of ordered pairs, and mappings. ☐ ALGII.18.4: Translate verbal phrases into a function. ☐ ALGII.18.5: Arrange data given as ordered pairs into a table and a table of values into ordered pairs. ☐ ALGII.18.6: Identify the x and y values in an ordered pair.	☐ Define domain, range, relation, function, table of values, and mappings. ☐ Determine the appropriate domain for a given function. ☐ Identify functions from information in tables, sets of ordered pairs, and mappings. ☐ Translate verbal phrases into a function. ☐ Graph a function on a coordinate plane. ☐ Arrange data given as ordered pairs into a table and a table of values into ordered pairs.

Algebra and Functions		
Focus 3: Functions		
Cluster	2019 Math COS Standard	
Functions can be represented graphically, and key features of the graphs, including zeros, intercepts, and, when relevant, rate of change and maximum/minimum values, can be associated with and interpreted in terms of the equivalent symbolic representation.	19. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. Extend to polynomial, trigonometric (sine and cosine), logarithmic, reciprocal, radical, and general piecewise functions.	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGII.19.1: Define average rate of change as slope. ☐ ALGII.19.2: Estimate the rate of change from a graph (rise/run). ☐ ALGII.19.3: Interpret the average rate of change. ☐ ALGII.19.4: Calculate the average rate of change. ☐ ALGII.19.5: Compute the slope of a line given two ordered pairs. ☐ ALGII.19.6: Identify the slope, given slope-intercept form.	☐ Define rate of change. ☐ Read data points on a table ☐ Understand slope is a divided by the change in the y values over the change in x values.

Algebra and Functions		
Focus 3: Functions		
Cluster	2019 Math COS Standard	
Functions can be represented graphically, and key features of the graphs, including zeros, intercepts, and, when relevant, rate of change and maximum/minimum values, can be associated with and interpreted in terms of the equivalent symbolic representation.	20. *Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Extend to polynomial, trigonometric (sine and cosine), logarithmic, reciprocal, radical, and general piecewise functions. - Graph polynomial functions expressed symbolically, identifying zeros when suitable factorizations are available, and showing end behavior. - Graph sine and cosine functions expressed symbolically, showing period, midline, and amplitude. - Graph logarithmic functions expressed symbolically, showing intercepts and end behavior. - Graph reciprocal functions expressed symbolically, identifying horizontal and vertical asymptotes. - Graph square root and cube root functions expressed symbolically. - Compare the graphs of inverse functions and the relationships between their key features, including but not limited to quadratic, square root, exponential, and logarithmic functions.	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGII.20.1: Graph functions expressed symbolically by hand in simple cases. ☐ ALGII.20.2: Graph functions expressed symbolically using technology for more complicated cases.	☐ Recall how to graph parent functions. ☐ Use a graphing calculator to graph a linear equation. ☐ Calculate the square and cube root of a number. ☐ Identify the intercepts of a graphed function.

	☐ ALGII.20.3: Solve polynomial function for their zeros. ☐ ALGII.20.4: Plot the zeros on a coordinate plane. ☐ ALGII.20.5: Recognize end behavior on a graph. ☐ ALGII.20.6: Review multiplicity of zeros. ☐ ALGII.20.7: Graph trigonometric functions showing period, midline, and amplitude. ☐ ALGII.20.8: Graph logarithmic functions showing intercepts and end behavior. ☐ ALGII.20.9: Graph reciprocal functions, identifying horizontal and vertical asymptotes. ☐ ALGII.20.10: Define square root and cube root. ☐ ALGII.20.11: Graph cube root functions. ☐ ALGII.20.12: Graph square root functions. ☐ ALGII.20.13: Define exponential function, logarithmic function, trigonometric function, intercepts, end behavior, period, midline, and amplitude. ☐ ALGII.20.14: Graph exponential functions showing intercepts and end behavior.	
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Algebra and Functions		
Focus 3: Functions		
Cluster	2019 Math COS Standard	
Functions can be represented graphically, and key features of the graphs, including zeros, intercepts, and, when relevant, rate of change and maximum/minimum values, can be associated with and interpreted in terms of the equivalent symbolic representation.	21. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle, building on work with non-right triangle trigonometry.	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGII.21.1: Define unit circle, trigonometric functions, periodic functions, and radians. ☐ ALGII.21.2: Apply special right triangles to trigonometric ratios. ☐ ALGII.21.3: Demonstrate periodicity of trigonometric functions. ☐ ALGII 21.4: Recall Law of Sines and Law of Cosines. ☐ ALGII.21.4: Recall Pythagorean Theorem.	☐ Recall how to find the missing side lengths of a right triangle using Pythagorean Theorem. ☐ Recall the basic trig ratios such as sine, cosine, and tangent. ☐ Identify the ratios of 30-60-90 and 45-45-90 triangles.

Algebra and Functions		
Focus 3: Functions		
Cluster	2019 Math COS Standard	
Functions model a wide variety of real situations and can help students understand the processes of making and changing assumptions, assigning variables, and finding solutions to contextual problems.	22. Use the mathematical modeling cycle to solve real-world problems involving polynomial, trigonometric (sine and cosine), logarithmic, radical, and general piecewise functions, from the simplification of the problem through the solving of the simplified problem, the interpretation of its solution, and the checking of the solution's feasibility.	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGII.22.1: Define the real-world problem. (i.e., what is the problem asking). ☐ ALGII.22.2: Make assumptions and define the variables (independent, dependent). ☐ ALGII.22.3: Assess the model and identify which function will be used (i.e., polynomial, trigonometric (sine and cosine), logarithmic, radical, and general piecewise functions). ☐ ALGII.22.4: Find the solution. ☐ ALGII.22.5: Interpret the results.	Note: One does not need to move through the modeling cycle in the same order, aspects of the cycle may be repeated. The Mathematical Modeling Cycle: ☐ Define the problem. ☐ Make assumptions/Define variables. ☐ Do the math/Get solutions. ☐ Assess the model and solutions. ☐ Iterate to refine and extend model. ☐ Implement and report results.

Data Analysis. Statistics, and Probability		
Focus 1: Quantitative Literacy		
Cluster	2019 Math COS Standard	
Mathematical and statistical reasoning about data can be used to evaluate conclusions and assess risks.	23. *Use mathematical and statistical reasoning about normal distributions to draw conclusions and assess risk; limit to informal arguments. <i>Example: If candidate A is leading candidate B by 2% in a poll which has a margin of error of less than 3%, should we be surprised if candidate B wins the election?</i>	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGII.23.1: Define margin of error and confidence interval. ☐ ALGII 23.2: Justify the mathematical and statistical reasoning.	☐ List the properties involved in solving a multi-step equation using deductive reasoning. ☐ Solve a multi-step equation using the properties, if the original equation has a solution. ☐ Define equation, inequality, and variable. ☐ Set up equations and inequalities to represent the given situation, using correct mathematical operations and variables. ☐ Calculate a solution or solution set by combining like terms, isolating the variable, and/or using inverse operations. ☐ Test the found number or number set for accuracy by substitution. ☐ Recall solving one step equations and inequalities.

Data Analysis. Statistics, and Probability		
Focus 1: Quantitative Literacy		
Cluster	2019 Math COS Standard	
Making and defending informed data-based decisions is a characteristic of a quantitatively literate person.	24. Design and carry out an experiment or survey to answer a question of interest, and write an informal persuasive argument based on the results. <i>Example: Use the statistical problem-solving cycle to answer the question, “Is there an association between playing a musical instrument and doing well in mathematics?”</i>	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGII.24.1: Determine your question of interest. ☐ ALGII.24.2: Design your study (experiment, survey, etc.). ☐ ALGII.24.3: Collect data. ☐ ALGII.24.4: Analyze results. ☐ ALGII 24.5: Interpret results. ☐ ALGII 24.6: Develop an informal persuasive argument. <i>OBJECTIVES FOLLOW THE STEPS OF THE STATISTICAL PROBLEM-SOLVING CYCLE.</i>	☐ Define categorical data. ☐ Write arguments to support claims with clear reasons and relevant evidence. ☐ Infer information from data distributions.

Data Analysis. Statistics, and Probability		
Focus 2: Visualizing and Summarizing Data		
Cluster	2019 Math COS Standard	
Distributions of quantitative data (continuous or discrete) in one variable should be described in the context of the data with respect to what is typical (the shape, with appropriate measures of center and variability, including standard deviation) and what is not (outliers), and these characteristics can be used to compare two or	25. *From a normal distribution, use technology to find the mean and standard deviation and estimate population percentages by applying the empirical rule. - Use technology to determine if a given set of data is normal by applying the empirical rule. - Estimate areas under a normal curve to solve problems in context, using calculators, spreadsheets, and tables as appropriate.	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGII.25.1: Define normal distribution, mean, standard deviation, and empirical rule. ☐ ALGII.25.2: Use technology to calculate mean and standard deviation. ☐ ALGII.25.3: Use technology (ex. calculator, Microsoft Excel, etc.) to estimate areas under the normal curve. ☐ ALGII 25.4: Analyze data sets to determine if appropriate.	☐ Calculate the mean. ☐ Define standard deviation. ☐ Know the empirical rule.

*Critical Standard

more subgroups with respect to a variable.		
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Data Analysis. Statistics, and Probability		
Focus 3: Statistical Inference		
Cluster	2019 Math COS Standard	
Study designs are of three main types: sample survey, experiment, and observational study.	26. Describe the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. <i>Examples: random assignment in experiments, random selection in surveys and observational studies.</i>	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGII.26.1: Define sample surveys, experiments, randomization, and observational studies.	☐ Define sample surveys. ☐ Define experiment. ☐ Define observational studies. ☐ Define random assignment.

Data Analysis. Statistics, and Probability		
Focus 3: Statistical Inference		
Cluster	2019 Math COS Standard	
The role of randomization is different in randomly selecting samples and in randomly assigning subjects to experimental treatment groups.	27. Distinguish between a statistic and a parameter and use statistical processes to make inferences about population parameters based on statistics from random samples from that population.	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGII.27.1: Define statistic, parameter, statistical process, and random sample.	☐ Define statistic, parameter, statistical process, and random sample.

Data Analysis. Statistics, and Probability		
Focus 3: Statistical Inference		
Cluster	2019 Math COS Standard	
The role of randomization is different in randomly selecting samples and in randomly assigning subjects to experimental treatment groups.	28. Describe differences between randomly selecting samples and randomly assigning subjects to experimental treatment groups in terms of inferences drawn regarding a population versus regarding cause and effect. <i>Example: Data from a group of plants randomly selected from a field allows inference regarding the rest of the plants in the field, while randomly assigning each plant to one of two treatments allows inference regarding differences in the effects of the two treatments. If the plants were both randomly selected and randomly assigned, we can infer that the difference in effects of the two treatments would also be observed when applied to the rest of the plants in the field.</i>	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGII.28.1: Define random selecting, random assigning, experimental treatment group, and control group. ☐ ALGII.28.2: Use data from a random sample to make an inference about a population. ☐ ALGII.28.3: Distinguish between random selecting and random assigning and between control group and experimental treatment group.	☐ Define randomization.

Data Analysis. Statistics, and Probability		
Focus 3: Statistical Inference		
Cluster	2019 Math COS Standard	
The scope and validity of statistical inferences are dependent on the role of randomization in the study design.	29. Explain the consequences, due to uncontrolled variables, of non-randomized assignment of subjects to groups in experiments. <i>Example: Students are studying whether or not listening to music while completing mathematics homework improves their quiz scores. Rather than assigning students to either listen to music or not at random, they simply observe what the students do on their own and find that the music-listening group has a higher mean quiz score. Can they conclude that listening to music while studying is likely to raise the quiz scores of students who do not already listen to music? What other factors may have been responsible for the observed difference in mean quiz scores?</i>	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGII.29.1: Define non-randomized assignment and categorical outcomes. ☐ ALGII.29.2: Analyze the data and explain the outcome.	☐ Define uncontrolled variables.

Data Analysis. Statistics, and Probability		
Focus 3: Statistical Inference		
Cluster	2019 Math COS Standard	
Bias, such as sampling, response, or nonresponse bias, may occur in surveys, yielding results that are not representative of the population of interest.	30. *Evaluate where bias, including sampling, response, or nonresponse bias, may occur in surveys, and whether results are representative of the population of interest. <i>Example: Selecting students eating lunch in the cafeteria to participate in a survey may not accurately represent the student body, as students who do not eat in the cafeteria may not be accounted for and may have different opinions, or students may not respond honestly to questions that may be embarrassing, such as how much time they spend on homework.</i>	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGII.30.1: Define bias (sampling, response, or nonresponse bias). ☐ ALGII.30.2: Interpret survey results. ☐ ALGII.30.3: Determine where bias may occur.	☐ Define bias (sampling, response, or nonresponse bias). ☐ Interpret survey results.

Data Analysis. Statistics, and Probability		
Focus 3: Statistical Inference		
Cluster	2019 Math COS Standard	
The larger the sample size, the less the expected variability in the sampling distribution of a sample statistic.	31. Evaluate the effect of sample size on the expected variability in the sampling distribution of a sample statistic. - Simulate a sampling distribution of sample means from a population with a known distribution, observing the effect of the sample size on the variability. - Demonstrate that the standard deviation of each simulated sampling distribution is the known standard deviation of the population divided by the square root of the sample size.	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGII.31.1: Define data, random sampling, population, variation, prediction, estimation, standard deviation and inference. ☐ ALGII 31.2: Calculate standard deviation of the samples. ☐ ALGII.31.3: Compare and contrast the random sampling data to the population. ☐ ALGII.31.4: Predict an outcome of the entire population based on random samplings. ☐ ALGII.31.5: Collect data from population randomly, choosing same size samples.	☐ Define bias (sampling, response, or nonresponse bias). ☐ Interpret survey results. ☐ Determine where bias may occur.

Data Analysis. Statistics, and Probability		
Focus 3: Statistical Inference		
Cluster	2019 Math COS Standard	
The sampling distribution of a sample statistic formed from repeated samples for a given sample size drawn from a population can be used to identify typical behavior for that statistic. Examining several such sampling distributions leads to estimating a set of plausible values for the population parameter, using the margin of	32. *Produce a sampling distribution by repeatedly selecting samples of the same size from a given population or from a population simulated by bootstrapping (resampling with replacement from an observed sample). Do initial examples by hand, then use technology to generate a large number of samples. a. Verify that a sampling distribution is centered at the population mean and approximately normal if the sample size is large enough. b. Verify that 95% of sample means are within two standard deviations of the sampling distribution from the population mean. c. Create and interpret a 95% confidence interval based on an observed mean from a sampling distribution.	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGII.32.1: Define data, random sampling, population, variation, prediction, estimation, normal, empirical rule, standard deviation and inference. ☐ ALGII 32.2: Calculate standard deviation of the samples by hand and using technology to justify the empirical rule. ☐ ALGII.32.3: Predict an outcome of the entire population based on random samplings. ☐ ALGII.32.4: Collect data from population randomly, choosing same size samples.	☐ Collect data from population randomly, choosing same size samples.

*Critical Standard

error as a measure that describes the sampling variability.		
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Data Analysis. Statistics, and Probability		
Focus 3: Statistical Inference		
Cluster	2019 Math COS Standard	
The sampling distribution of a sample statistic formed from repeated samples for a given sample size drawn from a population can be used to identify typical behavior for that statistic. Examining several such sampling distributions leads to estimating a set of plausible values for the population parameter, using	33. *Use data from a randomized experiment to compare two treatments; limit to informal use of simulations to decide if an observed difference in the responses of the two treatment groups is unlikely to have occurred due to randomization alone, thus implying that the difference between the treatment groups is meaningful. <i>Example: Fifteen students are randomly assigned to a treatment group that listens to music while completing mathematics homework and another 15 are assigned to a control group that does not, and their means on the next quiz are found to be different. To test whether the differences seem significant, all the scores from the two groups are placed on index cards and repeatedly shuffled into two new groups of 15 each, each time recording the difference in the means of the two groups. The differences in means of the treatment and control groups are then compared to the differences in means of the mixed groups to see how likely it is to occur.</i>	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGII.33.1: Define randomized experiment, simulation, and parameter. ☐ ALGII.33.2: Determine if differences in two parameters are significant.	☐ Define randomized experiment, simulation, and parameter.

*Critical Standard

the margin of error as a measure that describes the sampling variability.		
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Geometry and Measurement		
Focus 1: Measurement		
Cluster	2019 Math COS Standard	
When an object is the image of a known object under a similarity transformation, a length, area, or volume on the image can be computed by using proportional relationships.	34. Define the radian measure of an angle as the constant of proportionality of the length of an arc it intercepts to the radius of the circle; in particular, it is the length of the arc intercepted on the unit circle.	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGII.34.1: Define arc length, radian measure, and sector. ☐ ALGII.34.2: Prove the length of the arc intercepted by an angle is proportional to the radius by similarity. ☐ ALGII.34.3: Discuss the relationship between arc length and angles. ☐ ALGII.34.4: Apply the arc length formula.	☐ Define arc length, radian measure, and sector.

Geometry and Measurement		
Focus 4: Solving Applied Problems and Modeling in Geometry		
Cluster	2019 Math COS Standard	
Recognizing congruence, similarity, symmetry, measurement opportunities, and other geometric ideas, including right triangle trigonometry in real-world contexts, provides a means of building understanding of these concepts and is a powerful tool for solving problems related to the physical world in which we live.	35. Choose trigonometric functions (sine and cosine) to model periodic phenomena with specified amplitude, frequency, and midline.	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGII.35.1: Define amplitude, frequency, period, vertical and horizontal translation, and midline. ☐ ALGII.35.2: Calculate amplitude, frequency, period, vertical and horizontal translations, and midline from given data. ☐ ALGII.35.3: Graph the trigonometric function (sine and cosine) that model periodic phenomena. ☐ ALGII.35.4: Graph the sine and cosine parent functions.	☐ Recall a vertical and horizontal line. ☐ Identify the sine and cosine of a triangle.

Geometry and Measurement		
Focus 4: Solving Applied Problems and Modeling in Geometry		
Cluster	2019 Math COS Standard	
Recognizing congruence, similarity, symmetry, measurement opportunities, and other geometric ideas, including right triangle trigonometry in real-world contexts, provides a means of building understanding of these concepts and is a powerful tool for solving problems related	36. Prove the Pythagorean identity $\sin^2(8) + \cos^2(8) = 1$ and use it to calculate trigonometric ratios.	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGII.36.1: Define Pythagorean identity $\sin^2(8) + \cos^2(8) = 1$. ☐ ALGII.36.2: Identify the sine and cosine of special angles. ☐ ALGII.36.3: Identify trigonometric ratios (sine, cosine, and tangent). ☐ ALGII.36.4: Square fractions.	☐ Calculate the exponent of a fraction. ☐ Recall the basic trig ratios (sine, cosine, and tangent).

*Critical Standard

to the physical world in which we live.		
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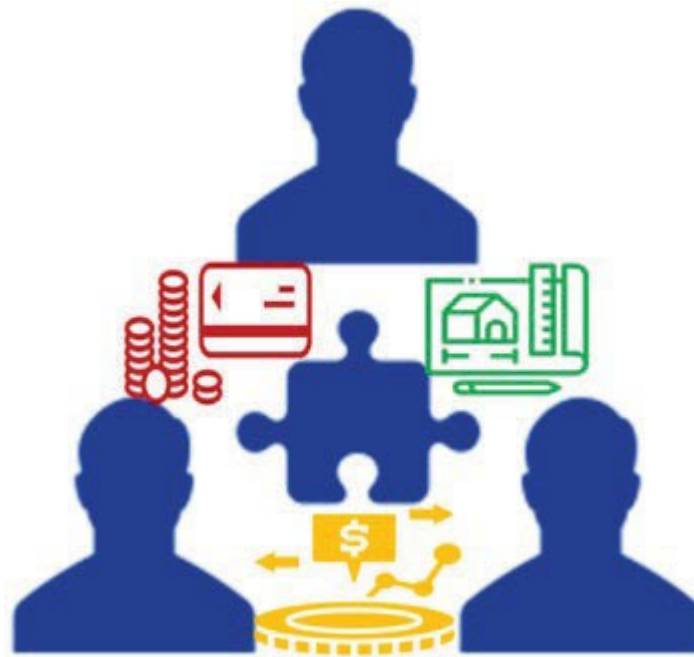
Geometry and Measurement		
Focus 4: Solving Applied Problems and Modeling in Geometry		
Cluster	2019 Math COS Standard	
Recognizing congruence, similarity, symmetry, measurement opportunities, and other geometric ideas, including right triangle trigonometry in real-world contexts, provides a means of building understanding of these concepts and is a powerful tool for solving problems related to the physical world in which we live.	37. Derive and apply the formula $A = \frac{1}{2} \cdot ab \cdot \sin(\theta)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side, extending the domain of sine to include right and obtuse angles.	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGII.37.1: Define the formula $A = \frac{1}{2} \cdot ab \cdot \sin(\theta)$ for the area of a triangle. ☐ ALGII.37.2: Derive the formula $A = \frac{1}{2} \cdot ab \cdot \sin(\theta)$ for the area of a triangle when given base and height. ☐ ALGII.37.3: Apply the formula $A = \frac{1}{2} \cdot ab \cdot \sin(\theta)$ for the area of a triangle.	☐ Recall how to find the area of a triangle. ☐ Calculate the missing value in an equation.

Geometry and Measurement		
Focus 4: Solving Applied Problems and Modeling in Geometry		
Cluster	2019 Math COS Standard	
Recognizing congruence, similarity, symmetry, measurement opportunities, and other geometric ideas, including right triangle trigonometry in real-world contexts, provides a means of building understanding of these concepts and is a powerful tool for solving problems related	38. *Derive and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles. Extend the domain of sine and cosine to include right and obtuse angles. Examples: surveying problems, resultant forces.	
	Learning Objectives	Prior Knowledge Skills
	☐ ALGII.38.1: Define the Law of Sines and the Law of Cosines. ☐ ALGII.38.2: Solve real world problems using the Law of Sines and the Law of Cosines. ☐ ALGII.38.3: Apply the Law of Sines and the Law of Cosines ☐ ALGII.38.4: Create an equation using the given information.	☐ Identify the basic trig functions (sine, cosine, and tangent). ☐ Solve an equation for a missing value.

*Critical Standard

to the physical world in which we live.		
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Mathematical Modeling



Mathematical Modeling

Modeling

Cluster	2019 Math COS Standard	
Mathematical modeling and statistical problem-solving are extensive, cyclical processes that can be used to answer significant real-world problems.	1. Use the full Mathematical Modeling Cycle or Statistical Problem-Solving Cycle to answer a real-world problem of student interest, incorporating standards from across the course. <i>Examples: Use a mathematical model to design a three-dimensional structure and determine whether design constraints are met; to decide under what conditions the purchase of an electric vehicle will save money; to predict the extent to which the level of the ocean will rise due to the melting polar ice caps; or to interpret the claims of a statistical study regarding the economy.</i>	
	Learning Objectives	Prior Knowledge Skills
	☐ MMOD.1.1: Define the mathematical model and the statistical problem-solving cycle. ☐ MMOD.1.2: Use the mathematical model or the statistical problem-solving cycle to solve a real-world problem. ☐ MMOD.1.3: Determine which model to use, mathematical modeling or statistical problem solving, in a real-world problem.	☐ Recall estimation strategies. ☐ Analyze the given word problem to set up a mathematical problem. ☐ Recall problem solving methods.

Financial Planning and Management

Cluster	2019 Math COS Standard	
Mathematical models involving growth and decay are useful in solving real-world problems involving borrowing and investing; spreadsheets are a frequently used and powerful tool to assist with modeling financial situations.	2. Use elements of the Mathematical Modeling Cycle to solve real-world problems involving finances.	
	Learning Objectives	Prior Knowledge Skills
	☐ MMOD.2.1: Analyze a personal budget. ☐ MMOD.2.2: Design a monthly budget, including investments, savings, borrowing and credit. ☐ MMOD.2.3: Differentiate the various modes of payment options (cash, check, money order, debit cards, credit cards). ☐ MMOD.2.4: Determine and prioritize personal needs and wants according to current or expected income (housing, food, clothing, transportation, wellness needs, healthcare, utilities, insurance, benefits).	<i>Life Skills</i> <i>Experience with checking and savings accounts.</i> <i>Real-world examples of credit cards</i> <i>Determine personal needs and contrast with wants.</i> ☐ Analyze data from tables. ☐ Summarize categorical data for two categories in two-way frequency tables. ☐ Recognize possible associations and trends in the data. ☐ Create a scatter plot and line of best fit using data from a spreadsheet. ☐ Organize numerical data in a spreadsheet. ☐ Create graphical representations from classroom-generated data to model consumer costs. ☐ Create graphical representations from classroom-generated data to predict future outcomes. ☐ Create graphical representations from equations to model consumer costs. ☐ Create graphical representations from equations to predict future outcomes. ☐ Create graphical representations from tables to model consumer costs. ☐ Create graphical representations from tables to predict future outcomes.

Financial Planning and Management

Cluster	2019 Math COS Standard	
Mathematical models involving growth and decay are useful in solving real-world problems involving borrowing and investing; spreadsheets are a frequently used and powerful tool to assist with modeling financial situations.	3. Organize and display financial information using arithmetic sequences to represent simple interest and straight-line depreciation.	
	Learning Objectives	Prior Knowledge Skills
	☐ MMOD.3.1: Define arithmetic sequences, simple interest, and straight-line depreciation. ☐ MMOD.3.2: Analyze the long-term costs of borrowing money. ☐ MMOD.3.3: Calculate straight-line depreciation. ☐ MMOD.3.4: Calculate simple interest. ☐ MMOD.3.5: Identify the formula to compute straight-line depreciation. ☐ MMOD.3.6: Identify the formula to compute simple interest.	☐ Define interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, and percent error. ☐ Apply definitions to context in Real-world problems.

Financial Planning and Management

Cluster	2019 Math COS Standard	
Mathematical models involving growth and decay are useful in solving real-world problems involving borrowing and investing; spreadsheets are a frequently used and powerful tool to assist with modeling financial situations.	4. Organize and display financial information using geometric sequences to represent compound interest and proportional depreciation, including periodic (yearly, monthly, weekly) and continuous compounding. a. Explain the relationship between annual percentage yield (APY) and annual percentage rate (APR) as values for r in the formulas $A=P(1+r)^t$ and $A=Pe^{rt}$.	
	Learning Objectives	Prior Knowledge Skills
	☐ MMOD.4.1: Define geometric sequence, compound interest, proportional depreciation, frequent compounding, continuous compounding, annual percentage yield and annual percentage rate. ☐ MMOD.4.2: Calculate proportional depreciation. ☐ MMOD.4.3: Identify the formula for proportional depreciation. ☐ MMOD.4.4: Calculate compound interest. ☐ MMOD.4.5: Calculate simple interest. ☐ MMOD.4.6: Compare compound and simple interest. ☐ MMOD.4.7: Identify the formula to compute compound interest. ☐ MMOD.4.8: Identify the formula to compute simple interest.	☐ Evaluate a function rule given the independent variable. ☐ Define arithmetic and geometric sequence, and input-output pairs. ☐ Define sequences and recursively defined sequences. ☐ Recognize that sequences are functions whose domain is the set of all positive integers and zero ☐ Calculate the common ratio of a geometric sequence.

Financial Planning and Management

Cluster	2019 Math COS Standard	
Mathematical models involving growth and decay are useful in solving real-world problems involving borrowing and investing; spreadsheets are a frequently used and powerful tool to assist with modeling financial situations.	5. Compare simple and compound interest, and straight-line and proportional depreciation.	
	Learning Objectives	Prior Knowledge Skills
	☐ MMOD.5.1: Define simple and compound interest, and straight-line and proportional depreciation. ☐ MMOD.5.2: Compare simple and compound interests. ☐ MMOD.5.3: Identify the formula to compute compound interest. ☐ MMOD.5.4: Identify the formula to compute simple interest. ☐ MMOD.5.5: Compare straight-line and proportional depreciation.	☐ Define interest, tax, markups and markdowns; gratuities, commissions, fees, percent increase and decrease; and percent error. ☐ Apply definitions to context in Real-world problems.

Financial Planning and Management

Cluster	2019 Math COS Standard	
Mathematical models involving growth and decay are useful in solving real-world problems involving borrowing and investing; spreadsheets are a frequently-used and powerful tool to assist with modeling financial situations.	6. Investigate growth and reduction of credit card debt using spreadsheets, including variables such as beginning balance, payment structures, credits, interest rates, new purchases, finance charges, and fees.	
	Learning Objectives	Prior Knowledge Skills
	☐ MMOD.6.1: Define previous balance, payments, credits, interest rate, finance charge fees, credit score, exponential growth, and exponential decay. ☐ MMOD.6.2: Use exponential growth and exponential decay to model given relationships between quantities. ☐ MMOD.6.3: Calculate cost of credit card interest with benefits. ☐ MMOD.6.4: Discuss cause and effect between use of credit and personal credit score. ☐ MMOD.6.5: Calculate a finance charge at various percentages. ☐ MMOD.6.6: Assess a monthly credit card statement. ☐ MMOD.6.7: Identify benefits associated with credit cards. ☐ MMOD.6.8: Identify the long-term costs of borrowing money.	<i>Life Skills</i> <i>Experience with checking and savings accounts.</i> <i>Real-world examples of credit cards.</i> ☐ Recall the formula of an exponential function. ☐ Recall the slope-intercept form of a linear function. ☐ Define b as growth or decay factor in the context of an exponential problem. ☐ Define k as the initial amount in the context of an exponential problem.

Financial Planning and Management

Cluster	2019 Math COS Standard	
Mathematical models involving growth and decay are useful in solving real-world problems involving borrowing and investing; spreadsheets are a frequently-used and powerful tool to assist with modeling financial situations.	7. Compare and contrast housing finance options including renting, leasing to purchase, purchasing with a mortgage, and purchasing with cash. - Research and evaluate various mortgage products available to consumers. - Compare monthly mortgage payments for different terms, interest rates, and down payments. - Analyze the financial consequence of buying a home (mortgage payments vs. potentially increasing resale value) versus investing the money saved when renting, assuming that renting is the less expensive option.	
	Learning Objectives	Prior Knowledge Skills
	☐ MMOD.7.1: Define mortgage and lease. ☐ MMOD.7.2: Identify various types of mortgages. Examples: 30-year loan, 15-year loan, fixed rate loans, adjustable-rate loans, VA loans, FHA loans. ☐ MMOD.7.3: Investigate housing costs in local area. ☐ MMOD.7.4: Identify housing options.	<i>Life Skills</i> ☐ Identify different types of housing in local community. ☐ Use the internet to find local house prices

Financial Planning and Management

Cluster	2019 Math COS Standard	
Mathematical models involving growth and decay are useful in solving real-world problems involving borrowing and investing; spreadsheets are a frequently used and powerful tool to assist with modeling financial situations.	8. Investigate the advantages and disadvantages of various means of paying for an automobile, including leasing, purchasing by cash, and purchasing by loan.	
	Learning Objectives	Prior Knowledge Skills
	☐ MMOD.8.1: Define depreciation and leasing. ☐ MMOD.8.2: Compare the cost of purchasing a vehicle by cash, leasing, and by loan. ☐ MMOD.8.3: Determine the cost of purchasing a vehicle with cash. ☐ MMOD.8.4: Determine the cost of purchasing a vehicle by leasing. ☐ MMOD.8.5: Determine the cost of purchasing a vehicle by loan.	<i>Life Skills</i> ☐ Use the internet to determine price of vehicles at local dealerships. ☐ Knowledge of payment types through commercials.

Design in Three Dimensions

Cluster	2019 Math COS Standard	
Two- and three-dimensional representations, coordinates systems, geometric transformations, and scale models are useful tools in planning, designing, and constructing solutions to real-world problems.	9. Use the Mathematical Modeling Cycle to solve real-world problems involving the design of three-dimensional objects.	
	Learning Objectives	Prior Knowledge Skills
	☐ MMOD.9.1: Define three-dimensional, scale factor, and transformations. ☐ MMOD.9.2: Define the problem to be answered. ☐ MMOD.9.3: Make assumptions to simplify the situation. ☐ MMOD.9.4: Identify variables in the situation and select those that represent essential features in order to formulate a mathematical model. ☐ MMOD.9.5: Analyze and perform operations to draw conclusions. ☐ MMOD.9.6: Assess the model and solutions in terms of the original situation. ☐ MMOD.9.7: Refine and extend the model as needed. ☐ MMOD.9.8: Report on the conclusions and the reasoning.	☐ Compare and contrast the random sampling data to the population. ☐ Analyze conclusions of the sample to determine its appropriateness for the population. ☐ Predict an outcome of the entire population based on random samplings. ☐ Justify the mathematical and statistical reasoning.

Design in Three Dimensions		
Cluster	2019 Math COS Standard	
Two- and three-dimensional representations, coordinates systems, geometric transformations, and scale models are useful tools in planning, designing, and constructing solutions to real-world problems.	10. Construct a two-dimensional visual representation of a three-dimensional object or structure. - Determine the level of precision and the appropriate tools for taking the measurements in constructing a two-dimensional visual representation of a three-dimensional object or structure. - Create an elevation drawing to represent a given solid structure, using technology where appropriate. - Determine which measurements cannot be taken directly and must be calculated based on other measurements when constructing a two-dimensional visual representation of a three-dimensional object or structure. - Determine an appropriate means to visually represent an object or structure, such as drawings on paper or graphics on computer screens.	
	Learning Objectives	Prior Knowledge Skills
	☐ MMOD.10.1: Define two-dimensional figure, three-dimensional figure, precision, area, elevation drawing, scale factor, model, and perimeter. ☐ MMOD.10.2: Calculate precise measurements. ☐ MMOD.10.3: Describe the relationship between two- and three-dimensional figures. ☐ MMOD.10.4: Identify appropriate tools for taking measurements of various objects.	☐ Define two-dimensional figure, three-dimensional figure, and plane section. ☐ List attributes of three-dimensional figures. ☐ List attributes of two-dimensional figures. ☐ Describe the relationship between two- and three-dimensional figures. ☐ Define scale factor, similarity, and proportions. ☐ Compare two figures in terms of similarity. ☐ Create proportional equations from given information. ☐ Solve proportional equations. ☐ Prove that equivalent ratios are proportions.

Design in Three Dimensions		
Cluster	2019 Math COS Standard	
Two- and three-dimensional representations, coordinates systems, geometric transformations, and scale models are useful tools in planning, designing, and constructing solutions to real-world problems.	11. Plot coordinates on a three-dimensional Cartesian coordinate system and use relationships between coordinates to solve design problems. - Describe the features of a three-dimensional Cartesian coordinate system and use them to graph points. - Graph a point in space as the vertex of a right prism drawn in the appropriate octant with edges along the x, y, and z axes. - Find the distance between two objects in space given the coordinates of each. <i>Examples: Determine whether two aircraft are flying far enough apart to be safe; find how long a zipline cable would need to be to connect two platforms at different heights on two trees.</i> - Find the midpoint between two objects in space given the coordinates of each. <i>Example: If two asteroids in space are traveling toward each other at the same speed, find where they will collide.</i>	
	Learning Objectives	Prior Knowledge Skills
	☐ MMOD.11.1: Define two-dimensional and three-dimensional Cartesian coordinate systems. ☐ MMOD.11.2: Determine how to graph a point in a three-dimensional coordinate system. ☐ MMOD.11.3: Calculate the distance between two objects in space. ☐ MMOD.11.4: Calculate the midpoint between two objects in space. ☐ MMOD.11.5: Compare and contrast a three-dimensional and two-dimensional Cartesian coordinate system. ☐ MMOD.11.6: Determine how to graph a point in a two-dimensional coordinate system. ☐ MMOD.11.7: Calculate the distance between two objects.	☐ Identify ordered pairs. ☐ Recognize ordered pairs. ☐ Define ordered pair and coordinate plane. ☐ Create linear equations with two variables. ☐ Graph linear equations on coordinate axes with labels and scales. ☐ Identify an ordered pair and plot it on the coordinate plane.

	☐ MMOD.11.8: Calculate the midpoint between two objects. ☐ MMOD.11.9: Identify a diagram that shows a two-dimensional and three-dimensional coordinate system.	
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Design in Three Dimensions

Cluster	2019 Math COS Standard	
Two- and three-dimensional representations, coordinates systems, geometric transformations, and scale models are useful tools in planning, designing, and constructing solutions to real-world problems.	12. Use technology and other tools to explore the results of simple transformations using three-dimensional coordinates, including translations in the x, y, and/or z directions; rotations of 90°, 180°, or 270° about the x, y, and z axes; reflections over the xy, yz, and xz planes; and dilations from the origin. <i>Example: Given the coordinates of the corners of a room in a house, find the coordinates of the same room facing a different direction.</i>	
	Learning Objectives	Prior Knowledge Skills
	☐ MMOD.12.1: Define translation, rotation, and dilation. ☐ MMOD.12.2: Use technology or other tools to interpret the results of transformations (translation, rotation, and dilation). ☐ MMOD.12.3: Use technology or other tools to rotate an object 90° or 180°. ☐ MMOD.12.4: Use technology or other tools to dilate an object. ☐ MMOD.12.5: Use technology or other tools to translate an object. ☐ MMOD.12.6: Use technology or other tools to reflect an object.	☐ Define dilation. ☐ Recall how to find scale factor. ☐ Give examples of scale drawings. ☐ Recognize translations. ☐ Recognize reflections. ☐ Recognize rotations.

Design in Three Dimensions

Cluster	2019 Math COS Standard	
Two- and three-dimensional representations, coordinates systems, geometric transformations, and scale models are useful tools in planning, designing, and constructing solutions to real-world problems.	13. Create a scale model of a complex three-dimensional structure based on observed measurements and indirect measurements, using translations, reflections, rotations, and dilations of its components. <i>Example: Develop a plan for a bridge structure using geometric properties of its parts to determine unknown measures and represent the plan in three dimensions.</i>	
	Learning Objectives	Prior Knowledge Skills
	☐ MMOD.13.1: Define observed measurements, indirect measurements, scale models, complex three-dimensional shapes, translations, reflections, rotations, and dilations. ☐ MMOD.13.2: Apply geometric concepts in modeling situations. ☐ MMOD.13.3: Perform all transformations (i.e., translations, reflections, rotations, dilations). ☐ MMOD.13.4: Calculate scale factor.	☐ Define dilation. ☐ Recall how to find scale factor. ☐ Give examples of scale drawings. ☐ Recognize translations. ☐ Recognize reflections. ☐ Recognize rotations.

Creating Functions to Model Change in the Environment and Society

Cluster	2019 Math COS Standard	
Functions can be used to represent general trends in conditions that change over time and to predict future conditions based on present observations.	14. Use elements of the Mathematical Modeling Cycle to make predictions based on measurements that change over time, including motion, growth, decay, and cycling.	
	Learning Objectives	Prior Knowledge Skills
	☐ MMOD.14.1: Define motion, growth, decay, and cycling. ☐ MMOD.14.2: Define the problem to be answered. ☐ MMOD.14.3: Make assumptions to simplify the situation. ☐ MMOD.14.4: Identify variables in the situation, and select those that represent essential features in order to formulate a mathematical model. ☐ MMOD.14.5: Analyze and performing operations to draw conclusions. ☐ MMOD.14.6: Assess the model and solutions in terms of the original situation. ☐ MMOD.14.7: Refine and extend the model as needed. ☐ MMOD.14.8: Report on the conclusions and the reasoning.	☐ Solve the equation represented by the real-world situation. ☐ Set up an equation to represent the given situation, using correct mathematical operations and variables. ☐ Given a contextual situation, interpret and defend the solution in the context of the original problem. ☐ Define equation, expression, variable, equality, and inequality.

Creating Functions to Model Change in the Environment and Society

Cluster	2019 Math COS Standard	
Functions can be used to represent general trends in conditions that change over time and to predict future conditions based on present observations.	15. Use regression with statistical graphing technology to determine an equation that best fits a set of bivariate data, including nonlinear patterns. <i>Examples: global temperatures, stock market values, hours of daylight, animal population, carbon dating measurements, online streaming viewership.</i> - Create a scatter plot with a sufficient number of data points to predict a pattern. - Describe the overall relationship between two quantitative variables (increase, decrease, linearity, concavity, extrema, inflection) or pattern of change. - Make a prediction based upon patterns.	
	Learning Objectives	Prior Knowledge Skills
	☐ MMOD.15.1: Define bivariate scatter plot, outlier, cluster, linear, nonlinear, positive, and negative association, slope, intercept, linear, equation, concave up, concave down, and bivariate. ☐ MMOD.15.2: Make a prediction based upon patterns. ☐ MMOD.15.3: Describe patterns found in a scatter plot. ☐ MMOD.15.4: Demonstrate how to label and plot information on a scatter plot (dot plot). ☐ MMOD.15.5: Distinguish the difference between positive and negative correlation. ☐ MMOD.15.6: When given data points, use technology to find the equation of a line.	☐ Define bivariate scatter plot, quantitative data, outlier, cluster, linear, nonlinear, and positive and negative association. ☐ Describe patterns found in a scatter plot. ☐ Demonstrate how to label and plot information on a scatter plot (dot plot). ☐ Distinguish the difference between positive and negative correlation. ☐ Recall how to describe the spread of the scatter plot (dot plot). ☐ Create a scatter plot and line of best fit using data from a spreadsheet. ☐ Organize and display bivariate quantitative data using a scatter plot and extend from simple cases by hand to more complex cases involving a large data set using technology. ☐ Create a scatter plot of data. ☐ Calculate the fit of the function to the data by examining residuals.

Creating Functions to Model Change in the Environment and Society

Cluster	2019 Math COS Standard	
Functions can be used to represent general trends in conditions that change over time and to predict future conditions based on present observations.	16. Create a linear representation of non-linear data and interpret solutions, using technology and the process of linearization with logarithms.	
	Learning Objectives	Prior Knowledge Skills
	☐ MMOD.16.1: Define linearization, linear, non-linear, exponential function, and logarithmic function. ☐ MMOD.16.2: Interpret solutions based on results. ☐ MMOD.16.3: Using technology create a linear representation of nonlinear data. ☐ MMOD.16.4: Using technology graph a logarithmic function. ☐ MMOD.16.5: Using technology graph an exponential function.	☐ Define logarithmic and exponential function. ☐ Recognize the inverse relationship of logarithmic function and exponential functions. ☐ Define ordered pair, coordinate plane, polynomial, trigonometric (sine and cosine), logarithmic, reciprocal, radical, and general piecewise functions. ☐ Create equations with two variables (polynomial, trigonometric (sine and cosine), logarithmic, reciprocal, radical, and general piecewise functions). ☐ Graph equations on coordinate axes with labels and scales (polynomial, trigonometric (sine and cosine), logarithmic, reciprocal, radical, and general piecewise functions.). ☐ Define linear function and exponential function. ☐ Distinguish between graphs of a line and an exponential function. ☐ Identify the graph of an exponential function.

Modeling to Interpret Statistical Studies

Cluster	2019 Math COS Standard	
Statistical studies allow a conclusion to be drawn about a population that is too large to survey completely or about cause and effect in an experiment.	17. Use the Statistical Problem-Solving Cycle to answer real-world questions.	
	Learning Objectives	Prior Knowledge Skills
	☐ MMOD.17.1: Formulate question. ☐ MMOD.17.2: Design study. ☐ MMOD.17.3: Collect data. ☐ MMOD.17.4: Communicate interpretations and limitations. ☐ MMOD.17.5: Interpret, refine variables and assumptions. ☐ MMOD.17.6: Analyze results.	☐ Solve the equation represented by the real-world situation. ☐ Set up an equation to represent the given situation, using correct mathematical operations and variables. ☐ Given a contextual situation, interpret and defend the solution in the context of the original problem. ☐ Define equation, expression, variable, equality, and inequality.

Modeling to Interpret Statistical Studies

Cluster	2019 Math COS Standard	
Statistical studies allow a conclusion to be drawn about a population that is too large to survey completely or about cause and effect in an experiment.	18. Construct a probability distribution based on empirical observations of a variable. <i>Example: Record the number of student absences in class each day and find the probability that each number of students will be absent on any future day.</i> - Estimate the probability of each value for a random variable based on empirical observations or simulations, using technology. - Represent a probability distribution by a relative frequency histogram and/or a cumulative relative frequency graph. - Find the mean, standard deviation, median, and interquartile range of a probability distribution and make long-term predictions about future possibilities. Determine which measures are most appropriate based upon the shape of the distribution.	
	Learning Objectives	Prior Knowledge Skills
	☐ MMOD.18.1: Define center, mean, median, spread, interquartile range, standard deviation, data set, dot plots, histograms, empirical observations, and box plots. ☐ MMOD.18.2: Make long-term predictions based on the calculations. ☐ MMOD.18.3: Find the mean, standard deviation, median, and interquartile range. Determine which measures are most appropriate based upon the shape of the distribution. ☐ MMOD.18.4: Represent the probability distribution by a relative frequency histogram and/or a cumulative relative frequency graph. ☐ MMOD.18.5: Find the probability of each value for the random variable.	☐ Define normal distribution, mean, standard deviation, and empirical rule. ☐ Use technology to calculate mean and standard deviation. ☐ Use technology (ex. calculator, Microsoft Excel, etc.) to estimate areas under the normal curve. ☐ Analyze data sets to determine if appropriate. ☐ Accurately find the center (median and mean) and spread (interquartile range and standard deviation) of data sets. ☐ Present viable arguments and critique arguments of others from the comparison of the center and spread of multiple data sets. ☐ Reason how standard deviation develops from the mean absolute deviation. ☐ Define probability, ratio, simple event, compound event, and independent event. ☐ Determine the probability of a compound event. ☐ Determine the probability of an independent event. ☐ Determine the probability of a simple event by expressing the probability as a ratio, percent, or decimal.

*Critical Standard

		☐ Identify the probability of an event that is certain as 1 or impossible as 0. ☐ Solve word problems involving probability. ☐ Use proportional relationships to solve multi-step ratio and percent problems. ☐ Recognize and represent proportional relationships as ratios between two quantities.
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Modeling to Interpret Statistical Studies

Cluster	2019 Math COS Standard	
Statistical studies allow a conclusion to be drawn about a population that is too large to survey completely or about cause and effect in an experiment.	19. Construct a sampling distribution for a random event or random sample. <i>Examples: How many times do we expect a fair coin to come up “heads” in 100 flips, and on average how far away from this expected value do we expect to be on a specific set of flips? What do we expect to be the average height for a random sample of students in a local high school given the mean and standard deviation of the heights of all students in the high school?</i> - Use the binomial theorem to construct the sampling distribution for the number of successes in a binary event or the number of positive responses to a yes/no question in a random sample. - Use the normal approximation of a proportion from a random event or sample when conditions are met. - Use the central limit theorem to construct a normal sampling distribution for the sample mean when conditions are met. - Find the long-term probability of a given range of outcomes from a random event or random sample.	
	Learning Objectives	Prior Knowledge Skills
	☐ MMOD.19.1: Define sample, validity, population, inference, random sampling, statistic, binomial theorem, binary event, generalization, normal approximation of a proportion, central limit theorem and normal sampling distribution. ☐ MMOD.19.2: Explain the validity of random sampling. ☐ MMOD.19.3: Differentiate the appropriate sampling method. ☐ MMOD.19.4: Analyze attributes of sample size. ☐ MMOD.19.5: Draw conclusions by finding the long-term probability of a given range of outcomes from a random event or random sample.	☐ Define mean, standard deviation, population, sample, and correlation coefficient. ☐ Define sample, validity, population, inference, random sampling, statistic, and generalization. ☐ Identify the nature of the attribute, how it was measured, and its unit of measure. ☐ Discuss real-world examples of valid sampling and generalizations. ☐ Compare sample size with population to check for validity. ☐ Analyze attributes of sample size. ☐ Differentiate between appropriate sampling methods. ☐ Explain the validity of random sampling.

Modeling to Interpret Statistical Studies

Cluster	2019 Math COS Standard	
Statistical studies allow a conclusion to be drawn about a population that is too large to survey completely or about cause and effect in an experiment.	20. Perform inference procedures based on the results of samples and experiments. - Use a point estimator and margin of error to construct a confidence interval for a proportion or mean. - Interpret a confidence interval in context and use it to make strategic decisions. <i>Example: short-term and long-term budget projections for a business</i> - Perform a significance test for null and alternative hypotheses. - Interpret the significance level of a test in the context of error probabilities and use the results to make strategic decisions. <i>Example: How do you reduce the rate of human error on the floor of a manufacturing plant?</i>	
	Learning Objectives	Prior Knowledge Skills
	☐ MMOD.20.1: Define samples, inference, experiments, point estimator, margin of error, confidence interval, proportion, mean, null and alternative hypotheses, significance test and error of probabilities. ☐ MMOD.20.2: Interpret the significance level of a test in the context of given error probabilities. ☐ MMOD.20.3: Differentiate the appropriate sampling method. ☐ MMOD.20.4: Given a point estimator and margin of error, determine confidence interval. ☐ MMOD.20.5: Given data, perform and interpret a significance test for null alternative hypotheses. ☐ MMOD.20.6: Use the given results to make strategic decisions. ☐ MMOD.20.7: Collect and organize data for analysis.	☐ Identify the attribute used to create the numerical set. ☐ Organize the data. ☐ Collect the data. ☐ Compare and contrast the center and variation. ☐ Define numerical data set, quantitative, measure of center, median, frequency distribution, and attribute. ☐ Define margin of error and confidence interval. ☐ Justify the mathematical and statistical reasoning.

Modeling to Interpret Statistical Studies		
Cluster	2019 Math COS Standard	
Statistical studies allow a conclusion to be drawn about a population that is too large to survey completely or about cause and effect in an experiment.	21. Critique the validity of reported conclusions from statistical studies in terms of bias and random error probabilities.	
	Learning Objectives	Prior Knowledge Skills
	☐ MMOD.21.1: Define validity, conclusions, bias, and random error probabilities. ☐ MMOD.21.2: Critique the validity of reported conclusions. ☐ MMOD.21.3: Describe processes that can be used to make fair decisions.	☐ Define and discuss bias. ☐ Compare and contrast statistical situations to determine if statistical bias exists. ☐ Define bias (sampling, response, or nonresponse bias). ☐ Interpret survey results. ☐ Determine where bias may occur.

Modeling to Interpret Statistical Studies

Cluster	2019 Math COS Standard	
Statistical studies allow a conclusion to be drawn about a population that is too large to survey completely or about cause and effect in an experiment.	22. Conduct a randomized study on a topic of student interest (sample or experiment) and draw conclusions based upon the results. <i>Example: Record the heights of thirty randomly selected students at your high school. Construct a confidence interval to estimate the true average height of students at your high school. Question whether or not this data provides significant evidence that your school's average height is higher than the known national average and discuss error probabilities.</i>	
	Learning Objectives	Prior Knowledge Skills
	☐ MMOD.22.1: Define sample, experiment, randomized study, outliers, and scatterplot. ☐ MMOD.22.2: Predict probabilities based on the effect of outliers on the data. ☐ MMOD.22.3: Evaluate and draw conclusions based on the collected data. ☐ MMOD.22.4: Create a model of a set of data. (i.e., Google form, table, curve, scatterplot)	☐ Identify outliers for the mean and standard deviation. ☐ Compare and contrast the random sampling data to the population. ☐ Analyze conclusions of the sample to determine its appropriateness for the population. ☐ Predict an outcome of the entire population based on random samplings. ☐ Justify mathematical and statistical reasoning.